Approximation for Minimum Multicast Route in Optical Network with Nonsplitting Nodes

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Abstract

Consider the problem of computing the minimum-weight multicast route in an optical network with both nonsplitting and splitting nodes. This problem can be reduced to the minimum Hamiltonian path problem when all nodes are nonsplitting, and the Steiner minimum tree problem when all nodes are splitting. Therefore, the problem is NP-hard. Previously, the best known polynomial-time approximation has the performance ratio 3. In this paper, we present a new polynomial-time approximation with performance ratio of $1 + \rho$, where ρ is the best known approximation performance ratio for the Steiner minimum tree in graph and it has been known that $\rho < 1.55$.

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1 Introduction

A potential infrastructure for a next generation network is to put mobile wireless access networks on top of an all-optical core network. The optical network in core provides the efficient high-speed communication with high bandwidth and low end-to-end delay. It is also desirable that the optical network layer provides multicast capability due to the requirement of many applications. By multicast, we mean that given a network topology, source of the multicast session, multicast members, finds a multicast route that spans all the members. In this paper, we consider the minimum-weight multicast problem, that is, we want to find a multicast route with the minimum total weight.

An optical network is usually formulated as a weighted graph with switches as nodes. We consider two types of switches, nonsplitting and splitting. Corresponding nodes are also said to be *nonsplitting* and *splitting*, respectively. A nonsplitting switch cannot split an input signal into several outputs. Therefore, in a multicast route, a signal may pass a nonsplitting node several times (Fig. 1), but cannot be split. If all nodes are nonsplitting, the minimum-weight problem can be reduced to the the minimum weight Hamiltonian path problem. The latter is well-known to be NP-hard [3] and to have a polynomial-time approximation with performance ratio 1.5 [1].

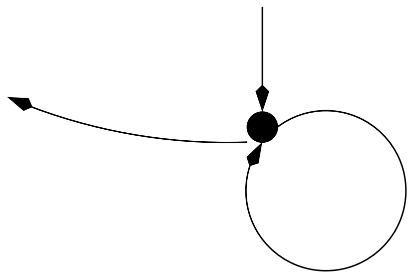

Figure 1: A nonsplitting node

If all nodes are splitting, then the minimum-weight multicast route is the minimum Steiner tree, which is also well-known to be NP-hard [3] and to have a polynomial-time approximation with performance ratio < 1.55 [5].

Clearly, when both nonsplitting and splitting nodes exist, the minimum-weight multicast problem is NP-hard and its polynomial-time approximation should be constructed with techniques from the study of both the Hamiltonian path and the Steiner minimum tree.

Yan, Deogun and Ali [6] gave the first approximation consisting of two steps. In the first step, a Steiner tree T is constructed to interconnect the source node and all multicast members under the assumption that all nodes are splitting. In the second step, a tour starting from the source node along the Steiner tree to reach all multicast members is constructed in the depth-first-search rule. Suppose ρ is the performance ratio of the best known polynomial-time approximation for the Steiner minimum tree. Then the approximation of Yan, Deogun and Ali has the performance ratio 2ρ (≈ 3.1). Du *et al.* [2] gave an improvement by pointing out that when all nodes are considered to be nonsplitting, the 1.5-approximation for the Hamiltonian path actually gives a 3-approximation for the minimum-weight multicast problem.

In this paper, we will present a new polynomial-time approximation with performance ratio $\rho + 1$ (< 2.55).

2 Preliminary

Motivated from Christofides' 1.5-approximation for the Hamiltonian cycle, it is naive to design an approximation for the minimum-weight multicast problem as follows:

Step 1. Construct an edge-weighted complete graph G for the source node, all multicast nodes and all splitting nodes where the weight of each edge is the length of the shortest path between the two endpoints in the input optical network.

Step 2. Construct a Steiner tree T for the source node and all multicast members in G.

Step 3. Construct a perfect matching M for all multicast members with odd degree, if the number of those members is even; or for the source node and all multicast members with odd degree, otherwise.

Step 4. Find a multicast route in the union of T and M.

However, this algorithm may stuck at Step 4 because the union sometimes does not contain a multicast route. A counterexample can be found in [2]. This is why Yan, Deogun and Ali [6] did not use the perfect matching and Du *et al* [2] use a minimum spanning tree instead of a Steiner minimum tree. In this paper, we will introduce a technique to solve this problem.

3 Main Result

Let us describe our new approximation algorithm.

First, construct a weighted complete graph G on the source node, all multicast members, and all splitting nodes where the weight of each edge (u, v) equals the total weight of the shortest path between u and v in the original optical network. Note that this weight function satisfies the triangular inequality in G. Then construct a Steiner tree T for the source node and all multicast nodes in G using a polynomial-time approximation algorithm [5, 4]. Suppose ρ is the performance ratio of this approximation of the Steiner minimum tree. All nodes other than the source node and multicast members are called *Steiner nodes*. They must be splitting.

Consider T as a tree rooted at the source node s. Then we can assign every edge in T a top-down direction coincided with a path from the root s to a leaf. All edges each of which is incident to at least one Steiner node form a forest F. Each connected component E of F is a rooted subtree. Let $p(E)$ be a path from the root to a leaf in E. Let $T \setminus F$ be the subforest of T, with edges in T but not in F. We union $T \setminus F$ together with all $p(E)$ for E over all connected components of F. The resultant subforest of T is denoted by K. Note that in K every Steiner node has even degree. Therefore, the number of multicast members with odd degree in K must be even.

Let O be the set of nodes with odd degree in K. Construct a minimum weight perfect matching M for O. Now, we show that $T \cup M$ contains a multicast route.

Theorem 1 *$T \cup M$ contains a multicast route using each edge at most once.*

Proof. Note that $K \cup M$ is a disjoint union of cycles; each cycle is a connected component of $K \cup M$. One of these cycle, say C, contains the source node s. From s, send a message along an edge of C, in the top-down direction, to an adjacent node. Later, every node will follow from the following rules to transmit the message.

(a) When a Steiner node receives a message, it will pass the message to all its children nodes. This may require to split the message.

(b) When a multicast member a receives a message at the first time and the message comes from an adjacent node in a cycle C of $K \cup M$, a will pass the message to the other adjacent node in C.

(c) When a multicast member a receives a message at the first time and the message does not come from an edge in $K \cup M$, a will pass the message to an adjacent node in $K \cup M$ along an edge in the top-down direction.

(d) When a multicast member receives a message not at the first time, it will do nothing.

This multicast route would use each edge at most once and all multicast nodes would receive the message because T is connected. □

We next estimate the total weight of $T \cup M$. To this end, it suffices to study the weight of M since the total weight of T is within a factor of ρ from the weight of a Steiner minimum tree, hence is upper-bounded by $\rho \cdot opt$ where opt is the minimum-weight of a multicast route.

Lemma 1 *The total weight of M is at most opt.*

Proof. Consider a minimum multicast tree T^* in the given optical network. Starting from the source node, travel along tree T^* in the depth-first search way. Then we would obtain a tour passing through the source node and all multicast members in the given optical network. Turn this tour into a cycle Q in graph G. The total weight of the cycle Q is at most $2opt$. Note that the source node and all multicast members are on the cycle Q. We consider those nodes with odd degree in K. Recall that those nodes form a set O. Along the cycle Q, connect nodes in O directly. We would obtain a cycle Q' on O with total wight at most $2opt$ since the edge-weight in G satisfies the triangular inequality. The cycle Q' can be decomposed into two disjoint perfect matchings for O. One of them must have the total weight at most opt. Therefore, M has the total weight at most opt. □

Theorem 2 *The total weight of* $T \cup M$ *is at most* $(1 + \rho)opt$.

Proof. It follow immediately from Lemma 1. □

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