# Fault Tolerant Topology Control for One-to-All Communications in Symmetric Wireless Networks

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## Abstract

This paper introduces the problem of fault tolerant topology control for one-to-all communications in symmetric wireless networks. We investigate two algorithms to address the problem, namely Minimum Weight Based Algorithm (MWBA) and Nearest Neighbor Augmentation Algorithm (NNAA), and prove that the former is a $4k$ approximation and the latter is a $(k+4)$ approximation. Through extensive simulations, we evaluate the average performance of these two algorithms and find that MWBA is slightly better than NNAA in terms of the total power consumption.

## 1 Introduction

Fault tolerant topology control is an important and challenging problem in wireless networks. First, wireless networks are usually deployed under harsh environments, thus wireless nodes and links could experience frequent failures. Therefore, fault tolerance must be considered for upper layer applications to function. Second, topology control in wireless network has been proved effective in saving node power and reduce the MAC layer contention. The main idea of topology control is that instead of using its maximal transmission power, each node sets its power to a certain level such that the global topology satisfies a certain constraint. To increase fault tolerance, nodes in the network will consume more power, while the goal of topology control is to save power, thus how to add fault tolerance while using as little power as possible is very interesting and challenging.

In wireless networks, the communication models can be categorized into four classes: i) all-to-all, which represents end-to-end communication of every pair of nodes in the network; ii) one-to-one, which represents the communication from a given source node to a given destination node; iii) all-to-one, which indicates the communication from all nodes to a given (root) node; and iv) one-to-all, which indicates the communication from the root to all the other nodes. In literature, [1][2][3] proposed approximations for constructing minimum power k-node disjoint paths between any two nodes, i.e., for all-to-all communication model. [4] gave an optimal solution to construct k-node disjoint paths between the given source and destination, i.e., for one-to-one communication model. [5] proposed two approaches for all-to-one and one-to-all communication model assuming asymmetric wireless links. In this paper, we focus on symmetric links, thus the all-to-one and one-to-all fault tolerant topology control are the same. To be specific, the problem we study in this paper is as follows: Given a network with symmetric links, find a topology that has k-node disjoint paths between one node and all the other nodes (called k-outconnected) and consume as little power as possible. We call it Minimum Power k-Outconnectivity Problem (MPOP). MPOP is NP-hard because when k=1, this problem becomes the minimum power assignment in symmetric networks which has been proved to be NP-hard.

We investigate two algorithms, namely Minimum Weight based Algorithm (MWBA) and Nearest Neighbor Augmentation Algorithm (NNAA). The main idea of MWBA is to utilize the minimum weight k-outconnectivity algorithm, while NNAA is to first construct a topology using small power, then augment it to k-outconnected[1]. We prove that the MWBA is a $4k$-approximation and NNAA is a $(k + 4)$-approximation for minimum power k-outconnectivity problem.

We also compare the average performance of the MWBA and NNAA through extensive simulations. The results show that MWBA is slightly better than NNAA in terms of total power consumption. For example, when node size is 16, MWBA outperforms NNAA 40 times over totally 100 runs, NNAA outperforms MWBA 33 times, and they perform equally 27 times. In most cases, NNAA generates k-outconnected graphs with smaller weight. In both algorithms, the total power over total weight ratio is around 1.2 for various node sizes.

The rest of this paper is organized as follows. Section 2 describes the related work. Section 3 discusses the network model we use and gives the formal problem definition. In section 4, we present the MWBA and NNAA algorithms and analyze their approximation ratios. The simulation results are illustrated in section 5. Section 6 concludes this paper.

## 2 Related Work

Previous work related to fault tolerant topology control in static wireless networks can be categorized into four classes: 1) finding all-to-all k-node connected subgraph consuming minimum total power, 2) finding all-to-all k-node connected subgraph consuming minimum maximal power, i.e., the maximal possible power used by each node is minimized, 3) finding one-to-one (also called source-destination) k-node connected subgraph consuming minimum total power, and 4) finding all-to-one and/or one-to-all k-node connected subgraph consuming minimum total power for asymmetric networks. This work is most similar to the last one, while we focus on the symmetric networks. Below we briefly discuss each of these classes.

**Minimum total power for all-to-all k-fault tolerance in symmetric networks:** [1] analyzed a linear programming approach and proved that its approximation ratio is at least $O(\frac{n}{k})$. In the same paper, the authors also analyzed the k-approximation minimum weight k-connectivity algorithm and proved its approximation ratio of $8k$ for all-to-all k-fault tolerant problem. [2] proposed a $3k$-approximation algorithm that first constructs $(k-1)$th nearest neighbor graph and then augments it to be k-connected using existing minimum weight k-connected subgraph algorithm.

The only localized algorithm is Fault-Tolerant Cone-Based Topology Control (FCBTC) [6] which generalized the well-known Cone-Based Topology Control (CBTC)It's approximation ratio is proved in [1] to be $O(\frac{n}{k})$.

**Minimum maximal power for all-to-all k-fault tolerance in symmetric networks:** [7] [3] fall in the second category, and they gave solutions for minimum maximal power consumption for 2-vertex and k-vertex connectivity, respectively. Both papers proposed greedy algorithms in the sense that at each iteration, the edge with minimum weight is chosen until the subgraph becomes 2 or $k$-connected. [3] also proposed a localized implementation of the centralized algorithm and proved its optimality in aspect to minimize maximal power consumption among all localized algorithms.

**Minimum total power for S-D k-fault tolerance in asymmetric networks:** [4] proposed an algorithm called Source Transmit Power Selection (STPS) based on the observation that each internal vertex on the k-vertex disjoint S-D paths has only one outgoing edge and only the source node has more than k outgoing edges. Thus their

[1] Minimum weight based approach and nearest neighbor augmentation approach have been proposed by [1][2] respectively. In this paper, we show how to apply them to construct k-outconnected subgraph and compare their performance under average case.

Table 1: Comparison of Fault Tolerant Topology Control Algorithms

| Problem | Reference | Scheme | Approximation Ratio |
|---|---|---|---|
| all-to-all | Centralized | Hajiaghayi *et al.* (LP) [1] | $O(\frac{n}{k})$ |
| | | Hajiaghayi *et al.* [1] | 8(k-1) |
| | | Jia *et al.* [2] | 3k |
| | Localized | Bahramgiri *et al.* (FCBTC) [6] | $\geq O(\frac{n}{k})$ |
| one-to-one | Centralized | Srinivas *et al.* [4] | Optimal |
| all-to-one | Centralized | Wang *et al.* [5] | k |
| one-to-all | Centralized | Wang *et al.* [5] | $\Delta^-$ |

algorithm is to try every possible power setting for the source node and then apply minimum weight k-vertex disjoint S-D path algorithm [8] for each setting and pick the one with minimum power consumption.

**Minimum total power for all-to-one and/or one-to-all k-fault tolerance in asymmetric networks and/or one-to-all k-fault tolerance in asymmetric networks:**[5] proposed six algorithms classified into two approaches: minimum weight approach and augmentation based approach. It is the first one to study the fault tolerance for all-to-one communication in asymmetric networks and proposed a $k$-approximation, and also the first one to study the fault tolerance for one-to-all communication, namely broadcast, in asymmetric networks and proposed a $\Delta^-$-approximation, where $\Delta^-$ is the maximum outdegree in the minimum power k-one-to-all connected topology.

Table 2 summarizes these forementioned studies on fault tolerant topology control in wireless networks and their approximation ratios.

## 3 Network Model and Problem Definition

In this paper, we use the following common network model. A wireless network consists of $N$ nodes, each of which is equipped with an omnidirectional antenna with a maximal transmission range of $r_{max}$. The power required for a node to attain a transmission range of $r$ is at least $Cr^{\alpha}$, where $C$ is a constant, $\alpha$ is the *power attenuation exponent* and usually chosen between 2 and 4. For any two nodes $u$ and $v$, there exists a link from $u$ to $v$ if the distance $d(u,v) \leq r_u$, where $r_u$ is the transmission range for node $u$, determined by its power level. If the links are asymmetric, the existence of a link from $u$ to $v$ does not guarantee the existence of a link from $v$ to $u$. In this paper, we consider *symmetric* links and assume the wireless network is static, i.e., the nodes in the network are stationary.

Given the coordination of the nodes in the plane and the transmission power of the nodes, the network can be mapped into a *cost graph* $G = (V, E, c)$, where $V$ denotes the set of wireless nodes, $E$ denotes the set of wireless links induced by the transmission power, and the weight $c$ for a given edge $(u, v)$ is computed as $Cd(u,v)^{\alpha}$, where $d$ is the distance. By this mapping, a symmetric wireless network is represented by an undirected graph.

Wireless Networks have an important feature called *Wireless Multicast Advantage (WMA)* because of its broadcast media. WMA is often utilized to save power. For a node to send data to multiple nodes in its transmission range, instead of sending data multiple times, it only needs to send it once and all nodes in its transmission range can receive the same data. In light of WMA, the power and weight are different in wireless networks.[2] Weight is link based, while power is node based. The power and weight are defined as follows: Given a cost graph $G = (V, E, c)$, let $p(u)$ be the power assignment of node $u$, $w(uv)$ be the cost of an edge $uv$, $c(G)$ be the weight of $G$, and $p(G)$ be the power of $G$, then we have

- $p(u) = max_{uv \in E} w(uv)$,

[2] In this paper, we will use cost and weight, power and energy interchangeably.

- $c(G) = \sum_{e \in E} w(e)$
- $p(G) = \sum_{v \in V} p(v)$.

Traditional problems in graph theory are link based with the goal of minimizing total weight. However, wireless networks call for node-based algorithms to minimize the total power consumption.

In the following, we give formal definition for Minimum Power k-Outconnectivity Problem:

**Definition 1** Minimum Power k-Outconnectivity Problem: *Given the cost graph of a network and a root node $r$, find the power assignment of each node such that there exist k-node disjoint paths between the root node $r$ and every other node in the induced spanning subgraph, and total node power assignment is minimized.*

In this paper, we focus on investigating node connectivity instead of link connectivity because node failure is more frequent than link failure in wireless network, thus node connectivity is more relevant to wireless networks. Our solutions for node connectivity can be applied directly to link connectivity. In the following, we will use *vertex* in the context of graph, and use *node* in the context of network.

## 4 Two Algorithms

In this section, we present two proposed algorithms, Minimum Weight Based Algorithm and Nearest Neighbor Augmentation Algorithm and give theoretical analysis.

### 4.1 Minimum Weight Based Algorithm

The main idea of Minimum Weight Based Algorithm (MWBA) is to construct a $k$-outconnected subgraph with the goal to minimize its weight, then analyze its performance for minimum power k-outconnectivity problem. Given $G = (V, E)$, $k$, and root node $r \in V$, MWBA utilizes an algorithm proposed by Frank *et al.* [9] (let's call it FT) which constructs a minimum weight directed $k$-outconnected subgraph for a directed graph to construct a undirected $k$-outconnected subgraph for undirected graph as follows:

1. Construct $G'$ by replacing each edge in $G$ with two opposite directed edges. The weight of each directed edges is the same weight as the original edge;
2. $D_{FT} = FT(G', k, r)$, $D_{FT}$ is the directed k-outconnected subgraph with optimal weight;
3. Construct the undirected version of $D_{FT}$, called $G_{FT}$. An undirected graph is constructed from a directed graph as follows: if there is a directed edge $uv$ in $D_{FT}$, then there exists an undirected edge $uv$ in $G_{FT}$. It is obvious that $G_{FT}$ is an undirected k-outconnected subgraph.

Note that [10] also gave an algorithm to construct undirected k-outconnected subgraph using the algorithm FT by adding a new root node. And they prove the weight of the subgraph constructed by their algorithm is less than twice the optimal weight of a k-connected subgraph. The difference of MWBA to [10] is that MWBA does not add a new vertex. We further prove the relationship of the weight of the output of MWBA to the optimal weight k-outconnected subgraph.

In the following, we give theoretical analysis of the MWBA algorithm.

**Lemma 1 ([1][2])** *For undirected graph $G$, $p(G) \leq 2c(G)$, and $c(G) \leq \Delta p(G)$, where $\Delta$ is the maximal degree of $G$.*
*For any tree $T$, $c(T) \leq p(T)$.*
*For any forest $F$, $c(F) \leq p(F)$.*

**Theorem 1 ([11])** *Let $S$ be a graph that is k-outconnected from a node $r$. In $S$, a cycle consisting of critical edges must be incident to a node $v \neq r$ such that $deg(v) = k$. A critical edge is the edge that if it is deleted, the graph is not k-outconnected any more.*

**Lemma 2 ([1])** *For any undirected graph $G$ which can be written as a union of $t$ forests, $c(G) \leq tp(G)$.*

**Lemma 3** *Let $G$ be any critically k-outconnected graph, we have $c(G) \leq kp(G)$, where critically k-outconnected means every edge is critical edge.*

*Proof.* We can split $G$ into $k$ forests in the same way as [1] except that [1] use Mader Theorem [12], while our construction utilize the result in Theorem 1. Then by Lemma 2, $c(G) \leq kp(G)$. □

**Lemma 4** *Let $G_{MWBA}$ be the output of MWBA, let $G_{wopt}$ be the undirected k-outconnected subgraph with minimum weight, we have $c(G_{MWBA}) \leq 2c(G_{wopt})$.*

*Proof.* Construct the bidirectional graph $B_{wopt}$ of $G_{wopt}$ by replacing each edge in $G_{wopt}$ with two opposite directed edges with the same weight, $c(B_{wopt}) = 2c(G_{wopt})$[3]. Let $D_{MWBA}$ be the directed version of $G_{MWBA}$ which is constructed by the FT algorithm, we have $c(G_{MWBA}) \leq c(D_{MWBA})$. We know that $D_{MWBA}$ has the optimal weight, thus $c(D_{MWBA}) \leq c(B_{MWBA}) = 2c(G_{wopt})$. Thus we have we have $c(G_{MWBA}) \leq 2c(G_{wopt})$. □

**Theorem 2** *Let $G_{popt}$ be an k-outconnected undirected subgraph consuming minimum power, we have $p(G_{MWBA}) \leq 4kp(G_{popt})$*

*Proof.* Let $G_{critical}$ be a subgraph of $G_{popt}$ that is critically $k$-outconnected. From Lemma 3, we have $c(G_{critical}) \leq kp(G_{critical})$. Also $p(G_{critical}) \leq p(G_{popt})$ since $G_{critical}$ is a subgraph of $G_{popt}$. Thus, we have

$$\begin{aligned} p(G_{MWBA}) &\leq 2c(G_{MWBA}) \\ &\leq 4c(G_{wopt}) \\ &\leq 4c(G_{critical}) \\ &\leq 4kp(G_{critical}) \\ &\leq 4kp(G_{popt}) \end{aligned}$$

□

To summarize, we present the MWBA algorithm and analyze its approximation ratio for minimum power k-outconnectivity problem as $4k$. Next, we introduce the NNAA algorithm.

[3] It is believed that minimum weight k-outconnectivity problem in undirected graph is NP-hard.

### 4.2 Nearest Neighbor Augmentation Algorithm

Before we present NNAA, first we introduce the definitions of $i$th nearest neighbor graph and k-outconnected augmentation.

**Definition 2** *$i$th nearest neighbor graph $G_i$: Given $G = (V, E)$, $G_i = (V, E')$ is its ith nearest neighbor graph, where for each $v \in G$, the first $i$ edges incident to $v$ with smaller weight are in $E'$.*

We can define the $i$th nearest outgoing neighbor graph $D_i$ of the bidirectional version of $G_i$ as containing the first $i$ outgoing edges incident to each $v$ with smaller weight.

**Definition 3** *k-outconnected augmentation: Given $G = (V, E)$ and a subgraph of $G$ called $H$, $F$ is called k-outconnected augmentation to $H$ if $H \cup F$ is k-outconnected spanning subgraph of $G$.*

The main idea of Nearest Neighbor Augmentation Algorithm (NNAA) is to first construct the $G_{k-1}$, then find a k-outconnected augmentation to $G_{k-1}$ by setting the weight of edges in $G_{k-1}$ to zero, then apply the MWBA to calculate the augmentation. This algorithm is the application of the algorithm in [2] to k-outconnectivity problem.

NNAA is illustrated as follows: Given an undirected graph $G = (V, E)$,

1. Construct the $(k-1)$th nearest neighbor graph $G_{k-1}$.
2. Set the weight of edges of $G$ in $G_{k-1}$ to zero
3. Construct $G'$ by replacing each edge in $G$ with two opposite directed edges with the same weight as the original edge;
4. output MWBA$(G')$

In the following, we give theoretical analysis for NNAA.

**Lemma 5 ([2])** *Let $G_{k-1}$ be the undirected $(k-1)$th nearest neighbor graph, $D_{k-1}$ be the directed $(k-1)$th nearest outgoing neighbor graph, $p(G_{k-1}) \leq kp(D_{k-1})$.*

**Lemma 6** *Let $G_{popt}$ be an undirected k-outconnected subgraph consuming optimal power,* $p(G_{k-1}) \leq kp(G_{popt})$

*Proof.* First, by definitions of $D_{k-1}$ and $G_{popt}$, the minimal degree of $G_{popt}$ is at least $k$, while the outgoing degree of each node in $D_{k-1}$ is $k-1$, thus $p(D_{k-1}) \leq p(G_{popt})$. By Lemma 5, $p(G_{k-1}) \leq kp(G_{popt})$. □

**Lemma 7** *For undirected graph, let $F_{popt} \subseteq G_{popt}$ be the minimum weight k-outconnected augmentation to $G_{k-1}$ among all augmentations which are subgraphs of $G_{popt}$, $c(F_{popt}) \leq p(G_{popt})$*[4].

*Proof.* Because $G_{popt}$ is k-outconnected from $r$, thus it contains a subgraph which is a k-outconnected augmentation to $G_{k-1}$. Next we prove $F_{popt}$ is a forest by contradiction. Suppose $F_{popt}$ is not a forest, then there exists a cycle $C$ in $F_{popt}$. Every edge in $F_{popt}$ is critical for $G_{k-1} \cup F_{popt}$ to be k-outconnected because $F_{popt}$ has the minimum weight among all augmentations which are subgraphs of $G_{popt}$. Applying Theorem 1, $C \subset F_{popt}$ must contain a node $v \neq r$ and the degree of $v$ is $k$. On the other hand, since $F_{popt}$ is the k-outconnected augmentation to $G_{k-1}$ in which each vertex degree is at least $k-1$, and $F_{popt}$ contains a cycle including $v$, the degree of $v$ must be at least $k-1+2 = k+1$, which contradicts that the degree of $v$ is $k$. Thus $F_{popt}$ is a forest. By Lemma 1, $c(F_{popt}) \leq p(F_{popt}) \leq p(G_{popt})$. □

**Lemma 8** *Let $F_{NNAA}$ be the k-outconnected augmentation constructed in algorithm NNAA,* $p(F_{NNAA}) \leq 4p(G_{popt})$

*Proof.* Let $F_{wopt}$ be the optimal weight k-outconnected augmentation to $G_{k-1}$. We have

$$\begin{aligned} p(F_{NNAA}) &\leq 2c(F_{NNAA}) \\ &\leq 4c(F_{wopt}) \\ &\leq 4c(F_{popt}) \\ &\leq 4p(G_{popt}) \end{aligned}$$

[4]The difference of this lemma to the proof in [2] is this lemma is for a k-outconnected augmentation to $G_{k-1}$, while [2] is for a k-connected augmentation.

The first two inequalities follow Lemma 1 and Lemma 4 respectively. The third inequality is true because $F_{wopt}$ has the optimal weight. The fourth inequality follows Lemma 7. □

**Theorem 3** $p(G_{NNAA}) \leq (k+4)p(G_{popt})$, *i.e., the algorithm NNAA has approximation ratio $k+4$.*

*Proof.* From Lemma 6 and 8, we have $p(G_{NNAA}) = p(G_{k-1} \cup F_{NNAA}) \leq p(G_{k-1}) + p(F_{NNAA}) \leq kp(G_{popt}) + 4p(G_{popt}) = (k+4)p(G_{popt})$. □

In this section, we propose a $4k$-approximation called MWBA and $(k+4)$-approximation called NNAA for minimum power k-outconnectivity problem. Although NNAA performs much better than MWBA in worst case, it is hard to predict their performance under average cases. We evaluate their average performance by simulation in the next section.

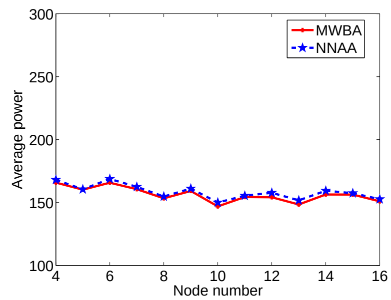
## 5 Performance Evaluation

In this section, we evaluate and compare the performance of MWBA and NNAA in terms of total power and total weight required for constructing undirected $k$-outconnected spanning subgraphs.

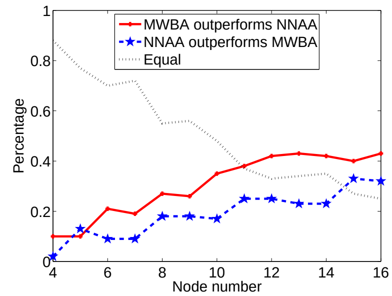
To set up the simulation environment, we randomly generate various number of nodes in a fixed area and construct a complete cost graph from these nodes by setting the weight of each edge $uv$ as $Cd^2(uv)$ and $C$ is randomly chosen between $(0.5, 1.5)$. For each node size, we run the simulation for 100 times.

Fig.1.(a) shows the comparison of average power consumption. We observe that MWBA always outperforms NNAA slightly for different node size. Moreover, the average power of both schmes tends to decrease as the node size increase. This reflects the effect of node density to total power. As the network gets denser, the total power to satisfy the connectivity requirement decreases because the power assignment of each node is reduced.

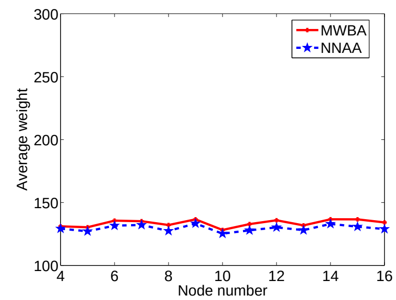
Besides total power consumption, we also show the number of times one algorithm outperforms



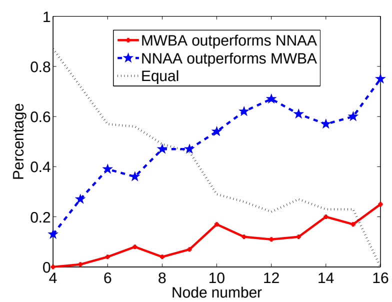

(a) The average power of MWBA vs. the average power of NNAA



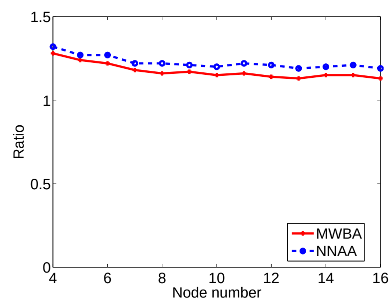

(b) Percentage of times that one algorithm uses less power than the other




(c) The average weight of MWBA vs. the average weight of NNAA




(d) Percentage of times that one algorithm uses less weight than the other




(e) The $\frac{power}{weight}$ ratio of MWBA vs. that of NNAA

Figure 1: Comparison of MWAB and NNAA

the other in terms of power in Fig.1.(b). The solid line illustates the percentage of times when MWBA outperforms NNAA, the dashed line with star illustrates the percentage of times when NNAA outperforms MWBA, and the dotted line illustates the percentage of times when both algorithms use the same power. MWBA achieves better performance in general than NNAA. This suggests that algorithms for minimum weight k-outconnectivity is also a good approximation for minimum power k-outconnectivity.

The comparison of total weight is shown in Fig.1.(c) and (d). Fig.1.(c) illustrates that NNAA has less weight than MWBA and Fig.1.(d), which uses the same legend as in Fig.1.(b), shows that NNAA uses less weight than MWBA for most of the runs. This conveys that although minimum weight directed k-outconnectivity has an optimal solution, its undirected version for undirected graphs does not guarantee small weight.

We also compare the ratio of power and weight of both algorithms. We define the ratio as follows:

$$Ratio = \frac{\sum_{i=1}^{\mathrm{r}} \frac{Power_i}{Weight_i}}{\mathrm{r}}$$

where $r$ is the number of runs.

As illustrated in Fig. 1.(e), MWBA and NNAA both show similar $\frac{power}{weight}$ ratio over various node size. MWBA has slightly smaller ratio of around 1.16 than that of NNAA, which is around 1.2. The reason is that output of MWBA has greater total weight than NNAA while their total power are almost the same.

To conclude, MWBA is slightly better than

NNAA in terms of total power consumption and NNAA is moderately better than MWBA in terms of total weight.

## 6 Conclusions

In this paper, we introduced the minimum power k-outconnectivity problem in symmetric networks and studied its hardness. We presented a $4k$-approximation called minimum weight based algorithm and a $(k + 4)$-approximation called nearest neighbor augmentation algorithm for this problem. In addition, we compare their performance under average case using extensive simulation. The simulation results show that although the NNAA outperms MWBA in the worst case, under average case, MWBA is slightly better than NNAA in terms of total power consumption.

Our future work will focus on the distributed and localized algorithms since if the networks are dynamic, distributed and localized algorithms are necessary.

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