Punishment, Rational Expectations, and Relative Payoffs

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An Experiment

- 2 subjects play anonymously
- Players have only 2 choices
- Choices are made simultaneously

<table>
<thead>
<tr>
<th></th>
<th>Player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Player 1</td>
<td>A</td>
</tr>
<tr>
<td>A</td>
<td>$x_1, x_1$</td>
</tr>
<tr>
<td>B</td>
<td>$x_2, x_3$</td>
</tr>
</tbody>
</table>
An Experiment

• Game 1

• Expected outcome:
  – Players choose B

• Results:
  – B selected in 74% of games

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<tr>
<td>Player 1</td>
<td>A</td>
</tr>
<tr>
<td>A</td>
<td>3, 3</td>
</tr>
<tr>
<td>B</td>
<td>5, 0</td>
</tr>
</tbody>
</table>
An Experiment

- **Game 2**

- **Expected outcome:**
  - Players choose A

- **Results:**
  - B selected in 72% of games

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<tr>
<td>Player 1</td>
<td>A</td>
</tr>
<tr>
<td>A</td>
<td>4, 4</td>
</tr>
<tr>
<td>B</td>
<td>3, 1</td>
</tr>
</tbody>
</table>
```
An Experiment

- Authors concluded that utility is driven by “the need to maximize the difference between one’s self and the other player”

Questions

• Are assumptions of rational choice theory correct?
  – Do people really only maximize payoffs?
  – Is rational choice theory framed in evolutionary terms?

• What does evolutionary biology tell us?
  – Natural selection is driven by *relative* fitness

• How do absolute models compare to relative models?
The simulation model

- Payoff maximization with **local interactions**
- Populations structured in **social networks**
- Prisoners dilemma with punishment
- Evolutionary algorithm with mutation
Continuous prisoners dilemma

- Not limited to 2 choices
  - Choose a cooperation level $x$ on $[0,1]$

- Given a game between $i$ and $j$
  - $p_i = 1 - x_i + r(x_i + x_j)/2$

- The dilemma
  - $p_i$ is maxed when $x_i = 0$
  - $(p_i + p_j)$ is maxed when $x_i$ and $x_j = 1$
Altruistic punishment

- Punishment is known to induce cooperation in laboratory experiments

- A reduction in payoffs to punish another with no material benefit in return

- When \( k \) punishes \( j \)
  - \( k \) is charged a cost of \( c \)
  - \( j \) is charged a fine of \( cM \)
Simulation example
Simulation example

- $i$ is selected to initiate a game
Simulation example

- $i$ interacts only within a local neighborhood
Simulation example

- $i$ randomly selects $j$ to play with

- $i$ and $j$ contribute $x_i$ and $x_j$ respectively
  $$x_i + x_j = G$$

- Each gets a payoff
  $$p_i = (1-x_i) + rG/2$$
  $$p_j = (1-x_j) + rG/2$$
Simulation example

- $i$ randomly chooses a neighbor $k$ to evaluate $i$

- If $i$’s offer was sufficient nothing happens

- If $i$’s offer was too low $k$ punishes $i$
  
  $$p_k = p_k - c$$
  $$p_i = p_i - cM$$
## Networks used

<table>
<thead>
<tr>
<th>Network Type</th>
<th>Description of Neighbors</th>
<th>Number of Neighbors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ring</td>
<td>Left, right</td>
<td>2</td>
</tr>
<tr>
<td>Von Neumann</td>
<td>Left, right, up, down</td>
<td>4</td>
</tr>
<tr>
<td>Hexagonal</td>
<td>Left, right, diagonals</td>
<td>6</td>
</tr>
<tr>
<td>Moore</td>
<td>Left, right, up, down, diagonals</td>
<td>8</td>
</tr>
<tr>
<td>Complete</td>
<td>All other agents</td>
<td>$N-1$</td>
</tr>
</tbody>
</table>
Behavior models

**Economic model**
- Agents maximize $p$
- Agents never punish
- No punishment = no cooperation

**Biological model**
- Agents maximize $p/P_n$
- Agents punish at $M > n$
- Punishment results in cooperation
Linear Network - Predictions

![Graph showing mean contribution versus punishment multiplier M. The graph is a step function that remains constant at 1.0 once the punishment multiplier exceeds 2.0.](image-url)

- The x-axis represents the punishment multiplier (M).
- The y-axis represents the mean contribution.

When the punishment multiplier is greater than 2.0, the mean contribution is consistently at 1.0, indicating that the behavior modeled is predicted to maximize the mean contribution once the punishment threshold is surpassed.

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Linear Network - Results

- Punishment multiplier $M$
- Mean contribution

Graph showing the relationship between the punishment multiplier $M$ and the mean contribution.
## Results ($N=400$)

Value of $M$ at which punishment drives the population to cooperate

<table>
<thead>
<tr>
<th>Network Type</th>
<th>Economic Expectation</th>
<th>Biological Expectation</th>
<th>Simulation Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ring</td>
<td>Never</td>
<td>2</td>
<td>1.5</td>
</tr>
<tr>
<td>Von Neumann</td>
<td>Never</td>
<td>4</td>
<td>1.8</td>
</tr>
<tr>
<td>Hexagonal</td>
<td>Never</td>
<td>6</td>
<td>2.2</td>
</tr>
<tr>
<td>Moore</td>
<td>Never</td>
<td>8</td>
<td>2.8</td>
</tr>
<tr>
<td>Complete</td>
<td>Never</td>
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Why is this important?

- International treaties are often designed to convert a dilemma to a “win-win” scenario.

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<td>4, 4</td>
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<td>B</td>
<td>5, 0</td>
<td>1, 3</td>
</tr>
<tr>
<td>A</td>
<td>0, 5</td>
<td>1, 3</td>
</tr>
<tr>
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- If relative payoffs drive behavior the treaty has no effect.
Conclusion

- Behavior in networked populations is better predicted by a biological, relative payoff model.

- The importance of relative vs. absolute payoffs is dependent on network topology.

- If relative payoffs drive behavior, restructuring payoffs may not have intended effects.
Thank you

• Questions?

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