Chapter 11 Review

1. Solve the proportion. Check for extraneous solutions.
   a. \( \frac{16}{4} = \frac{12}{x} \)
   b. \( \frac{r + 4}{3} = \frac{r}{5} \)
   c. \( \frac{x + 6}{3} = \frac{x - 5}{2} \)
   d. \( \frac{2}{3t} = \frac{t - 1}{t} \)
   e. \( \frac{-2}{q} = \frac{q + 1}{q^2} \)
   f. \( \frac{2}{6x + 1} = \frac{2x}{1} \)

2. You are shopping and find a coat that is on sale for 30% off. It is regularly priced at $80. Your friend tells you that she saw the same coat for $80 in another store, but it was 20% off plus an additional 10% off. Will you save money by going to the other store? Explain why or why not.

3. Use the table to the right. It shows the results of a survey in which 100 students were asked how they spent money last week.

<table>
<thead>
<tr>
<th>Item</th>
<th>Number of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food</td>
<td>78</td>
</tr>
<tr>
<td>Clothes, accessories</td>
<td>20</td>
</tr>
<tr>
<td>Books, magazines, comics</td>
<td>15</td>
</tr>
<tr>
<td>Toys, stickers, games</td>
<td>14</td>
</tr>
<tr>
<td>Movie tickets</td>
<td>14</td>
</tr>
<tr>
<td>Arcade games</td>
<td>14</td>
</tr>
<tr>
<td>Gifts</td>
<td>13</td>
</tr>
<tr>
<td>Movie rentals</td>
<td>13</td>
</tr>
<tr>
<td>Music</td>
<td>12</td>
</tr>
<tr>
<td>Footwear</td>
<td>11</td>
</tr>
<tr>
<td>Grooming products</td>
<td>11</td>
</tr>
</tbody>
</table>

   a. Estimate the number of students out of 500 that bought clothes or accessories in the last week.
   b. Choose 3 items that were bought by different numbers of students. Based on the survey, how many students out of 20 would you predict to have bought each item?
4. Researchers studying fish populations at Dryden Lake in New York caught, marked, and then released 232 Chain pickerel. Later a sample of 329 Chain pickerel were caught and examined. Of these, 16 were found to be marked. Use the proportion below to estimate the total Chain pickerel population in the lake.

\[
\frac{\text{marked pickerel in sample}}{\text{total pickerel in sample}} = \frac{\text{marked pickerel in lake}}{\text{total pickerel in lake}}
\]

5. Solve the percent problem.
   a. What number is 25% of 80?
   b. 52 is 12.5% of what number?
   c. 16% of what number is 8?
   d. 62 hours is what percent of 3 days?
   e. 2 percent of what amount is $200?
   f. $240 is what percent of $50?

6. The histogram shows how 861 people answered a survey question about when they usually change the oil in their cars.
   a. How many of the people change their oil between 3001 and 4000 miles?
   b. How many of the people change their oil between 4001 and 6000 miles?
   c. If you surveyed 2500 people, about how many people do you expect to answer “2001 to 3000 miles?”

7. The variables \(x\) and \(y\) vary directly. Use the given values to write an equation that relates \(x\) and \(y\).
   a. \(x = 3, \ y = 9\)
   b. \(x = 36, \ y = 12\)
   c. \(x = 45, \ y = 81\)
8. The variable $x$ and $y$ vary inversely. Use the given values to write an equation that relates $x$ and $y$.
   a. $x = 2$, $y = 5$
   b. $x = 5$, $y = \frac{1}{3}$
   c. $x = 10.5$, $y = 7$
   d. $x = 45$, $y = \frac{3}{5}$

9. State whether the variables have direct variation, inverse variation, or neither.
   a. The area $B$ of the base and height $h$ of a prism with a volume of 10 cubic units are related by the equation $Bh = 10$
   b. The mass $m$ and the volume $V$ of a substance are related by the equation $2V = m$, where 2 is the density of the substance.
   c. Alicia cut a pizza into 8 pieces. The number of pieces $d$ that Alicia ate for dinner and the number of pieces $g$ that she can eat for breakfast are related by the equation $b = 8 - d$.
   d. The number of hours $h$ that you must work to earn $\$480$ and your hourly rate of pay $p$ are related by the equation $ph = 480$.

10. When a person walks, the pressure $P$ on each boot sole varies inversely with the area $A$ of the sole. Denise is walking through deep snow, wearing boots that have a sole area of 29 square inches each. The boot-sole pressure is 4 pounds per square inch when she stands on one foot.
   a. The constant of variation is Denise’s weight in pounds. What is her weight?
   b. If Denise wears snowshoes, each with an area 11 times that of her boot soles, what is the snowshoe pressure when she stands on one foot?

11. You are taking a trip on a highway in a car that gets a gas mileage of about 26 miles per gallon for highway driving. You start with a full tank of gasoline.
   a. Find your rate of gas consumption (the gallons of gas used to drive 1 mile).
   b. Use part a to write an equation relating the number of gallons of gas in your tank $g$ and the number of miles $m$ that you have driven on your trip.
   c. Do the variables $g$ and $m$ vary directly, inversely, or neither? Explain.
12. Simplify the expression if possible
   a. \[ \frac{4x}{20} \]
   b. \[ \frac{-18x^2}{12x} \]
   c. \[ \frac{42x - 6x^3}{36x} \]
   d. \[ \frac{5 - x}{x^2 - 8x + 15} \]
   e. \[ \frac{2x^2 + 11x - 6}{x + 6} \]
   f. \[ \frac{x^2 + x - 20}{x^2 + 2x - 15} \]
   g. \[ \frac{x^3 - x}{x^3 + 5x^2 - 6x} \]

13. For what values of the variable is the rational expression undefined?
   a. \[ \frac{11}{x - 8} \]
   b. \[ \frac{4}{x^2 - 1} \]
   c. \[ \frac{x - 3}{x^2 + 5x - 6} \]

14. You are designing a game for a school carnival. Players will drop a coin into a basin of water, trying to hit a target on the bottom. The water is kept moving randomly, so the coin is equally likely to land anywhere. You use a rectangular basin twice as long as it is wide. You place the blue rectangular target an equal distance from each end.
   a. Express the two dimensions of the target in terms of the variables \( x \) and \( y \).
   b. Write a model that gives the probability that the coin will land on the target.
c. You want players to win about half the time. Give a set of values you could use for \( x \) and \( y \) if the basin's area is between 72 and 120 square inches.

15. A meteorite is equally likely to hit anywhere on Earth. The probability that a meteorite lands in the Torrid Zone is \[ \frac{\text{Area of Torrid Zone}}{\text{Total surface area of Earth}}. \] (The Torrid Zone is the area of the Earth between the Tropic of Cancer and Tropic of Capricorn.) Let \( R \) represent Earth's radius.

a. Write an expression to estimate the area of the Torrid Zone. You can think of the distance between the tropics (about 3250 miles) as the height of a cylindrical belt around Earth at the equator. The length of the belt is Earth's circumference \( 2\pi R \).

b. The surface area of a sphere with radius \( R \) is \( 4\pi R^2 \). Write and simplify an expression for the probability that a meteorite lands in the Torrid Zone.

c. Find the probability in part b. Use 3963 miles for Earth's radius.

e. \( x^2 - 9x + 18 = 2x \)

16. Simplify the expressions.

a. \[ \frac{4x \cdot 1}{3x} \]

b. \[ \frac{25x^2}{10x} \div \frac{5x}{10x} \]

c. \[ \frac{2(x+2)}{5(x-3)} \div \frac{4(x-2)}{5x-15} \]

d. \[ \frac{x}{3x^2 + 2x - 8} \cdot (3x - 4) \]

e. \( (4x^2 + x - 3) \cdot \frac{1}{(4x + 3)(x-1)} \)

f. \[ \frac{x^2 - 8x + 15}{x^2 - 3x} \div (3x - 15) \]

g. \[ \left( \frac{2x^2 - 5}{3x} \right) \div \frac{6x^2}{25} \]
17. The models below are based on data collected by the Bureau of Economic
Analysis from 1990 to 1997 in the United States. Let \( t \) represent the number
of years since 1990.

Total sales (in billions of dollars) of services: \( S = \frac{1055 + 23t}{1 - 0.04t} \)

Total sales (in billions of dollars) of hotel services: \( H = \frac{46 + 0.7t}{1 - 0.04t} \)

Total sales (in billions of dollars) of auto repair services: \( H = \frac{48 - t}{1 - 0.06t} \)

a. Find the total sales given by each model in 1990.

b. Find a model for the ratio of hotel service sales to total service industry
sales. Was this ratio increasing or decreasing from 1990 to 1997?
Explain.

c. Find a model for the ratio of auto service sales total service industry
sales. Was this ratio increasing or decreasing from 1990 to 1997?
Explain.

d. What do your answers in parts b and c tell you about how the sales of the
service industry were changing in the period form 1990 to 1997?

18. Simplify the expression.

a. \( \frac{4}{x+1} + \frac{2x-2}{x+1} \)

b. \( \frac{2}{3x-1} - \frac{5x}{3x-1} \)

c. \( \frac{2}{x+3} + \frac{4x}{2x+6} \)

d. \( \frac{4x}{5x-2} - \frac{2x}{5x+1} \)

e. \( \frac{x+8}{3x-1} + \frac{x+3}{x+1} \)

f. \( \frac{2}{x+1} + \frac{3}{x-2} + \frac{3}{x+4} \)
19. Find an expression for the perimeter of the rectangle shown below:

\[
\begin{array}{c}
\frac{3x + 5}{2x - 1} \\
\frac{2x - 1}{x - 3}
\end{array}
\]

20. A boat moves thorough still water at \( x \) kilometers (kn) per hours. It travels 24 km upstream against a current of 2 km per hour and then returns with the current. The rate upstream is \( x - 2 \) because the boat moves against the current, the rate downstream is \( x + 2 \) because the boat moves with the current.
   a. Write an expression for the total time for the round trip.
   b. Write your answer to part a as a single rational expression.
   c. Use your answer to part b to find how long the round trip will take if the boat travels 10 kilometers per hour through still water.

21. Divide
   a. \( (8x + 13) \div 2 \)
   b. \( (10x^2 - 7x - 12) \div (2x - 3) \)
   c. \( (a^2 - 3a + 2) \div (a - 1) \)
   d. \( (5 - 7m + 3m^2) \div (m - 3) \)
   e. \( (x^2 + 9) \div (-x - 4) \)
   f. \( (c^2 - 7c + 21) \div (2c - 6) \)
   g. \( (16a^2 - 25) \div (5 + 4a) \)

22. The area and one dimension of the rectangle to the right are shown. Find the missing dimension.

Area is: \( 7x^2 + 17x + 6 \)

\( x + 2 \)
23. Use the models below that approximate spending in the United States from 1988 to 1997. Let $t$ represent the number of years since 1988.

Dollars spent on exercise equipment (in millions); $E = 200t + 1400$
Total dollars spent on sports equipment (in millions); $S = 900t + 9900$

a. Write a rational model for the ratio of the money spent on exercise equipment to the total money spent on sports equipment. Simplify the model by dividing out the greatest common factor.

b. Use long division to write the model from part a in another form.

c. Fill in the table below. Use the rewritten model from part b. Round to the nearest thousandth.

<table>
<thead>
<tr>
<th>Year $t$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio of $E$ to $S$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

d. Use the table. Was the ratio increasing or decreasing from 1988 to 1997?

e. Look at the model you found in part b. How can you tell from the model if the ratio is increasing or if it is decreasing?

24. Solve the equation by cross multiplying.

a. $\frac{x}{5} = \frac{7}{3}$

b. $\frac{4}{x} = \frac{12}{5(x+2)}$

c. $\frac{6}{x+2} = \frac{x}{4}$

25. Solve the equation by multiplying each side by the least common denominator.

a. $\frac{7}{3x-12} - \frac{1}{x-4} = \frac{2}{3}$

b. $\frac{1}{x-4} + 1 = -\frac{7}{x^2+x-20}$

c. $\frac{1}{x-4} + \frac{1}{x+4} = \frac{22}{x^2-16}$
26. Solve the equation.
   a. \( \frac{x - 8}{9} = \frac{1}{x} \)
   b. \( \frac{x + 42}{x} = x \)
   c. \( \frac{10 - 3}{x + 3} = \frac{10x + 1}{3x + 9} \)
   d. \( \frac{x - 1}{x - 11} = \frac{22}{x^2 - 5x - 66} \)
   e. \( \frac{2x - x}{x + 3} = \frac{x^2 - 1}{x + 7} = \frac{x^2 + 10x + 21}{x^2 + 10x + 21} \)

27. Graph the function. Describe the domain.
   a. \( y = \frac{1}{x} + 4 \)
   b. \( y = \frac{6}{x + 9} - 7 \)
   c. \( y = \frac{-5x + 19}{x - 3} \)

28. You have taken 3 tests and have an average of 72 points. If you score 100 points on each of the rest of your tests, how many more tests do you need to take to raise your average to 88 points?