Chapter 12 Review

1. Evaluate the function for the given value of \( x \). Round your answer to the nearest tenth.
   a. \( y = \sqrt{x - 7}; \quad 9 \)
   b. \( y = \sqrt{8x^2 + \frac{3}{2}}; \quad \frac{1}{4} \)
   c. \( y = \sqrt{21 - 2x}; \quad -2 \)

2. Find the domain of the function.
   a. \( y = \sqrt{x + 5} \)
   b. \( y = \sqrt{x} - 3 \)
   c. \( y = \sqrt{3x - 10} \)
   d. \( y = x\sqrt{x} \)
   e. \( y = \frac{\sqrt{x - 4}}{x} \)

3. Find the domain and the range of the function. Then sketch the graph of the function.
   a. \( y = 7\sqrt{x} \)
   b. \( y = \sqrt{x} + 4 \)
   c. \( y = \sqrt{x} - 6 \)
   d. \( y = \sqrt{2x} + 5 \)
   e. \( y = x\sqrt{8x} \)
4. An accident reconstructionist is responsible for finding how fast cars were going before an accident. To do this, a reconstructionist uses the model below where $S$ is the speed of the car in miles per hour, $d$ is the length of the tires' skid marks in feet, and $f$ is the coefficient of friction for the road.

Car speed model: $S = \sqrt{30df}$

a. In an accident, a car makes skid marks 74 feet long. The coefficient of friction is 0.5. A witness says that the driver was traveling faster than the speed limit of 45 miles per hour. Can the witness's statement be correct? Explain your reasoning.

b. How long would the skid marks have to be in order to know that the car was traveling faster than 45 miles per hour?

5. Simplify the expression.
   
a. $5\sqrt{7} + 2\sqrt{7}$
   
b. $\sqrt{32} + \sqrt{2}$
   
c. $\sqrt{147} - 7\sqrt{3}$
   
d. $\sqrt{243} - \sqrt{75} + \sqrt{300}$
   
e. $11\sqrt{3} - 12\sqrt{3}$
   
f. $\sqrt{3} \cdot \sqrt{12}$
   
g. $(1 + \sqrt{13})(1 - \sqrt{13})$
   
h. $\sqrt{3}(5\sqrt{2} + \sqrt{3})$
   
i. $(\sqrt{c} + d)(3 + \sqrt{5})$
   
j. $\frac{2}{\sqrt{2}}$
   
k. $\frac{6}{10 + \sqrt{2}}$
   
l. $\frac{\sqrt{3}}{\sqrt{3} - 1}$
   
m. $\frac{4 + \sqrt{3}}{a - \sqrt{b}}$
6. Solve the quadratic equation.
   a. \( x^2 + 10x + 13 = 0 \)
   b. \( x^2 - 6x - 1 = 0 \)
   c. \( 4x^2 - 2x - 1 = 0 \)

7. The average speed of an object \( S \) (in feet per second) that is dropped a certain distance \( d \) (in feet) is given by the following equation.

   Falling object model: \( S = \frac{d}{\left(\frac{\sqrt{d}}{4}\right)} \)

   a. Rewrite the equation with the right hand side in the simplest form.
   b. Use either equation to find the average speed of an object that is dropped from a height of 400 feet.
   c. Graph both the falling object model and our equation from part a on the same screen to check that you simplified correctly.

8. Solve the equation. Check for extraneous solutions.
   a. \( \sqrt{x} - 9 = 0 \)
   b. \( \sqrt{6x} - 13 = 23 \)
   c. \( \sqrt{5x + 1} + 2 = 6 \)
   d. \( \frac{1}{5} \sqrt{x} - 2 - \frac{1}{10} = \frac{7}{10} \)
   e. \( x = \sqrt{-4x - 4} \)
   f. \( \frac{2}{3} x = \sqrt{24x - 128} \)

9. Two numbers and their geometric mean are given. Find the value of \( a \).
   a. 12 and \( a \); geometric mean 27
   b. 8 and \( a \); geometric mean 104
10. Find the value of $x$ if the perimeter of the rectangle is 30.

11. A ride at an amusement park spins in a circle of radius $r$ (in meters). The centripetal force $F$ experiences by a passenger on the ride is modeled by the equation below, where $t$ is the number of seconds the ride takes to complete one revolution and $m$ is the mass (in kilograms) of the passenger.

$$t = \sqrt{\frac{4\pi^2 mr^2}{F}}$$

a. A person whose mass is 67.5 kilograms is on a ride that is spinning in a circle at a rate of 10 seconds per revolution. The radius of the circle is 6 meters. How much centripetal force does the person experience?

b. A person whose mass is 54 kilograms is on the ride. A mass of 54 kilograms is 80% of 67.5 kilograms. Is the centripetal force experiences by the person 80% of the force in part a? Explain.

12. Find the term that should be added to the expression to create a perfect square trinomial.

a. $x^2 + 12x$

b. $x^2 - 0.3x$

c. $x^2 + \frac{2}{3}x$

d. $x^2 - 5.2x$
13. Solve the equation by completing the square.
   a. \( x^2 + 10x = 39 \)
   b. \( x^2 - 8x + 12 = 0 \)
   c. \( x^2 + \frac{3}{5}x - 1 = 0 \)
   d. \( 4x^2 + 4x - 11 = 0 \)
   e. \( 5x^2 - 20x - 20 = 5 \)
   f. \( 6x^2 + 24x - 41 = 0 \)

14. Solve the quadratic equation.
   a. \( 9a^2 - 25 = 0 \)
   b. \( x^2 - x - 20 = 0 \)
   c. \( c^2 + 2c - 26 = 0 \)
   d. \( 4p^2 - 12p + 5 = 0 \)
   e. \( 7z^2 - 46z = 21 \)

15. The path of a diver diving from a 10-foot high diving board is
    \[ h = -0.44x^2 + 2.61x + 10 \]

    where \( h \) is the height of the diver above the water (in feet) and \( x \) is the horizontal distance (in feet) from the end of the board. How far from the end of the board will the diver enter the water?

16. Find the missing length of the right triangle if \( a \) and \( b \) are the lengths of the legs and \( c \) is the length of the hypotenuse.
    a. \( a = 3, b = 4 \)
    b. \( a = 14, c = 21 \)
    c. \( b = 3, c = 7 \)
17. Find the missing length.

a. 

b. 

c. 

18. Determine whether the given lengths are the sides of a right triangle. Explain your reasoning.

a. 2, 10, 11
b. 7, 24, 26
c. 5, 12, 13
d. 9.9, 2, 10.1

19. You have just planted a new tree. To support the tree in bad weather, you attach guy wires from the trunk of the tree to stakes in the ground. You cut 30 feet of wire into four equal lengths to make the guy wires. You attach four guy wires, evenly spaced around the tree. You put the stakes in the ground five feet from the base of the trunk. Approximately how far up the trunk should you attach the guy wires?

20. Find the distance between the two points. Round the result to the nearest hundredth if necessary

a. \((2,0)(8,-3)\)
b. \(\left(\frac{1}{2},\frac{1}{4}\right)(2,1)\)
c. \((3.5,6)(-3.5,-2)\)
d. \((-6,-2)(-3,-5)\)
21. Graph the points. Decide whether they are vertices of a right triangle.
   a.  \((4,0)\) \((2,1)\) \((-1,-5)\)
   b.  \((-2,2)\) \((3,4)\) \((4,2)\)
   c.  \((-1,1)\) \((-3,3)\) \((-7,-1)\)

22. Find the midpoint between the two points.
   a.  \((3,0)\) \((-5,4)\)
   b.  \((5,-5)\) \((-5,1)\)
   c.  \((-4,-3)\) \((-1,-5)\)

23. A trapezoid is isosceles if its two opposite nonparallel sides have the same length.
   a. Draw a polygon whose vertices are \(A(1, 1), B(5, 9), C2, 8), and D(0, 4)\).
   b. Show that the polygon is a trapezoid by showing that only two sides are parallel.
   c. Use the distance formula to show that the trapezoid is isosceles.

24. You and a friend go hiking. You hike 3 miles north and 2 miles west. Starting from the same point, your friend hikes 4 miles east and 1 mile south.
   a. How far apart are you and your friend? (Hint: Draw a diagram on a grid.)
   b. If you and your friend want to meet for lunch, where could you meet so that both of you hike the same distance

25. Find the sine, cosine and tangent of \(\angle R\) and of \(\angle S\)
26. Find the missing lengths of the sides of the triangle. Round your answers to the nearest hundredth. Use the Pythagorean Theorem to check.

![Diagram of triangle with labeled sides]

a. 

b. 

c. 

27. You are standing 197 feet from the base of the world’s largest catsup bottle located in Collinsville, Illinois. You estimate that the angle between your eye level and the line from your eyes to the top of the bottle is 40°. If you are 5 feet tall, about how high is the top of the bottle?

28. Complete the proof of the statement: For all real numbers $a$ and $b$,

\[
(a + b) - b = a
\]

\[
(a + b) - b = (a + b) + (-b)
\]

Definition of subtraction

\[
(a + b) - b = a + \left[ b + (-b) \right]
\]

Associative property of addition

\[
(a + b) - b = a + 0
\]

\[
(a + b) - b = a
\]

29. Find a counterexample to show that the statement is not true.

a. If $a$ and $b$ are real numbers, then $(a + b)^2 = a^2 + b^2$.

b. If $a$, $b$, and $c$ are nonzero real numbers, then $(a ÷ b) ÷ c = a ÷ (b ÷ c)$.

c. If $a$ and $b$ are integers, the $a ÷ b$ is an integer.
30. If \( p \) is an integer and \( p^2 \) is divisible by 2, then \( p \) is divisible by 2. \( (\text{Hint: An odd number can be written as } 2n+1, \text{ where } n \text{ is an integer. An even number can be written as } 2n. \text{ Use an indirect proof.}) \)

31. Let \( D \) represent the midpoint between \( B \) and \( C \), as shown at the right. Prove that for any right triangle, the midpoint of its hypotenuse is equidistant from the three vertices of the triangle.  
In order to prove this, you must first find the distance, \( BC \), between \( B \) and \( C \). Use the distance formula to get \( BC = \sqrt{x^2 + y^2} \), so \( BD \) and \( CD \) must be \( \frac{1}{2} \sqrt{x^2 + y^2} \). To help you with your proof, use the distance formula to find \( AD \).