## CHAPTER 4

## Solutions for Exercises

### 4.1 Chapter 1 Exercise Solutions

Exercise 1.1. Arthrodax Company
i) Figure 4.1 shows the decision tree.
ii) As the decision tree shows, the preferred alternative is to accept the order and purchase the injection molder, with an expected profit of $\$ 154.4$ thousand.

Exercise 1.2. Arthrodax Company (con't)
i) Figure 4.2 shows the decision tree.
ii) As the decision tree shows, the preferred alternative is to accept the order and purchase the cases, with an expected profit of $\$ 99.9$ thousand.

Exercise 1.3. Arthrodax Company (con't)
i) Figure 4.3 shows the decision tree. Note that in addition to the alternatives shown in this decision tree, it would be possible to immediately purchase the injection molder, or to immediately reject the purchase of the injection molder, without waiting to determine how many units will be ordered. These alternatives are shown in Figure 4.2 for the preceding exercise. Of course, these alternatives cannot be more preferred than the alternative of waiting to see how many units are ordered, since it costs nothing to wait, and you gain further information by waiting.
ii) As the decision tree shows, the preferred strategy is to accept the order, and if Ranger orders 100 units then purchase the molder while if Ranger orders 50 units purchase the cases. The expected profit for this strategy is $\$ 102.1$ thousand.

Exercise 1.4. Aba Manufacturing
i) This would potentially avoid the second $\$ 250,000$ setup cost.
ii) Figure 4.4 shows the decision tree for this part.
iii) From the decision tree, we see that the preferred alternative is to manufacture all $200,000 \mathrm{PC}$ boards now.


Figure 4.1 Arthrodax Company decision tree (dollar amounts in thousands)


Figure 4.2 Arthrodax uncertain units decision tree (dollar amounts in thousands)
Exercise 1.5. Kezo Systems
i) The decision tree is shown in Figure 4.5.
ii) The expected value calculations are shown on the decision tree for part a, and these show that Kezo should order all 500,000 PAL chips from KEC.

Exercise 1.6. Intermodular Semiconductor Systems
i) The decision tree for this exercise is shown in Figure 4.6.
ii) The expected values are shown on the Figure 4.6 tree for each of the four possible alternatives. The preferred alternative is to bid $\$ 5,000$.


Figure 4.3 Arthrodax delay molder decision tree (dollar amounts in thousands)


Figure 4.4 Aba Manufacturing decision tree (dollar amounts and quantities in thousands)


Figure 4.5 Kezo Systems decision tree (dollar amounts and quantities in thousands)

### 4.2 Chapter 2 Exercise Solutions

Exercise 2.1. Aba Manufacturing. The decision tree with utilities and certainty equivalents is shown in Figure 4.7. In contrast to Exercise 1.4, the preferred alternative is now to manufacture only 100,000 units. This shows that the preferred alternative changes when risk attitude is taken into account.

Exercise 2.2. Kezo Systems. The analysis using an exponential utility function is shown in Figure 4.8. Recall that since we are concerned with costs, the utility function is $u(x)=1-e^{x / R}$ and $\mathrm{CE}=R \times \ln (1-\mathrm{EU})$. In contrast to Exercise 1.4, the preferred alternative is now to immediately manufacture only 100,000 units. This shows that taking risk attitude into account shifts the decision to the less risky alternative of ordering all 500,000 chips from AM.
Exercise 2.3. Kezo cancellation option
The decision tree for this problem can be simplified by some initial "side" analysis. In particular, we will look at what Kezo should do assuming that it has ordered from KEC and that an antidumping tax is imposed. The costs of sticking with KEC or switching to AM depend on the initial quantities ordered


Figure 4.6 Intermodular Semiconductor Systems (net profits in thousands of dollars)
from KEC, and the size of the antidumping tax. The table in Figure 4.9 shows the per-chip costs and benefits for each possible course of action.

From this table, we can see that if Kezo originally orders 250,000 chips from KEC, it is less costly to stick with KEC if the antidumping tax is $50 \%$, but Kezo should switch to AM if the tax is higher. On the other hand, if Kezo originally orders 500,000 chips from KEC, then it is less costly to switch to AM only if the antidumping tax is $200 \%$.

A decision tree that takes Figure 4.9 into account is shown in Figure 4.10. This tree shows that the addition of the cancellation option is sufficient to change Kezo's decision so that it orders all the chips from KEC. (Note, however, that there is a $0.6 \times(1 / 3)=0.2$ probability that Kezo will ultimately end up canceling the order and paying the cancellation fee.) In this tree, the entries of $-\$ 675$ and - $\$ 450$ under "Cost: Tax or Cancel" are the portion of the amount that was to


Figure 4.7 Aba Manufacturing decision tree with utilities (\$'s and quantities in thousands)


Figure 4.8 Kezo Systems utility analysis (dollar amounts and order quantities in thousands)
be paid to KEC that does not have to be paid if the order is canceled. This is equal to the initial purchase price minus the cancellation fee.

### 4.3 Chapter 3 Exercise Solutions

## Exercise 3.1. Aba Manufacturing

The partial decision tree for the perfect information alternative is shown in Figure 4.11. This shows that the perfect information alternative has an expected value of $\$ 200,000$. Since the best alternative without perfect information has an expected value of $\$ 100,000$, then the expected value of perfect information is $\$ 200,000-\$ 100,000=\$ 100,000$.

Exercise 3.2. Aba Manufacturing (continued)
i) The partial decision tree for the research and development (R \& D) alternative is shown in Figure 4.12. (Of course it only makes sense to do the R \& D if Aba delays manufacturing the second 100,000 PC boards.) This tree shows that the expected value of this alternative is $\$ 90,000$. Since the analysis in Exercise 1.4 showed that the alternative of manufacturing all 200,000 boards now has an expected value of $\$ 100,000$, Aba should not undertake the R \& D.
ii) The phrase "learning for certain" means that we would have perfect information, and therefore we are asked to find the expected value of perfect information about the R \& D outcome. We know from the analysis in Exercise 1.4 that the best alternative without the $\mathrm{R} \& \mathrm{D}$ is to immediately manufacture all 200,000 PC boards, with an expected value of $\$ 100,000$. Therefore, if the perfect information reports that the $\mathrm{R} \& \mathrm{D}$ will not be successful, then Aba should build all of the PC boards immediately.

On the other hand, even if the $\mathrm{R} \& \mathrm{D}$ is going to be successful, then it still might not be worth waiting to build the second 100,000 boards. This is because there is a 0.5 probability that they will be needed, and the savings in the fixed cost from avoiding the second setup ( $\$ 75,000$, including the cost of the $\mathrm{R} \& \mathrm{D}$ ) might outweigh the cost savings on that setup from doing the $\mathrm{R} \& \mathrm{D}$. But if this is true, the perfect information about the R \& D outcome will have zero value, since it will not be used. Hence, this possibility can be ignored. Taking these points into account, the partial decision tree for the perfect information alternative is shown in Figure 4.13. This shows that the perfect information alternative has an expected value of $\$ 120,000$. Since the best alternative without perfect information has an expected value of $\$ 100,000$, the value of perfect information is $\$ 20,000$.

## Exercise 3.3. Kezo Systems

This is asking for the value of perfect information about whether the antidumping tax will be imposed. The tree for this analysis can be substantially simplified by examining the tree constructed to answer Exercise 1.5. That analysis shows that ordering all 500,000 chips from KEC is preferred even with the 0.6 probability that the antidumping tax will be imposed. If it is known that the tax will

| Quantity Ordered <br> from KEC | Antidumping <br> Tax | KEC Chip <br> Price | Cancellation <br> Fee | AM Chip <br> Price | Total <br> Cost |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 250,000 | $50 \%$ | $\$ 3.00$ | $\$ 0.20$ | $\$ 3.45$ | $\$ 3.65$ |
| $"$ | $100 \%$ | $\$ 4.00$ | $"$ | $\$ 3.60$ | $\$ 3.80$ |
| $"$ | $200 \%$ | $\$ 6.00$ | $"$ | $\$ 3.75$ | $\$ 3.95$ |
| 500,000 | $50 \%$ | $\$ 2.25$ | $\$ 0.15$ | $\$ 3.45$ | $\$ 3.60$ |
| $"$ | $100 \%$ | $\$ 3.00$ | $"$ | $\$ 3.60$ | $\$ 3.75$ |
| $"$ | $200 \%$ | $\$ 4.50$ | $"$ | $\$ 3.75$ | $\$ 3.90$ |

Figure 4.9 Kezo Systems per-chip cancellation analysis


Figure 4.10 Kezo Systems cancellation decision tree


Figure 4.11 Aba Manufacturing perfect information tree (\$'s and quantities in thousands)

| R\&D | Immediate |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cost | Quantity |  |  |  |  |  |  |
|  | Manufactured | Revenue |  | R\&D | Successful? | Future <br> Quantity <br> Required | Cost |



Figure 4.12 Aba Manufacturing $R$ \& $D$ (dollar amounts and quantities in thousands)
not be imposed, then this alternative becomes even more desirable, and therefore it is the only alternative that needs to be considered if the perfect information source predicts that the antidumping tax will not be imposed. Similarly, the previous analyses show that ordering all 500,000 chips from AM has the lowest expected cost if the antidumping tax is imposed ( $\$ 1500$ versus $\$ 1625$ or $\$ 1833$ ), and therefore this is the only alternative that needs to be considered if the perfect information source predicts that the antidumping tax will be imposed.

Taking these points into account, the simple tree in Figure 4.14 can be used to analyze the expected value of perfect information. This tree shows that the expected value for the perfect information alternative is $\$ 1,200,000$. Since the expected value of the best alternative without perfect information is $\$ 1,275,000$, then the expected value of perfect information is $\$ 75,000$.


Figure 4.13 Aba Manufacturing perfect information on $R \mathcal{G} D$ ( $\$$ 's/quantities in thousands)


Figure 4.14 Kezo Systems perfect information decision tree (\$'s and quantities in thousands)
Exercise 3.4. Drug Testing
i, ii, iii) Parts i through iii of this exercise are most easily answered by setting up a probability tree and flipping it. The necessary trees are shown in Figures 4.15 and 4.16. Figure 4.15 shows the probabilities described in the exercise. The three probabilities needed to answer parts i, ii, and iii of this exercise are marked by A, B, and C, respectively on Figure 4.16. From Figure 4.16, A $=0.092+0.036=$ 0.128 . Therefore, since $\mathrm{A} \times \mathrm{B}=0.092$, then $\mathrm{B}=0.092 / 0.128=0.719$. Of course, $1-\mathrm{A}=1-0.128=0.872$, and therefore $\mathrm{C}=0.008 / 0.872=0.009$.
iv) An advantage of this drug test is that it doesn't miss many actual drug users (probability C, which is 0.009 ), but a disadvantage is that someone who tests positive only has a 0.719 probability (probability B) of actually being a drug user. Presumably other tests would be run on the person to more accurately determine whether he or she actually was a drug user, but the stigma is likely to hang around. The test accuracy numbers in this exercise are based on results of studies of actual drug tests in real-world use by testing labs. These figures have led some organizations to be cautious about introducing drug testing.


Figure 4.15 Drug testing initial tree


Figure 4.16 Drug testing flipped tree
Exercise 3.5. Intermodular Semiconductor Systems value of information
i) The decision tree for the perfect information alternative is shown in Figure 4.17. This shows that the perfect information alternative has an expected value of $\$ 85,000$. Since the best alternative without perfect information was shown in Exercise 1.6 to have an expected value of $\$ 65,000$, then the expected value of perfect information is $\$ 20,000$.
ii) The complete decision tree for this part is rather large, including all of the branches in the decision tree for the preceding exercise, as well as the experiment branch. The experiment has two possible outcomes, and following each of those possible outcomes, there will be a set of branches that has the same structure as the complete decision tree for the preceding exercise, except that the probabilities for the production cost branches will differ depending on the experimental outcome. Those probabilities must be determined, and it is probably easiest to work them out in a spreadsheet. Spreadsheet versions of the probability trees to do this are shown in Figure 4.18. Note that the only probabilities that were


Figure 4.17 Intermodular Semiconductor Systems perfect information (\$'s in thousands)

|  | B | C | D | E | F | G | H | I | J | K | L |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Prob | Cost <br> $\left(\begin{array}{ll}\text { Material) }\end{array}\right.$ | Prob | Test Result | Path <br> Prob |  | Prob | Test Result | Prob | Cost <br> Material) | Path <br> Prob |
| 2 | 0.25 | $\$ 6,000$ | 0.9 | Expensive | 0.225 |  | 0.475 | Expensive | 0.474 | $\$ 6,000$ | 0.225 |
| 3 |  |  | 0.1 | Inexpensive | 0.025 |  |  |  | 0.421 | $\$ 4,000$ | 0.200 |
| 4 | 0.50 | $\$ 4,000$ | 0.4 | Expensive | 0.200 |  |  |  | 0.105 | $\$ 2,000$ | 0.050 |
| 5 |  |  | 0.6 | Inexpensive | 0.300 |  | 0.525 | Inexpensive | 0.048 | $\$ 6,000$ | 0.025 |
| 6 | 0.25 | $\$ 2,000$ | 0.2 | Expensive | 0.050 |  |  |  | 0.571 | $\$ 4,000$ | 0.300 |
| 7 |  |  | 0.8 | Inexpensive | 0.200 |  |  |  | 0.381 | $\$ 2,000$ | 0.200 |

Figure 4.18 Intermodular Semiconductor Systems probability trees
manually entered are those shown in the boxes on this spreadsheet in cells D 2 , D5, and D7, which are shown in boxes on the spreadsheet.

The exercise does not ask for a decision tree, and drawing this without specialized decision analysis software would be tedious. You may wish to suggest to your students that they set up the calculations in a spreadsheet, which will substantially reduce the work. They can create a partial structure that represents the decision tree for the preceding exercise, and then copy this and modify the probabilities to obtain the expected value for this alternative. A possible spreadsheet is shown in Figure 4.19. (In the actual spreadsheet, this was included on the same worksheet as the probability calculations shown in Figure 4.18, and the probabilities in the "decision tree" portion of this worksheet were tied directly to the probabilities in the probability tree by equations.)

In Figure 4.19, the numbers in the columns marked "EV" are calculated by the appropriate formulas. The top part of the spreadsheet, in range C11:J25, shows the results from the solution to Exercise 1.6 for the situation without any experiment. The bottom portion of the spreadsheet, in range A27:K57, shows the calculations for the situation with the experiment. This shows that if the experimental outcome is "expensive," then the preferred alternative is to bid $\$ 7,000$, with an expected value of $\$ 26.9$ thousand, while if the experimental outcome is "inexpensive," then the preferred alternative is to bid $\$ 5,000$, with an expected value of $\$ 101$ thousand.

The overall expected value for the experiment is $\$ 66$ thousand, and hence this is somewhat more preferred than the best alternative without the experiment, which has an expected value of $\$ 65$ thousand.

|  | A | B | C | D | E | F | G | H | I | J | K |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 9 | EV | Prob | ISS Bid | EV | Prob | GE Bid | EV | Prob | $\begin{gathered} \text { Production } \\ \text { Costs } \\ \hline \end{gathered}$ | Net Profit |  |
| 10 |  |  |  |  |  |  |  |  |  |  |  |
| 11 |  |  | NO EXPERIMENT |  |  |  |  |  |  |  |  |
| 12 |  |  | 7,000 | 45 | 0.85 | 4K/6K |  |  |  | 0 |  |
| 13 |  |  |  |  |  |  |  |  |  |  |  |
| 14 |  |  |  |  | 0.15 | 8K | 300 | 0.25 | 6000 | 100 |  |
| 15 |  |  |  |  |  |  |  | 0.50 | 4000 | 300 |  |
| 16 |  |  |  |  |  |  |  | 0.25 | 2000 | 500 |  |
| 17 |  |  | 5,000 | 65 | 0.35 | 4K |  |  |  | 0 |  |
| 18 |  |  |  |  |  |  |  |  |  |  |  |
| 19 |  |  |  |  | 0.65 | 6K/8K | 100 | 0.25 |  | -100 |  |
| 20 |  |  |  |  |  |  |  | 0.50 |  | 100 |  |
| 21 |  |  |  |  |  |  |  | 0.25 |  | 300 |  |
| 22 |  |  | 3,000 | -100 |  |  | -100 | 0.25 |  | -300 |  |
| 23 |  |  |  |  |  |  |  | 0.50 |  | -100 |  |
| 24 |  |  |  |  |  |  |  | 0.25 |  | 100 |  |
| 25 |  |  | No Bid | 0 |  |  |  |  |  | 0 |  |
| 26 |  |  |  |  |  |  |  |  |  |  | Net Profit (without experiment cost) |
| 27 |  |  | "EXPENSIVE" RESULT |  |  |  | EXPERIMENT COST: |  |  | 7 |  |
| 28 | 66.0 | 0.475 | 7,000 | 26.9 | 0.85 | 4K/6K |  |  |  | -7 | 0 |
| 29 |  |  |  |  |  |  |  |  |  |  |  |
| 30 |  |  |  |  | 0.15 | 8K | 219.3 | 0.474 | 6000 | 93 | 100 |
| 31 |  |  |  |  |  |  |  | 0.421 | 4000 | 293 | 300 |
| 32 |  |  |  |  |  |  |  | 0.105 | 2000 | 493 | 500 |
| 33 |  |  | 5,000 | 10.1 | 0.35 | 4K |  |  |  | -7 | 0 |
| 34 |  |  |  |  |  |  |  |  |  |  |  |
| 35 |  |  |  |  | 0.65 | 6K/8K | 19.3 | 0.474 |  | -107 | -100 |
| 36 |  |  |  |  |  |  |  | 0.421 |  | 93 | 100 |
| 37 |  |  |  |  |  |  |  | 0.105 |  | 293 | 300 |
| 38 |  |  | 3,000 | -181 |  |  | -180.7 | 0.474 |  | -307 | -300 |
| 39 |  |  |  |  |  |  |  | 0.421 |  | -107 | -100 |
| 40 |  |  |  |  |  |  |  | 0.105 |  | 93 | 100 |
| 41 |  |  | No Bid | -7 |  |  |  |  |  | -7 | 0 |
| 42 |  |  |  |  |  |  |  |  |  |  |  |
| 43 |  |  | "INEXPENSIVE" RESULT |  |  |  |  |  |  |  |  |
| 44 |  |  | 7,000 | 48 | 0.85 | 4K/6K |  |  |  | -7 | 0 |
| 45 |  |  |  |  |  |  |  |  |  |  |  |
| 46 |  |  |  |  | 0.15 | 8K | 359.7 | 0.048 | 6000 | 93 | 100 |
| 47 |  |  |  |  |  |  |  | 0.571 | 4000 | 293 | 300 |
| 48 |  |  |  |  |  |  |  | 0.381 | 2000 | 493 | 500 |
| 49 |  | 0.525 | 5,000 | 101 | 0.35 | 4K |  |  |  | -7 | 0 |
| 50 |  |  |  |  |  |  |  |  |  |  |  |
| 51 |  |  |  |  | 0.65 | 6K/8K | 159.7 | 0.048 |  | -107 | -100 |
| 52 |  |  |  |  |  |  |  | 0.571 |  | 93 | 100 |
| 53 |  |  |  |  |  |  |  | 0.381 |  | 293 | 300 |
| 54 |  |  | 3,000 | -40 |  |  | -40.3 | 0.048 |  | -307 | -300 |
| 55 |  |  |  |  |  |  |  | 0.571 |  | -107 | -100 |
| 56 |  |  |  |  |  |  |  | 0.381 |  | 93 | 100 |
| 57 |  |  | No Bid | -7 |  |  |  |  |  | -7 | 0 |

Figure 4.19 Intermodular Semiconductor Systems experiment

