New Product Dynamics

Illustrative System Dynamics Models

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Preface

These notes provide illustrative examples of models for new product market dynamics based on "word-of-mouth" modeling concepts. Both differential equation and Vensim system dynamics model notation are presented for these models. A version of Vensim is available free for instructional use over the World Wide Web at www.vensim.com. Further support material related to system dynamics is available from my web site at www.public.asu.edu/∼kirkwood.

CHAPTER 1

Total Market Dynamics

This chapter discusses system dynamics models for the dynamics of a market for an innovative durable good. Specifically, the models address the growth of the total market for a newly developed product that is purchased once by a customer. The models presented in this chapter are useful in themselves and form basic building blocks for models of competitive markets presented in the next chapter.

The models in this chapter take a "contagion" view of the process by which customers purchase a product. The basic idea is that potential customers "catch" the desire to purchase a product from those who have already purchased the product, and therefore the rate at which the product is purchased depends on 1) how many customers have already purchased the product, 2) how many potential customers remain for the product, and 3) how persuasive the current customers are in presenting the virtues of the product when they contact potential customers. This can also be viewed as a "predator-prey" situation, where those who have purchased the product are predators on the potential customers (the "prey") and attempt to convert them to the "purchased" state. (Or you may prefer thinking of those who have purchased the product as zombies who attempt to convert the potential customers into the zombie-like state of being actual customers for the product.) A more neutral term for this type of model is "word-of-mouth," which carries the implication that positive word-of-mouth from satisfied current customers leads potential customers to make a purchase.

While these images help in remembering the structure the models, the actual situation for most products is more complex. Word of mouth models provide a relatively simple description of the complex process that occurs when a new product type is introduced. The customers who have purchased the product may not literally walk around and attempt to sell the product to potential customers, but the degree of attention paid to a new product by potential customers and those who communicate with them tends to depend on both the number of people who have bought the product and the number of potential customers. Thus, as the product becomes more widely purchased, it receives more attention in the trade and general press, and is generally more talked about by potential customers. Hence, word-of-mouth sales models do not necessarily imply that current customers literally talk directly to potential customers and "make the sale." The communication may be more in the nature of a general "buzz" about the product that is "in the air," and which makes a potential customer more likely to purchase the product.

In the final analysis, the usefulness of a model should be judged by its value in providing insights into a situation, and also by its ability to match empirical data. By those criteria, the models in this chapter, while relatively simple, are useful.

1.1 Infinite Potential Customers

We call those who have already purchased the new product of interest "Actual Customers" and those who might purchase the product "Potential Customers." In this section, we consider situations where the population of Potential Customers for the new product is so much larger than the population of Actual Customers that the number of Potential Customers remains essentially the same even when the number of Actual Customers grows.

The model assumes that for a short period of time Δt each Actual Customer converts $c\Delta t$ Potential Customers into Actual Customers, where c is a positive constant (the "conversion coefficient") that encodes how effective Actual Customers are at converting Potential Customers. If at time t there are $n(t)$ Actual Customers, each of which converts $c\Delta t$ Potential Customers into Actual Customers during the next Δt time period, then at time $t + \Delta t$ the number of Actual Customers will be $n(t + \Delta t) = n(t) + n(t)c\Delta t$.

By the usual calculus arguments, this equation can be converted to the differential equation

$$\frac{dn(t)}{dt} = cn(t) \tag{1.1}$$

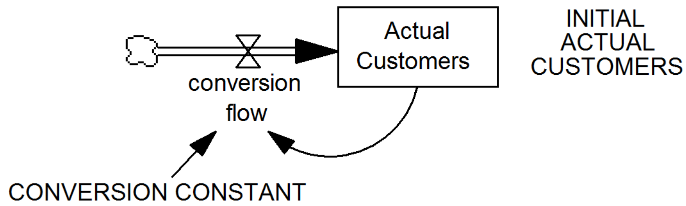
or the integral equation

$$n(t) = n_o + \int_0^t cn(\tau)\, d\tau \tag{1.2}$$

where n_o is the number of Actual Customers at time $t = 0$. The solution to these equations is well known to be the familiar exponential

$$n(t) = n_o e^{ct}, \quad t \geq 0 \tag{1.3}$$

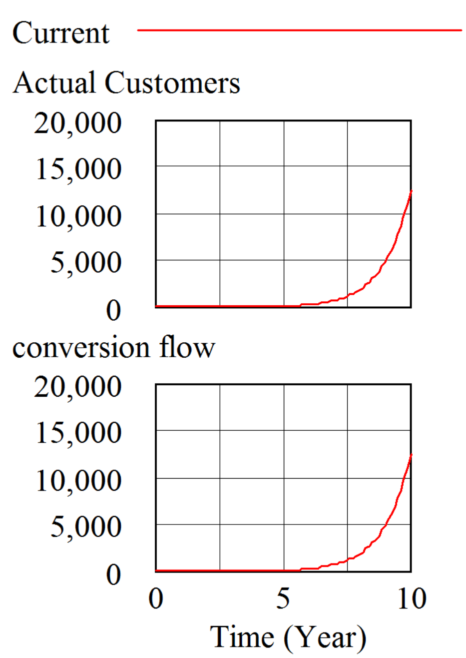
The Vensim equivalent of equation 1.2 is shown in Figure 1.1, and the model output displayed in this figure shows the familiar exponential growth pattern for both the number of Actual Customers and the flow of Potential Customers into Actual Customers.



a. Stock and Flow Diagram

```
(1) Actual Customers = INTEG (conversion flow,
                  INITIAL ACTUAL CUSTOMERS)
(2) CONVERSION CONSTANT = 1
(3) conversion flow = CONVERSION CONSTANT * Actual Customers
(4) FINAL TIME = 10
(5) INITIAL ACTUAL CUSTOMERS = 1
(6) INITIAL TIME = 0
(7) SAVEPER = TIME STEP
(8) TIME STEP = 0.125
```

b. Vensim Equations



c. Actual Customer Growth

Figure 1.1 *Infinite Potential Customers (exponential growth)*

1.2 Finite Potential Customers

In realistic markets, the exponential growth pattern shown in Figure 1.1 cannot continue forever because ultimately all the Potential Customers for a product will be converted to Actual Customers. This section considers a model that addresses this reality, and thus develops a model that more closely matches empirical data for many new product types.

Assume that the total number of Potential Customers for the product is M. Then the total number of remaining Potential Customers at time t is $M - n(t)$, where $n(t)$ is the number of Actual Customers at time t.

Further assume that c represents the conversion rate per Actual Customer when there are M Potential Customers. To complete the model specification, we must make an assumption about how the conversion rate for each Actual Customer changes when the number of Potential Customers drops as they are converted to Actual Customers. A reasonable assumption to make is that this conversion rate is linearly proportional to the total number of Potential Customers who remain at any time. Thus, for example, if half of the Potential Customers remain, then the conversion rate for each Actual Customer is $c/2$, and if a quarter of the Potential Customers remain, then the conversion rate for each Actual Customer is $c/4$.

With this assumption, the number of Potential Customers converted by each Actual Customer in a time interval Δt is $\{[M - n(t)]/M\} \times c\Delta t$, and hence the number of Potential Customers converted by all $n(t)$ Actual Customers is $n(t) \times \{[M - n(t)]/M\} \times c\Delta t$. By analogous arguments to those used to derive equation 1.1, these assumptions lead to the differential equation

$$\frac{dn(t)}{dt} = c \times \frac{M - n(t)}{M} \times n(t) \tag{1.4}$$

or the equivalent integral equation

$$n(t) = n_o + \int_0^t c \times \frac{M - n(\tau)}{M} \times n(\tau)\, d\tau \tag{1.5}$$

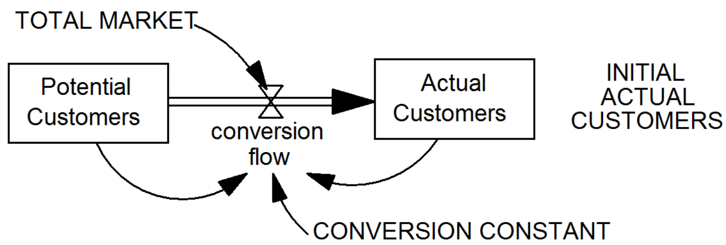
where n_o is the number of Actual Customers at time $t = 0$.

The solution to equation 1.4 or 1.5 is known to be

$$n(t) = \frac{M}{1 + [(M - n_o)/n_o]e^{-ct}}, \quad t \geq 0 \tag{1.6}$$

(Roughgarden 1998). The curves represented by equation 1.6 are called *logistic curves*.

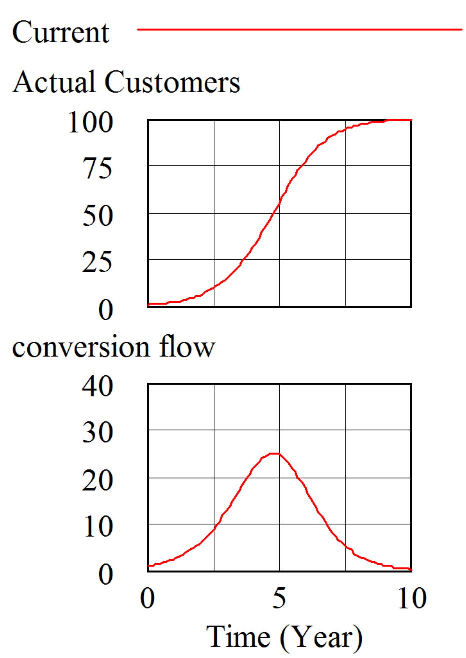
The Vensim equivalent of equation 1.5 is shown in Figure 1.2. The curve for Actual Customers as a function of time that is shown in Figure 1.2c displays the "s-shaped" pattern that is shown by all logistic growth curves, and the curve shown for conversion flow demonstrates the symmetric bell-shaped curve that is shown by all logistic conversion flows.



a. Stock and Flow Diagram

```
(01) Actual Customers= INTEG (conversion flow,
                          INITIAL ACTUAL CUSTOMERS)
(02) CONVERSION CONSTANT = 1
(03) conversion flow = CONVERSION CONSTANT
      *(Potential Customers / TOTAL MARKET) * Actual Customers
(04) FINAL TIME = 10
(05) INITIAL ACTUAL CUSTOMERS = 1
(06) INITIAL TIME = 0
(07) Potential Customers= INTEG (-conversion flow,
                 TOTAL MARKET - INITIAL ACTUAL CUSTOMERS)
(08) SAVEPER = TIME STEP
(09) TIME STEP = 0.125
(10) TOTAL MARKET = 100
```

b. Vensim Equations



c. Actual Customer Growth

Figure 1.2 *Finite Potential Customers (logistic growth)*

Properties of Logistic Curves

While this is not obvious from a casual inspection, equation 1.6 reduces to equation 1.3 in the limit as M becomes very large, and equation 1.6 is also approximately equal to equation 1.3 when $n(t)$ is much less than M. These facts can be shown by multiplying the top and bottom of equation 1.6 each by $[n_o/(M-n_o)]e^{ct}$ to yield

$$n(t) = \frac{M \times [n_o/(M-n_o)]e^{ct}}{[n_o/(M-n_o)]e^{ct}+1}. \tag{1.7}$$

For sufficiently small t and/or sufficiently large M, the first term in the denominator of equation 1.7 will be much less than 1, and hence the denominator will reduce to approximately 1. In the numerator, $M/(M-n_o)$ will be approximately 1, and hence equation 1.7 will reduce to equation 1.3.

The practical implication of these facts is that when the number of Actual Customers is much less than the number of Potential Customers, logistic growth appears to be just the same as exponential growth. However, there is a limit out there somewhere in the future!

The rate at which the number of Actual Customers grows is given by the derivative of $n(t)$, and the time t_{max} at which this is a maximum can be found by taking the derivative of the growth rate [that is, the second derivative of $n(t)$] and setting this equal to zero. The arithmetic is messy, but straightforward, and the result is

$$t_{max} = \left(\frac{1}{c}\right) \times \ln\left(\frac{M-n_o}{n_o}\right) \tag{1.8}$$

Substituting this into equation 1.6 demonstrates that $n(t_{max}) = M/2$. That is, the growth rate of Actual Customers is at its greatest value when the number of Actual Customers is equal to half of all the possible Actual Customers.

Also, it is straightforward to show by direct substitution that the growth rate of Actual Customers is symmetric in time around $t = t_{max}$. That is, $dn(t)/dt = dn(2t_{max} - t)/dt$. Furthermore, as t approaches either minus infinity or plus infinity, $dn(t)/dt$ approaches zero, while $n(t)$ approaches zero as t approaches minus infinity and $n(t)$ approaches M as t approaches plus infinity. Thus, all of the Potential Customers are eventually converted to Actual Customers.

A final property that can be useful for plotting logistic growth curves is to make the transformation $r(t) = n(t)/[M - n(t)]$. This defines $r(t)$ as the ratio of the Actual Customers to the remaining Potential Customers at time t. By direct substitution using equation 1.6, it is straightforward to show that

$$\ln[r(t)] = \ln\left(\frac{n_o}{M-n_o}\right) + ct \tag{1.9}$$

That is, $r(t)$ plots against time as a straight line on semi-logarithmic graph paper.

1.3 Determining Logistic Model Parameters

Examples of actual data displaying logistic growth patterns are shown in Farrell (1993), Fisher and Pry (1971), and Modis (1998). To use a logistic model, it is necessary to determine values for the three model parameters c, n_0, and M. It is straightforward to use standard data analysis procedures to fit the logistic curve in Figure 1.6 to existing data, and the procedure will be discussed below. However, in some applications, it is desired to use the logistic growth model to forecast future sales for a new product, and in such situations there will not be a complete times series of data to use in fitting the curve. (Putting this another way, if you already have the complete sales data for a new product, then you probably don't have much need to develop a growth model!)

The process of developing a logistic model when you do not have complete sales data is helped by the fact that all three of the model parameters have intuitive interpretations: The constant M is the total potential market for the product, c is the rate of sales per Actual Customer when the number of Actual Customers is very small relative to the total potential market, and n_o is the initial number of customers. Thus, data on early sales can be directly used to estimate c and n_o.

For example, suppose that sales for a new product are anticipated to ramp up to a relatively large quantity over a period of several years, and that sales during the first year are 1,000 units, with a growth to 3,000 the next year. Then, an estimate for n_o could be 1,000, with an estimate of c being $(3,000 - 1,000)/1,000 = 2$. That is, at a low saturation of the market, each Actual Customer generates 2 sales per year. (It should be noted that in most realistic situations growth will not be as smooth as predicted by the logistic curve, and therefore estimates of n_o and c based on early limited data should be used with caution since the data may be "noisy.")

Unfortunately, early sales data gives little information about the ultimate market M for the product. As discussed above, in the early stages of a new product, logistic growth is identical to exponential growth, and thus the early sales data does not indicate much about the size of the total market M. Estimating potential market size for a new product is a standard topic in marketing research, and the interested reader is referred to that literature for further information.

Fitting Data

In situations where there is substantial sales data available, standard curve fitting methods can be used to determine the parameters for the logistic growth curve. As an example, consider the data shown in Figure 1.3 for the market value of Microsoft and Intel as a percentage of the total market value for Microsoft, Intel, IBM, and Digital over the period 1984 through 1994 (Modis, 1998, Figure 1-4). The market value data is presumably correlated with product sales, and therefore

Year	Market Value
1984	3.0
1985	2.5
1986	4.0
1987	7.5
1988	7.0
1989	13.0
1990	17.0
1991	29.0
1992	46.5
1993	50.0
1994	49.5

Figure 1.3 *Microsoft/Intel market value (percentage, as defined in text)*

it is reasonable to think that a logistic curve of the form of equation 1.6 might fit this data.

The method of least squares can be used to fit the parameters n_o, c, and M for the logistic curve to this data. Figure 1.4 shows an Excel spreadsheet to implement this procedure. The data from Figure 1.3 is entered in column A and C of this spreadsheet. Since the logistic curve assumes a starting time of zero, in column B of the spreadsheet, the years 1984 through 1994 are transformed into years 0 through 10.

The logistic curve parameters are in range D1:D3, and the logistic equation is entered in range D5:D15. (This equation can be entered once in cell D5 and copied down, provided that absolute references are used for the logistic parameters.) Range E5:E15 shows the square of the error between the logistic curve estimate for each year in column D and the actual sales for that year in column C. Finally, cell E16 shows the sum of these squared errors.

Trial and error could be used to determine parameter values that give a reasonable fit to the data, and this was used to determine the parameter values shown in Figure 1.4. However, the Excel Solver can be used to determine a best fit (that is, the parameter values that minimize the sum of the squared errors). To do this, set Solver to minimize the Target Cell E16 by Changing Cells D1:D3. The result is shown in Figure 1.5a, and a graph of the data in column B of this spreadsheet, as well as the logistic curve in column D, is shown in Figure 1.5b.

	A	B	C	D	E
1			c =	0.4	
2			M =	55	
3			no =	3	
4	Year	Delta Year	Sales	Logistic	Sq Error
5	1984	0	3.0	3.000	0
6	1985	1	2.5	4.359	3.454201
7	1986	2	4.0	6.258	5.099791
8	1987	3	7.5	8.841	1.799486
9	1988	4	7.0	12.223	27.28467
10	1989	5	13.0	16.438	11.82303
11	1990	6	17.0	21.380	19.18827
12	1991	7	29.0	26.776	4.94402
13	1992	8	46.5	32.229	203.6654
14	1993	9	50.0	37.323	160.6993
15	1994	10	49.5	41.747	60.11448
16				SUM =	498.0726

a. Spreadsheet

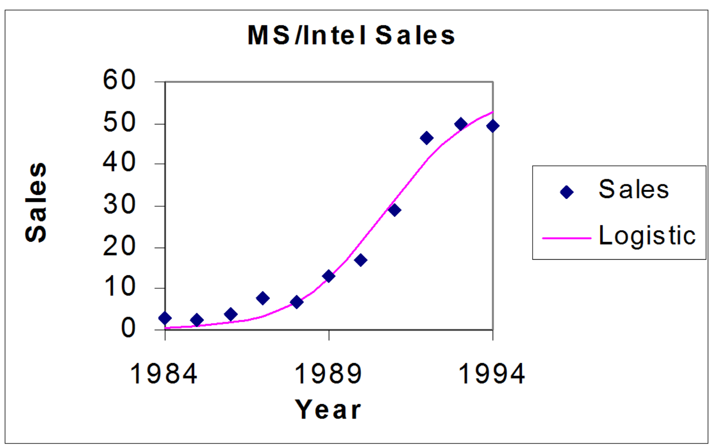
	A	B	C	D	E
1			c =	0.4	
2			M =	55	
3			no =	3	
4	Year	Delta Year	Sales	Logistic	Sq Error
5	1984	=A5-$A$5	3	=$D$2/(1+(($D$2-$D$3)/$D$3)*EXP(-$D$1*B5))	=(D5-C5)^2
6	1985	=A6-$A$5	2.5	=$D$2/(1+(($D$2-$D$3)/$D$3)*EXP(-$D$1*B6))	=(D6-C6)^2
7	1986	=A7-$A$5	4	=$D$2/(1+(($D$2-$D$3)/$D$3)*EXP(-$D$1*B7))	=(D7-C7)^2
8	1987	=A8-$A$5	7.5	=$D$2/(1+(($D$2-$D$3)/$D$3)*EXP(-$D$1*B8))	=(D8-C8)^2
9	1988	=A9-$A$5	7	=$D$2/(1+(($D$2-$D$3)/$D$3)*EXP(-$D$1*B9))	=(D9-C9)^2
10	1989	=A10-$A$5	13	=$D$2/(1+(($D$2-$D$3)/$D$3)*EXP(-$D$1*B10))	=(D10-C10)^2
11	1990	=A11-$A$5	17	=$D$2/(1+(($D$2-$D$3)/$D$3)*EXP(-$D$1*B11))	=(D11-C11)^2
12	1991	=A12-$A$5	29	=$D$2/(1+(($D$2-$D$3)/$D$3)*EXP(-$D$1*B12))	=(D12-C12)^2
13	1992	=A13-$A$5	46.5	=$D$2/(1+(($D$2-$D$3)/$D$3)*EXP(-$D$1*B13))	=(D13-C13)^2
14	1993	=A14-$A$5	50	=$D$2/(1+(($D$2-$D$3)/$D$3)*EXP(-$D$1*B14))	=(D14-C14)^2
15	1994	=A15-$A$5	49.5	=$D$2/(1+(($D$2-$D$3)/$D$3)*EXP(-$D$1*B15))	=(D15-C15)^2
16				SUM =	=SUM(E5:E15)

b. Spreadsheet equations

Figure 1.4 *Spreadsheet to determine logistic parameters*

	A	B	C	D	E
1			c =	0.728908	
2			M =	57.76107	
3			no =	0.414548	
4	Year	Delta Year	Sales	Logistic	Sq Error
5	1984	0	3.0	0.415	6.684564
6	1985	1	2.5	0.853	2.713545
7	1986	2	4.0	1.740	5.107751
8	1987	3	7.5	3.494	16.05024
9	1988	4	7.0	6.801	0.039775
10	1989	5	13.0	12.515	0.234742
11	1990	6	17.0	21.049	16.39699
12	1991	7	29.0	31.368	5.606954
13	1992	8	46.5	41.084	29.3325
14	1993	9	50.0	48.302	2.88326
15	1994	10	49.5	52.775	10.72609
16				SUM =	95.77642

a. Spreadsheet with best fit logistic curve parameters



b. Plots of data and logistic curve

Figure 1.5 *Logistic fit to Figure 1.3 data*

1.4 References

C. J. Farrell, "A Theory of Technological Progress," *Technological Forecasting and Social Change*, Vol. 44, pp. 161–178 (1993).

J. C. Fisher and R. H. Pry, "A Simple Substitution Model of Technological Change," *Technological Forecasting and Social Change*, Vol. 3, pp. 75–88 (1971).

T. Modis, *Conquering Uncertainty: Understanding Corporate Cycles and Positioning Your Company to Survive the Changing Environment*, McGraw-Hill, New York, 1998.

J. Roughgarden, *Primer of Ecological Theory*, Prentice Hall, Upper Saddle River, New Jersey, 1998.

1.5 Exercises

1.1 The following table shows the percentage of total revenue for Arthur Anderson generated by consulting activities over the period 1980 through 1995 (Modis, 1998, Figure 1-5).

Year	Revenue
1980	48.5
1981	50.0
1982	50.5
1983	51.0
1984	52.5
1985	53.5
1986	57.0
1987	60.0
1988	63.5
1989	67.0
1990	68.5
1991	71.0
1992	72.0
1993	71.5
1994	75.5
1995	76.0

A plot of this data shows that it has a logistic curve shape, except that the initial value of this curve is not at zero. This can be explained by the following hypothesis: Prior to approximately 1980, consulting services were an approximately stable portion of total Arthur Anderson revenue. Then, starting in 1980, consulting services started to be purchased by a new market (for example, computer systems consulting), and the growth of this new market followed a logistic process.

For the remainder of this exercise, assume that the "base revenue" for consulting prior to 1980 was 45 per cent of total revenue, and work with the revenue increase above this base revenue amount, rather than with the raw revenue numbers given in the table above.

(i) Follow the procedure demonstrated in Section 1.3 to determine the "best fit" parameters for a logistic curve fit to the revenue increase over the period 1980 through 1995.

(ii) Plot the actual revenue increase data shown in the table above on the same graph with the logistic curve fit determined in the preceding question.

(ii) Determine the logistic curve estimate for the ultimate percentage of Arthur Anderson total revenue that consulting will attain.

CHAPTER 2

Competitive Market Dynamics

In this chapter, the models developed in the preceeding chapter are extended to situations where there are competitors who are each attempting to sell a durable good to the same set of Potential Customers. We assume that the Potential Customers only purchase the durable good once, and therefore if a Potential Customer purchases from one of the competitors, that Potential Customer is lost forever to the other competitors.

We will use the same word-of-mouth modeling approach as in the preceding chapter for each of the competitors. That is, we will assume that Potential Customers become Actual Customers for each of the competitors in proportion to the number of Actual Customers for that competitor. We will restrict ourselves to situations with two competitors, although generalizing to more than two competitors is straightforward.

2.1 Infinite Potential Customers

First, consider the situation where the number of Potential Customers is infinitely large compared to the number of Actual Customers. Assume that for a short period of time Δt each Actual Customer of competitor number i converts $c_i \Delta t$ Potential Customers into Actual Customers for competitor number i, where c_i is a positive constant that encodes how effective Actual Customers for competitor i are at converting Potential Customers.

If at time t there are $n_i(t)$ Actual Customers for competitor i, each of which converts $c_i \Delta t$ Potential Customers into Actual Customers for competitor i during the next Δt time period, then at time $t + \Delta t$ the number of Actual Customers for competitor i will be $n_i(t + \Delta t) = n_i(t) + n_i(t) c_i \Delta t$.

This leads to the differential equations

$$\frac{dn_1(t)}{dt} = c_1 n_1(t) \quad (2.1)$$

$$\frac{dn_2(t)}{dt} = c_2 n_2(t) \quad (2.2)$$

or the corresponding integral equations

$$n_1(t) = n_{1o} + \int_0^t c_1 n_1(\tau)\, d\tau \tag{2.3}$$

$$n_2(t) = n_{2o} + \int_0^t c_2 n_2(\tau)\, d\tau \tag{2.4}$$

where n_{io} is the number of Actual Customers for competitor i at time $t = 0$. Since equations 2.1 and 2.2 (or equivalently, equations 2.3 and 2.4) are independent of each other, then $n_1(t)$ and $n_2(t)$ will not depend on each other. Hence, from the arguments in the preceding chapter, the solution to these equations are

$$n_i(t) = n_{io} e^{c_i t}, \quad t \geq 0, \quad i = 1, 2 \tag{2.5}$$

Thus, in a situation with an infinite number of Potential Customers, the two competitors can happily ignore each other and blissfully attract new customers.

2.2 Finite Potential Customers

The situation is different when there are a finite number of Potential Customers M. We will make an analogous assumption to that in the preceding chapter with respect to how Potential Customers are converted into Actual Customers for each competitor. That is, the number of Potential Customers converted by each Actual Customer of competitor i into additional Actual Customers for competitor i in a time interval Δt when there are M remaining Potential Customers is $c_i \Delta t$, and the number converted varies linearly with the remaining proportion $[M - n_1(t) - n_2(t)]/M$ of unconverted Potential Customers. [Both $n_1(t)$ and $n_2(t)$ appear in the numerator of this expression because both of the competitors are simultaneously removing customers from the Potential Customer pool.]

With this assumption, the number of Potential Customers converted by each Actual Customer of competitor i into additional Actual Customers of competitor i in a time interval Δt is $\{[M - n_1(t) - n_2(t)]/M\} \times c_i \Delta t$, and hence the number of Potential Customers converted by all $n_i(t)$ Actual Customers of competitor i is $n_i(t) \times \{[M - n_1(t) - n_2(t)]/M\} \times c_i \Delta t$.

Thus, using analogous arguments to those presented above for equations 2.1 through 2.4, these assumptions lead to the differential equations

$$\frac{dn_1(t)}{dt} = c_1 \times \frac{M - n_1(t) - n_2(t)}{M} \times n_1(t) \tag{2.6}$$

$$\frac{dn_2(t)}{dt} = c_2 \times \frac{M - n_1(t) - n_2(t)}{M} \times n_2(t) \tag{2.7}$$

or the equivalent integral equations

$$n_1(t) = n_{1o} + \int_0^t c_1 \times \frac{M - n_1(\tau) - n_2(\tau)}{M} \times n_1(\tau)\, d\tau \tag{2.8}$$

$$n_2(t) = n_{2o} + \int_0^t c_2 \times \frac{M - n_1(\tau) - n_2(\tau)}{M} \times n_2(\tau)\, d\tau \tag{2.9}$$

These equations are special cases of the Lotka-Volterra competition equations that have been extensively studied in ecology (Roughgarden 1998).

There are not closed form solutions to these equations, but before we turn to numerical solutions of them, we will generalize equation 2.7 to the following:

$$\frac{dn_2(t)}{dt} = \begin{cases} 0, & t < t_o \\ c_2 \times \frac{M - n_1(t) - n_2(t)}{M} \times n_2(t), & t \geq t_o \end{cases} \tag{2.10}$$

That is, we assume that competitor number 2 does not start selling its product until time $t = t_o$. (Of course, when $t_o = 0$, equation 2.10 reduces to equation 2.7.)

The stock and flow diagram and Vensim equations for this model are shown in Figure 2.1. For the base case analysis, the values of c_1 and c_2 are set equal, as are the values of n_{1o} and n_{2o}. The results of running this model are shown in Figure 2.2a, and these show the not very surprising result that the two competitors split the market equally.

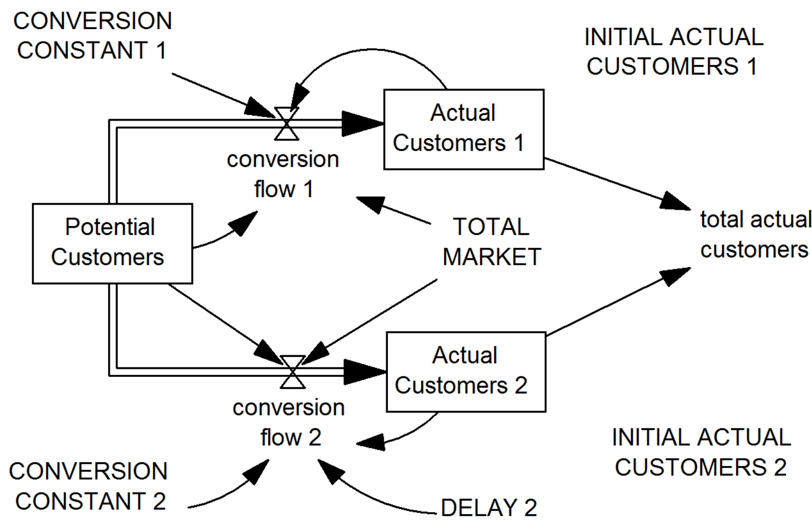
Figure 2.2b shows how things change if the relative attractiveness of competitor number 1 increases by twenty per cent to $c_1 = 1.2$. (Note that the vertical scales are different for the two graphs in Figure 2.2b.) Not surprisingly, competitor number 1 attracts substantially more customers in this situation. However, the impact of the twenty per cent increase on the final market share is more than twenty per cent. Competitor number 1 ends up with about sixty-six per cent of the market, while competitor number 2 only has about thirty-four per cent. Thus, the twenty per cent advantage in c_1 translates into a thirty-two per cent $[(66 - 50)/50]$ improvement in market share, or a $66/34 = 1.94$ ratio in market share advantage.

Figure 2.3 illustrates how other asymmetries between the two competitors impact the market share. In Figure 2.3a, the initial number of Actual Customers n_{1o} for competitor number 1 is increased from the base case of one to two. As the figure shows, this has a similar impact on market share to increasing c_1 to 1.2.

In Figure 2.3b, competitor number 2 is delayed in starting to market a product by 0.75 years (nine months), and this also has a similar impact on market share to increasing c_1 to 1.2.

2.3 Bandwagon Markets

The model of competition in the preceding section assumes that the effectiveness of Actual Customers for each competitor in recruiting Potential Customers remains constant over time in the following sense: The rate at which any particular Actual Customer recruits any particular Potential Customer is c_1/M for Actual Customers of competitor number 1 and c_2/M for Actual Customers of competitor number 2. This remains constant over time and regardless of the number of Actual or Potential Customers. (That is, the variation in the flow of new Actual Customers results from the fact that the numbers of Potential and

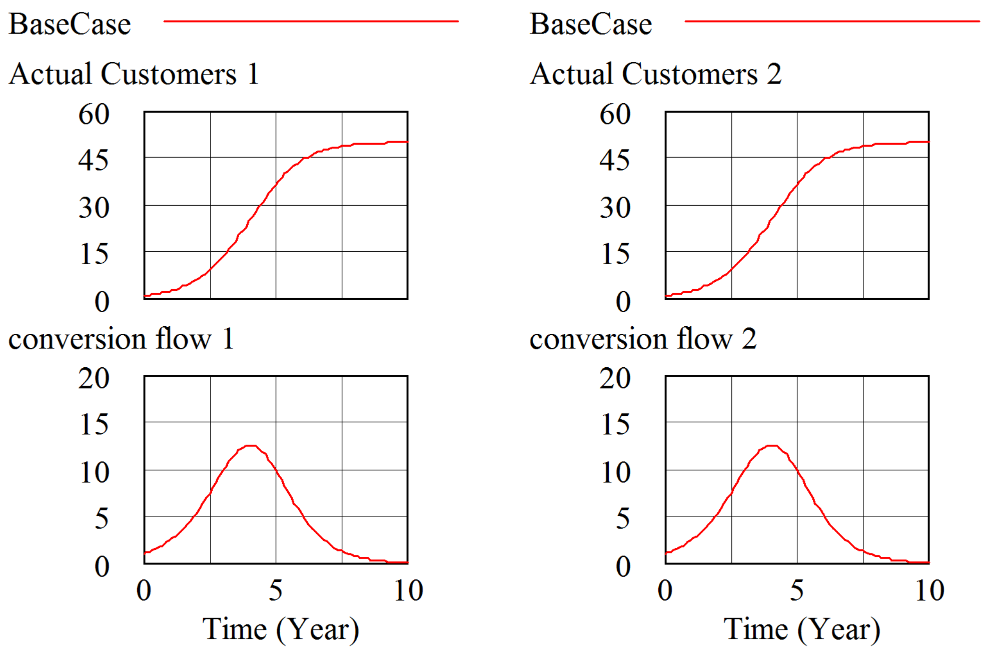


a. Stock and Flow Diagram

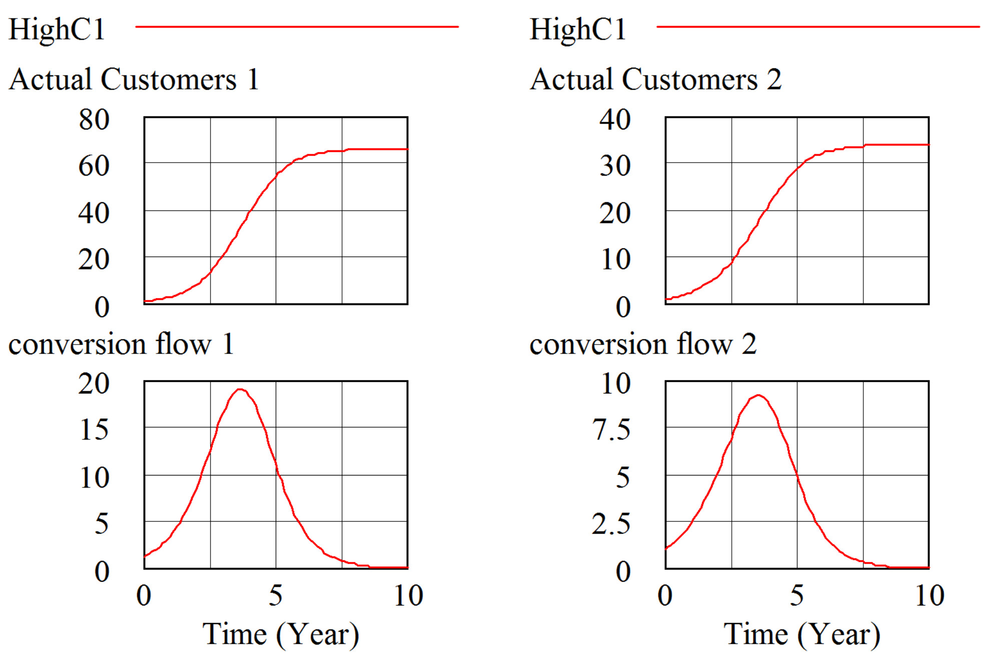
```
(01) Actual Customers 1 = INTEG (conversion flow 1,
                          INITIAL ACTUAL CUSTOMERS 1)
(02) Actual Customers 2 = INTEG (conversion flow 2,
                          INITIAL ACTUAL CUSTOMERS 2)
(03) CONVERSION CONSTANT 1 = 1
(04) CONVERSION CONSTANT 2 = 1
(05) conversion flow 1 = CONVERSION CONSTANT 1
     * (Potential Customers / TOTAL MARKET) * Actual Customers 1
(06) conversion flow 2 = STEP(CONVERSION CONSTANT 2
     * (Potential Customers / TOTAL MARKET)
     * Actual Customers 2, DELAY 2)
(07) DELAY 2 = 0
(08) FINAL TIME = 10
(09) INITIAL ACTUAL CUSTOMERS 1 = 1
(10) INITIAL ACTUAL CUSTOMERS 2 = 1
(11) INITIAL TIME = 0
(12) Potential Customers= INTEG (-conversion flow 1
       - conversion flow 2, TOTAL MARKET
       - INITIAL ACTUAL CUSTOMERS 1 - INITIAL ACTUAL CUSTOMERS 2)
(13) SAVEPER = TIME STEP
(14) TIME STEP = 0.125
(15) total actual customers = Actual Customers 1
                            + Actual Customers 2
(16) TOTAL MARKET = 100
```

b. Vensim Equations

Figure 2.1 *Competitive markets*

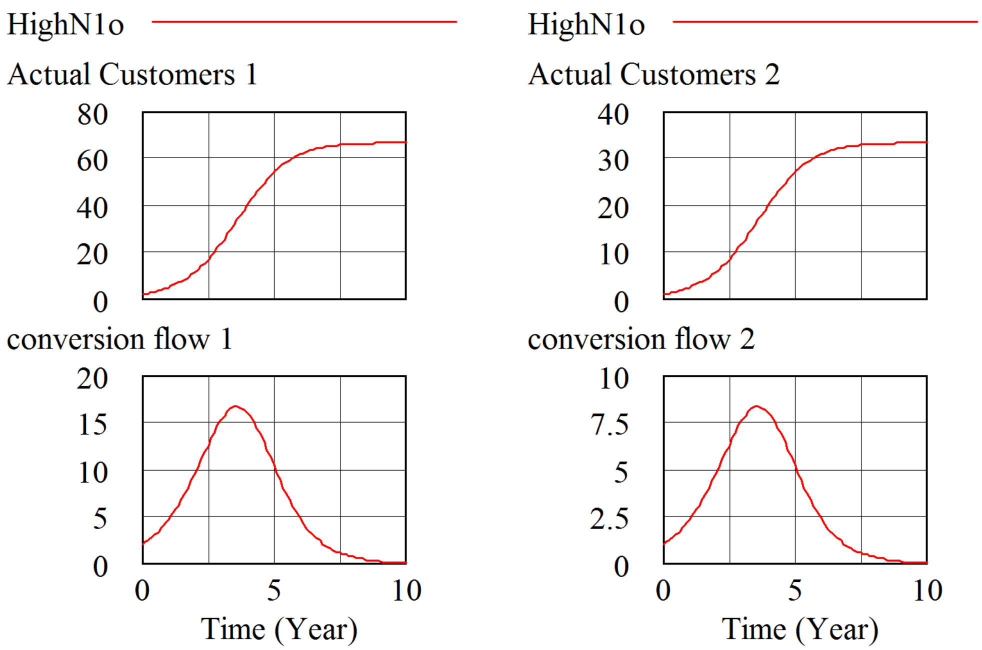


a. Base Case Results

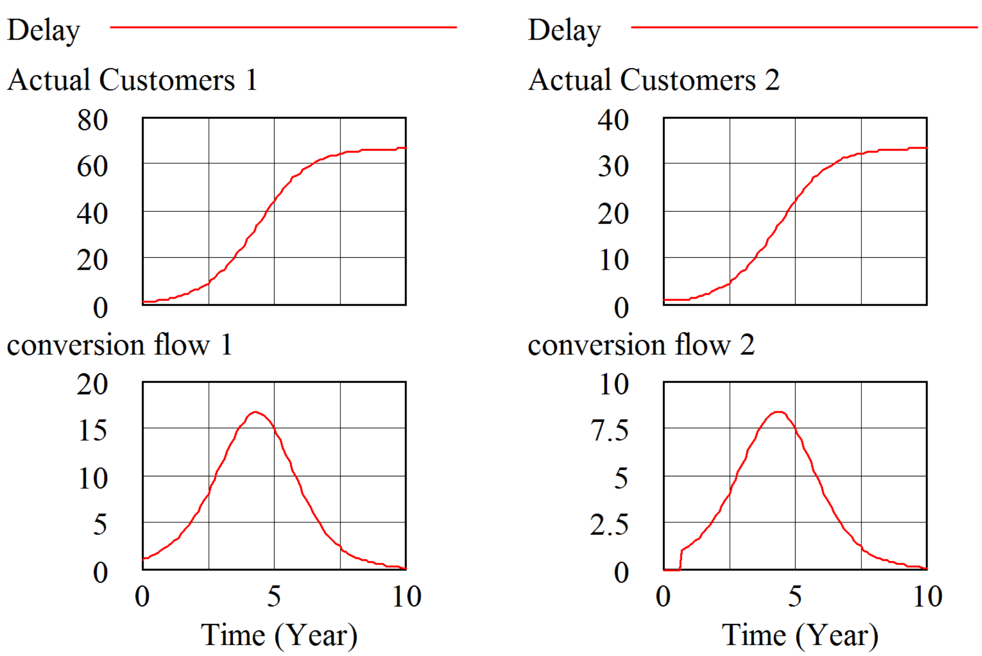


b. Sensitivity Analysis: $c_1 = 1.2$, $c_2 = 1.0$

Figure 2.2 *Competitive markets, part 1*



a. Sensitivity Analysis: $n_{1o} = 2$, $n_{2o} = 1$



b. Sensitivity Analysis: $t_o = 0.75$

Figure 2.3 *Competitive markets, part 2*

Actual Customers is varying, and not from a variation in the effectiveness of the "word of mouth" for Actual Customers of the two competitors.)

However, it has been pointed out (Arthur 1990) that in some markets the effectiveness of recruiting appears to change depending on the number of Actual Customers. There is a so-called "bandwagon effect" where the effectiveness of the recruiting increases as the number of Actual Customers for a particular competitor increases. This can be explained by saying that Potential Customers want to "go with a winner," or are afraid of purchasing a less popular product that might then be orphaned.

This phenomenon can be represented by replacing equations 2.6 and 2.10 with the following:

$$\frac{dn_1(t)}{dt} = c_1 \times f_1[n_1(t)/M] \times \frac{M - n_1(t) - n_2(t)}{M} \times n_1(t) \tag{2.11}$$

$$\frac{dn_2(t)}{dt} = \begin{cases} 0, & t < t_o \\ c_2 \times f_2[n_2(t)/M] \times \frac{M - n_1(t) - n_2(t)}{M} \times n_2(t), & t \geq t_o \end{cases} \tag{2.12}$$

In these equations, $f_1[n_1(t)/M]$ and $f_2[n_2(t)/M]$ represent the impact of the bandwagon effect. Of course, if $f_i[n_i(t)/M] = 1$, then equations 2.11 and 2.12 reduce to 2.6 and 2.10, respectively.

To investigate the impact of the bandwagon effect, assume that f_i varies linearly with respect to its argument. Specifically,

$$f_i(x) = s_i \times x + (1 - s_i/2), \tag{2.13}$$

where s_i is the slope of a straight line. Note that with this equation form, $f_i(1/2) = 1$ regardless of the value of s_i.

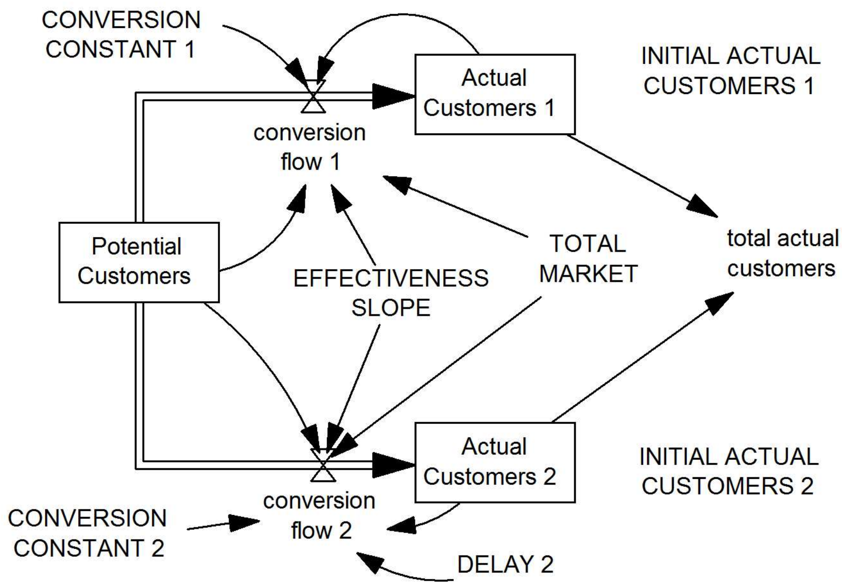
We might expect that in many cases, $s_1 = s_2$, and a Vensim model of equations 2.11 and 2.12 for this case is shown in Figure 2.4, with $s_i = 1.5$.

The results of simulation runs for this model corresponding to those shown in Figure 2.2 and Figure 2.3 are shown in Figure 2.5 and Figure 2.6. Note that with bandwagon effects, a differential advantage of one competitor translates into a greater market share advantage than in the models of the preceding section. The result is more of a "winner take all" situation.

2.4 References

W. B. Arthur, "Positive Feedbacks in the Economy," *Scientific American*, Vol. 262, No. 2, pp. 92–99 (February 1990).

J. Roughgarden, *Primer of Ecological Theory*, Prentice Hall, Upper Saddle River, New Jersey, 1998.

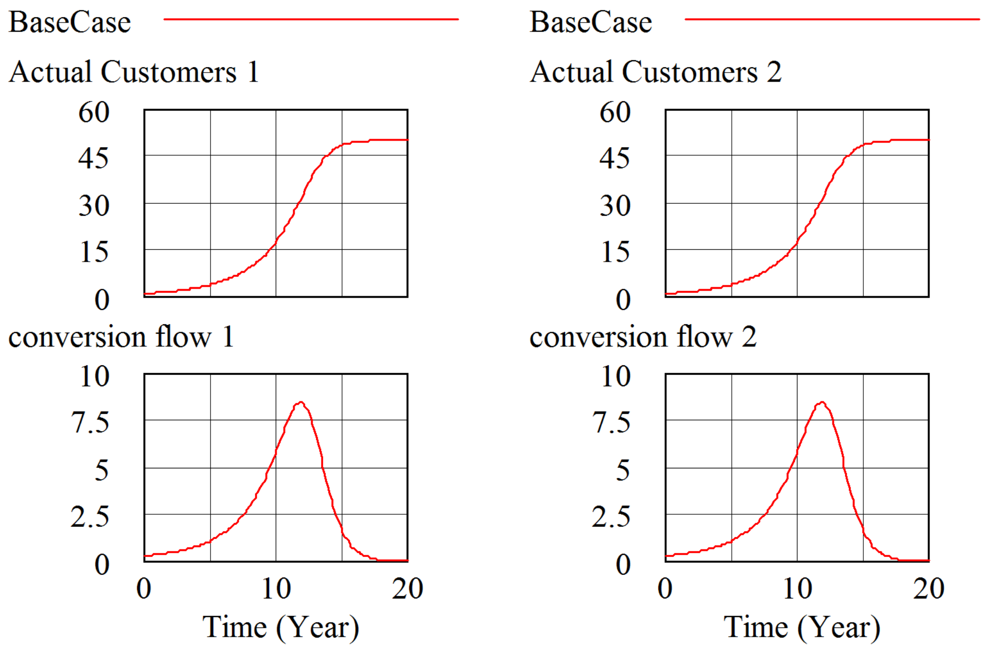


a. Stock and Flow Diagram

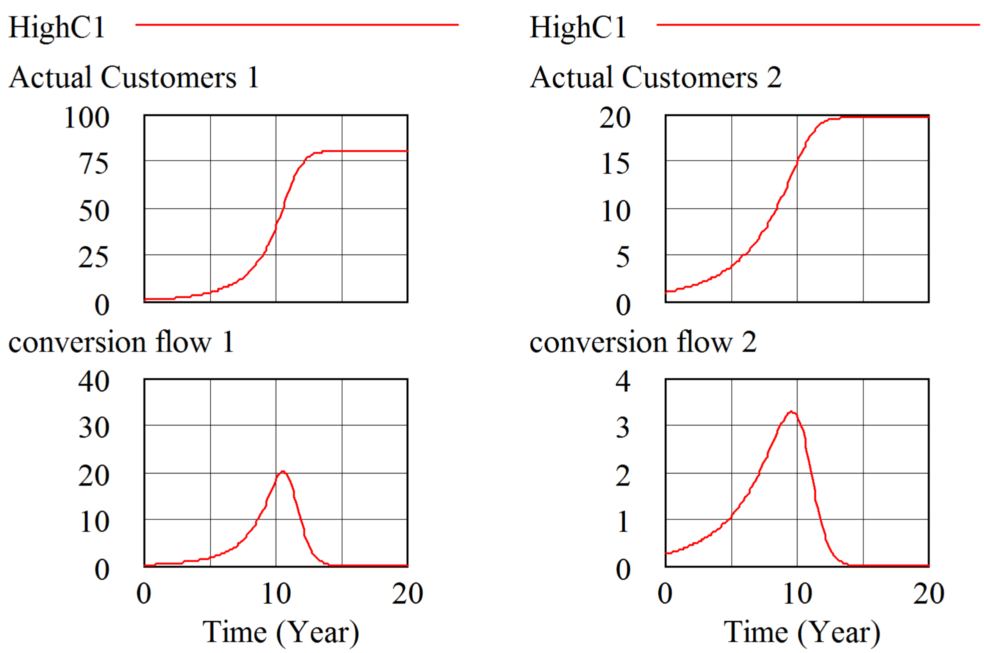
```
(01) Actual Customers 1 = INTEG (conversion flow 1,
                          INITIAL ACTUAL CUSTOMERS 1)
(02) Actual Customers 2 = INTEG (conversion flow 2,
                          INITIAL ACTUAL CUSTOMERS 2)
(03) CONVERSION CONSTANT 1 = 1
(04) CONVERSION CONSTANT 2 = 1
(05) conversion flow 1 = CONVERSION CONSTANT 1
     * (EFFECTIVENESS SLOPE * (Actual Customers 1
        / TOTAL MARKET) + (1 - (EFFECTIVENESS SLOPE / 2)))
     * (Potential Customers / TOTAL MARKET) * Actual Customers 1
(06) conversion flow 2 = STEP(CONVERSION CONSTANT 2
     * (EFFECTIVENESS SLOPE * (Actual Customers 2 / TOTAL MARKET)
     + (1 - (EFFECTIVENESS SLOPE / 2))) * (Potential Customers /
 TOTAL MARKET) * Actual Customers 2, DELAY 2)
(07) DELAY 2 = 0
(08) EFFECTIVENESS SLOPE = 1.5
(09) FINAL TIME = 20
(10) INITIAL ACTUAL CUSTOMERS 1 = 1
(11) INITIAL ACTUAL CUSTOMERS 2 = 1
(12) INITIAL TIME = 0
(13) Potential Customers= INTEG (-conversion flow 1
       - conversion flow 2, TOTAL MARKET
       - INITIAL ACTUAL CUSTOMERS 1 - INITIAL ACTUAL CUSTOMERS 2)
(14) SAVEPER = TIME STEP
(15) TIME STEP = 0.125
(16) total actual customers = Actual Customers 1
                            + Actual Customers 2
(17) TOTAL MARKET = 100
```

b. Vensim Equations

Figure 2.4 *Bandwagon effect*

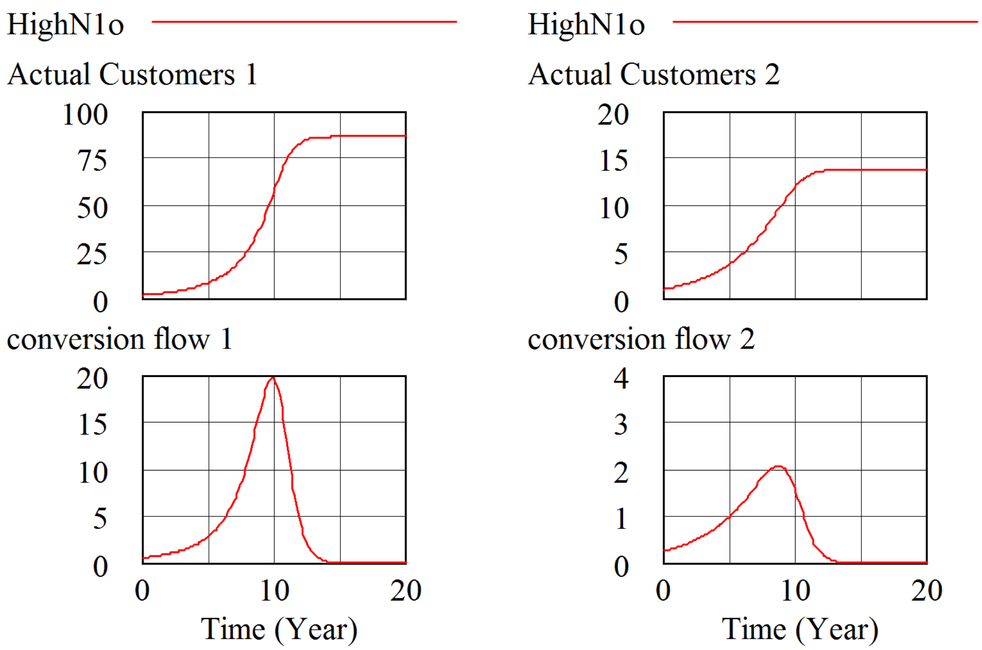


a. Base Case Results

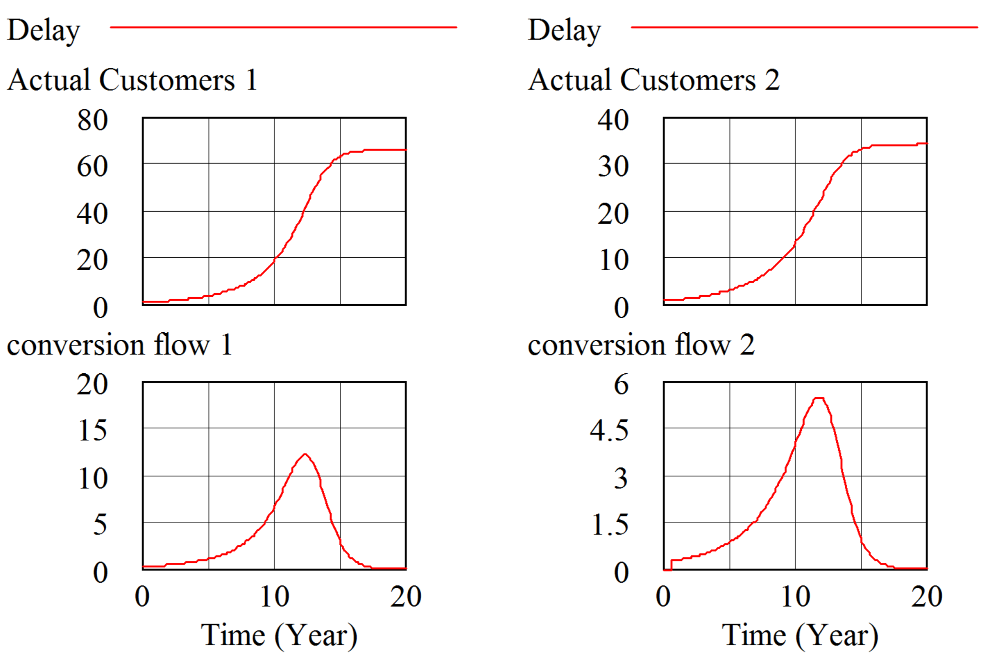


b. Sensitivity Analysis: $c_1 = 1.2$, $c_2 = 1.0$

Figure 2.5 *Bandwagon effects, part 1*



a. Sensitivity Analysis: $n_{1o} = 2$, $n_{2o} = 1$



b. Sensitivity Analysis: $t_o = 0.75$

Figure 2.6 *Bandwagon effects, part 2*