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The Content of Physics Self-Explanations

Michelene T. H. Chi
and

Kurt A. VanLehn

Learning Research and Development Center
University of Pittsburgh

Several earlier studies have found the amount learned while studying worked-out examples is proportional to the number of self-explanations generated while studying examples. A self-explanation is a comment about an example statement that contains domain-relevant information over and above what was stated in the example line itself. This article analyzes the specific content of self-explanations generated by students while studying physics examples. In particular, the content is analyzed into pieces of constituent knowledge that were used in the comments. These were further analyzed in order to trace the source of knowledge from which self-explanations could be generated. The results suggest that there are two general sources for self-explanations. The first is deduction from knowledge acquired earlier while reading the text part of the chapter, usually by simply instantiating a general principle, concept, or procedure with information in the current example statement. The second explanation is generalization and extension of the example statements. Such construction of the content of the example statements yield new general knowledge that helps complete the students' otherwise incomplete understanding of the domain principles and concepts. The relevance of this research for instruction and models of explanation-based learning is discussed.

Chi, Bassok, Lewis, Reimann, and Glaser (1989) investigated how college students learn Newtonian particle dynamics from studying four chapters of a physics textbook. During the study, students engaged in three activities

Requests for reprints should be sent to Micki Chi, Learning Research and Development Center, University of Pittsburgh, Pittsburgh, PA 15260.

(besides taking pre- and posttests): studying the prose parts of the text,¹ studying the examples presented in the target chapter, and solving problems. The results presented in the previous article analyzed primarily the data from the second and third activity, particularly focusing on quantitative analyses of how students studied the examples and how they used the examples in solving problems. This article investigates primarily what is learned from studying the examples. In addition, more data is reported about the students' profile and their study of the text.

The goal for the initial study was to understand the competence underlying problem-solving performance by examining how students acquire the knowledge they subsequently used. Thus, instead of investigating problem solving in the conventional way of analyzing the processes of problem solving per se, we instead investigated the way students study and understand worked-out examples presented in the textbook, assuming that these worked-out examples provided the major inputs from which students acquired conceptual and procedural knowledge needed for solving problems. Figure 1 provides a sample of a worked-out example taken directly out of Halliday and Resnick (1981), and used in our study.

Analyses of worked-out examples in standard textbooks makes it clear that they seldom explicate the underlying rationale for many of their component steps (see sample analyses in Chi & Bassok, 1989, pp. 259-264). In order to understand fully a worked-out example, students must provide their own explanations for the whys and wherefores of each component step in an example solution, so that the derivation of one action of the example solution from another makes sense. (Each component step or action corresponds roughly to one example line, as shown in the numbered lines of Figure 1.) We referred to these inferences generated by the students as *self-explanations* (cf. Schank, 1986).

In order to assess what kind of self-explanations students implicitly generate while studying an example, we asked them to talk aloud as they studied each line of an example. These protocols displayed their understanding in terms of the kind of self-explanations they generated and other kinds of statements, such as monitoring comments, and so forth.

The previous article (Chi et al., 1989) presents our analysis of how students studied and used examples in solving problems. There were basically three results. First, a fundamental phenomenon was identified: In learning sessions averaging 20 hr of study over several weeks, it was found that good solvers (those who subsequently were more successful at solving problems) not only provided more comments in general, but specifically, they provided more self-explanations. Self-explanations were comments that pertained to the content of physics (but were not paraphrases), as

¹Henceforth, the "text" refers to the prose parts of the text, versus the "examples."

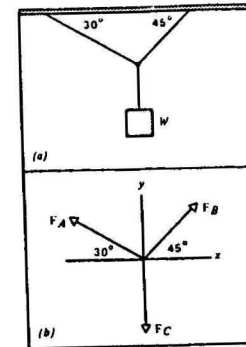


Figure 5-6 Example 5. (a) A block of weight W is suspended by strings. (b) A free-body diagram showing all the forces acting on the knot. The strings are assumed to be weightless.

1. Figure 5-6a shows an object of weight W hung by massless strings.
2. Consider the knot at the junction of the three strings to be "the body".
3. The body remains at rest under the action of the three forces shown in Fig. 5-6.
4. Suppose we are given the magnitude of one of these forces.
5. How can we find the magnitude of the other forces?
6. F_A , F_B , and F_C are all the forces acting on the body.
7. Since the body is unaccelerated, $F_A + F_B + F_C = 0$.
8. Choosing the x - and y -axes as shown, we can write this vector equation as three scalar equations:
 9. $F_{Ax} + F_{Bx} = 0$,
 10. $F_{Ay} + F_{By} + F_{Cy} = 0$
 11. using Eq. 5-2. The third scalar equation for the x -axis is simply:
 12. $F_{Ax} = F_{Bx} = F_{Cx} = 0$.
 13. That is, the vectors all lie in the xy plane so that they have no z components.
 14. From the figure we see that
 15. $F_{Ax} = -F_A \cos 30^\circ = -0.866F_A$,
 16. $F_{Ay} = F_A \sin 30^\circ = 0.500F_A$,
 17. and
 18. $F_{Bx} = F_B \cos 45^\circ = 0.707F_B$,
 19. $F_{By} = F_B \sin 45^\circ = 0.707F_B$.

FIGURE 1 The strings example. The data from this figure are from *Fundamentals of Physics* (p. 70) by D. Halliday & R. Resnick, 1981, New York: Wiley. Copyright 1981 by Wiley, Reprinted with permission.

opposed to monitoring statements that commented on their states of comprehension, or other miscellaneous statements (e.g., about mathematical manipulations). In contrast, poor solvers (those who were less successful at solving problems) often did not explain the steps of the example exercises to themselves. Notice that self-explanations are somewhat different from *elaborations*, a term used in the traditional psychological literature. Self-explanations are generated in the context of learning something new, whereas elaborations generally refer to the use of existing knowledge to embed or embellish a piece of information in a larger context

so that it is more memorable. Hence, the issue in self-explanation is how the explanations are generated, whereas the issue in elaborations is the appropriateness or facilitativeness of a particular context or elaboration.

The second result of the Chi et al. (1989) study was that good solvers were quite accurate at assessing their own comprehension. That is, they knew when they understood or failed to understand parts of an example, whereas poor solvers almost always thought they understood. The third result was that, as a result of effective study, good solvers used examples in a different way than poor solvers: The good solvers used the examples as prompts and a source of reference, whereas the poor solvers re-read them in a rote search for an exact solution procedure (i.e., the good solvers used the examples as a reference source from which they can check the accuracy of a particular equation, etc., whereas poor students used the examples as a template by which to solve the problems analogically).

The form and content of self-explanations were described briefly and qualitatively in the previous work. It was noted that self-explanations tended to take the following forms: They either refined and expanded the conditions under which an action was taken (an action corresponds to a step in the example solution) or extrapolated the consequences of an action beyond those stated in the example statements. In addition, some self-explanations provided a goal for a set of actions, and others explained the meaning of a set of quantitative expressions. We tend to believe that all self-explanations take this form, irrespective of who generated them. In contrast, other researchers who have replicated this result have contrasted the self-explanations of good and poor students as more semantic oriented or deep versus syntax oriented or shallow (Ferguson-Hessler & de Jong, 1990; Pirolli & Bielaczyc, 1989).

Among the three studies that have obtained fundamentally the same set of results (Chi et al., 1989; Ferguson-Hessler & de Jong, 1990; Pirolli & Bielaczyc, 1989), only one set of analyses pertained to the content of self-explanations. A preliminary analysis found that about a quarter of the self-explanations related an action in the example to principles (e.g., Newton's second law) and concepts stated in the text. Although students initially had only partial understanding of the principles prior to studying the examples, generating such principle-related self-explanations seemed to further enhance the good solvers' (but not the poor solvers') understanding of the principles (Chi et al., 1989). This tantalizing result suggests that it is possible for students to generate explanations without complete understanding of the domain principles, and moreover, that the processes of generating self-explanations further induced greater understanding of the domain principles.

This article presents additional analyses that clarify the role of self-explanations in the development of deeper understanding of principles,

concepts, and problem-solving procedures. We address four questions: (a) Do both the good and the poor solvers indeed have initially only partial understanding of the physics principle from having studied the text?; (b) What is the content of self-explanations?; (c) From what knowledge are self-explanations generated (i.e., are they generated via deductions from complete understanding of the principles gained from the text, or are they inferences derived directly from the example lines)?; and finally, (d) How might self-explanations subsequently enhance one's understanding of the domain principles (i.e., is such greater subsequent understanding achieved by inductive generalization across the self-explanations)? Although our analyses may not give conclusive answers to any of these questions, they suggest possible mechanisms for how self-explanations can be generated and what role they play in enhancing students' understanding of domain principles and concepts.

DESIGN OF THE STUDY

This section provides a brief description of the original study, including some data that were not previously reported. This allows the reader to comprehend this article without necessarily referring to the original study.

The study consisted of five components: a pretest, reading from text, studying examples, solving problems, and posttest (Figure 2 outlines the five phases of the study). A pretest consisted of the Bennett's Mechanical Ability Test and Conceptual Questions, some of which were taken directly from McCloskey (1983). The Bennett's Mechanical Test assessed students' mechanical ability. The Conceptual Questions assessed students' naive intuitive knowledge about the materials covered in the study. In the second

- I) Pre-Test:
 - A) Bennett's Mechanical Ability Test
 - B) Conceptual Questions
- II) Read Text:
 - A) Chapters 1-3
 - B) Answer Definitional, Qualitative and Quantitative Questions (pertaining to each chapter)
 - C) Chapter 5 (first part, up to the 3 examples)
 - D) Answer Definitional and Qualitative Questions Only
- III) Study 3 Examples:
 - (Taken from Chapter 5)
- IV) Solve Problems:
 - A) 12 Isomorphic Problems
 - B) 7 End-of-the-Chapter Problems
 - C) Subprocedure Problems
- V) Post-Test (Identical to Pre-Test)

FIGURE 2 Five phases of the original study.

text-reading phase (using Halliday & Resnick, 1981), students studied the necessary background materials, covering the topics of measurement, vectors, and motion in one dimension, using chapters 1 to 3. At the end of each chapter, they were asked to answer a set of definitional questions, a set of qualitative questions (usually labeled as *think* questions in physics textbooks and not requiring the manipulations of equations), and a set of quantitative problems. Students were not permitted to proceed to the next chapter until they had reached the criterion of answering these questions and solving the quantitative problems correctly. If they could not reach the criterion, they were told to re-study the chapter. After reaching criterion, students studied the first parts of chapter 5, an introduction to mechanics, including Newton's three laws. Chapter 4 was omitted because it was not directly relevant to the materials covered in chapter 5. After reading the text part of chapter 5, students did not solve any quantitative problems (as they did after reading chapters 1 to 3) until Phase IV of the study. They simply answered the definitional questions and qualitative questions. While reading the text, they were also told to highlight the parts that they thought were relevant and important.

After students had studied the prose material from the text of chapter 5 to the criterion specified, they studied three worked-out examples taken directly from chapter 5. (Figure 1 is an actual example of a worked-out solution referred to as Example 5 in the text.) Each worked-out solution represented a type of problem. There were three types: an inclined plane problem, a pulley problem, and a strings problem. (Diagrams corresponding to each type of problem are shown in Figure 3, Panels a, b, c.) Students were asked to say whatever they were thinking about as they read each statement of the example solution, and their protocols were taped.

Students then solved three sets of problems (Phase IV of Figure 2). The first set consisted of 12 isomorphic problems, with four problems corresponding to each of the three types of example. The four problems isomorphic to each example type were specially designed so that they had decreasing degrees of similarity to the example solutions. Figure 4 depicts the four levels of isomorphic problems corresponding to the strings example as shown in Figure 1. A second set of seven problems were taken directly from the end of chapter 5. They were considerably more difficult than the isomorphic problems. The combined inclined-plane-pulley problem, shown in Panel d of Figure 3, is an example of one. Verbal protocols were taken at all problem-solving sessions.

A third set of problems assessed the procedural subskills that students needed in order to be able to solve problems successfully. We wanted to tap students' subskills without embedding them in a problem-solving context. Examples of procedural subskills were: finding all the forces in a given situation (e.g., two blocks, one sitting on top of another), finding the

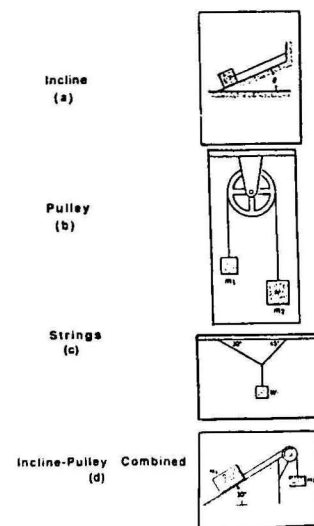


FIGURE 3 Diagrams depicting the three types of examples, and a type of the end-of-the chapter problem.

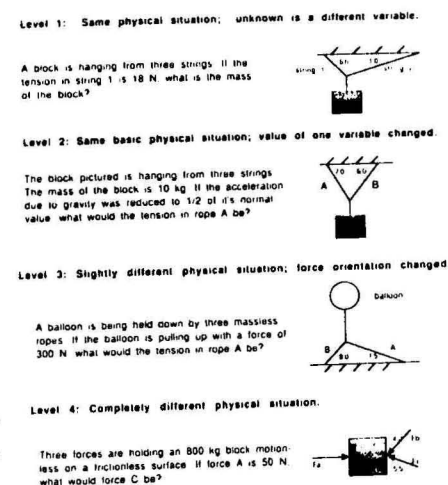


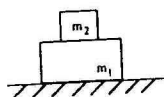
FIGURE 4 The four levels of isomorphs corresponding to the example shown in Figure 1.

reference frame given to vector forces, and decomposing forces. Figure 5 illustrates two of these subprocedure problems, tapping the subskills of finding all the forces on each body, and finding the reference frame. Finally, a posttest, identical to the pretest was again administered.

Ten students were selected from campus advertisements. Physical science majors were excluded. No further restrictions applied. We aimed for a nonhomogeneous group of undergraduates in terms of their profile, such as grade point average (GPA), Scholastic Aptitude Test (SAT) scores, and prior physics and mathematics courses taken.

Find all Forces on Each Body

On the following problems, please draw in all the forces acting on the masses. If ropes appear in a problem, draw in the forces acting on the knot where the ropes meet.



Find the Reference Frame

Please choose what you think would be the best reference frame for each of the following diagrams, draw that reference frame onto the diagram.



FIGURE 5 Examples of two sub-procedure problems.

RESULTS

If students are ranked according to their successes at solving the isomorphic problems (the black solid bar of each student's score in Figure 6), one can see that there is a continuous range of performances among the nine students. (The poorest student's data had to be discarded because this student did not solve any problems correctly.) The successes at solving the isomorphic (solid) and end-of-chapter (blank) problems (both shown in the first column of Figure 6) correlated highly with how well they solved the subprocedure problems (hatched lines). The correlation between the first and second columns in Figure 6 is 0.82.

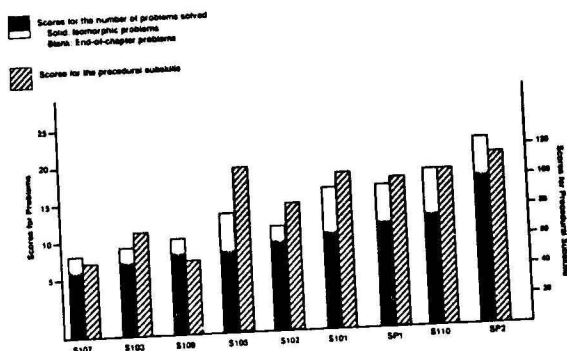


FIGURE 6 Each student's successes at solving problems ranked according to their scores for the isomorphic problems.

TABLE 1
Mean Scores for the Number of Problems Correctly Solved
By the Good and Poor Solvers

	Isomorphic Problems	End-of-Chapter Problems	Subprocedure Problems (out of 115 points)
Good solvers	10.6*	4.9*	106**
Poor solvers	7.6	2.7	64

* $p < .05$. ** $p < .01$.

Despite the continuous range, the students' successes on problem solving varied significantly. Such variability can be highlighted by a contrastive analysis, using a median split on their problem-solving scores. In order to obtain two groups of equal size, the good solvers and the poor solvers, the data of student SP1 was excluded (because she was the only student who had taken college physics).² Table 1 shows that all the differences in the number of problems correctly solved by the good and the poor solvers were significant at either the .05 or the .01 level.

The students' overall problem-solving successes were traced to five possible sources: (a) their entering abilities, (b) their naive intuitions, (c) what they encoded from studying the text, (d) what they learned from studying the examples, and (e) what they learned from solving problems. Because naive intuitions are difficult to score in a deep way, that issue is addressed in a separate article. What students learn during problem solving (learning from doing) is also addressed elsewhere. The remaining three factors (a, c, and d) are discussed next, with the greatest emphasis on the content of self-explanations.

Entering Abilities

Students could have come into the study with different entering abilities, so that better or high-ability students generated more explanations. Table 2

²In the contrastive analysis reported in Chi et al. (1989), we excluded the data of S102 because she was the intermediate one. In this analyses, we chose to exclude the data of SP1 from the good solvers because SP1 is the only student who had one semester of college physics. This factor became important here because we are more concerned with the issue of the source of knowledge from which self-explanations could be generated. Thus, it is important to control for the amount of prior exposure to physics instruction. S102 is therefore included as a good solver, along with S101, S110, and SP2. One could argue on the basis of the scores for the end-of-the-chapter problems that the roles of S102 and S105 should be reversed. The pattern of the data, however, will remain the same. Henceforth, all contrastive analyses carried out in this article involve only eight students (excluding SP1), and all other noncontrastive analyses include data from all nine students.

TABLE 2
Entering Abilities

	High School Physics	College Physics	GPA	Bennett Mechanical Ability Test (out of 29 points)	
				Pretest	Posttest
Good solvers	Yes	No	2.9	22.3	23.0
Poor solvers	Yes	No	3.4	23.3	22.8

shows that there are basically no apparent differences between the good and the poor solvers' prior knowledge and abilities, as gathered from their profiles, such as whether or not they have had high school or college physics, their overall GPA, and their scores on the Bennett's Mechanical Ability Test. All of them have taken high school physics, with grades varying from A to C among both the good and the poor solvers. If anything, the poor solvers had better overall GPA (3.4) than the good solvers (2.9). None of them have taken college physics, assuming we exclude the data of SP1. But most importantly, there are no differences between the good and the poor solvers in the Bennett's Mechanical Ability Test, either during pre- or posttest. Bennett's Mechanical Ability Test has been used as a sensitive discriminator of students' ability to solve mechanical type of problems, as required in physics (Hegarty, Just, & Morrison, 1988). It basically reflects students' spatial abilities. Not finding differences among our sample of good and poor solvers ensures us that we can attribute any differences we do find to factors other than this source of ability differences. Thus, on the basis of test and grade scores, there appears to be no blatant differences in the entering abilities of the good and poor solvers. This is why we are careful to always refer to these students as good and poor solvers or learners, and never as good and poor students (as did Pirolli & Bielaczyc, 1989; as well as Ferguson-Hessler & de Jong, 1990), because the latter implies a difference in ability, effort, motivation, and so forth.

Encoding Text Information

How well students can encode information from the text can be another possible source responsible for students' ultimate greater generation of self-explanations. This can be captured in two ways: the frequency with which students re-read the text in order for them to master the materials to the criteria that we had set (of being able to answer definitional, qualitative, and quantitative questions pertaining to that chapter, see Figure 2, Phase

II-B), and the number of lines that were highlighted for the text part of chapter 5, to see whether all the students have basically encoded the same materials. (We are assuming that highlighted lines are the parts that were declaratively encoded, although they may not be remembered or accessed.) Basically, there were no differences between the good and the poor solvers in the frequency with which they re-read the text part of the background materials (for all the chapters, see Columns 1 and 2 of Table 3). Unexpectedly, there was a difference in the number of lines highlighted for the text part of chapter 5: The poor solvers actually highlighted a greater number of lines (145) than the good solvers (88). Of these highlighted lines, 64 of them overlapped: That is, the good and the poor solvers highlighted the same 64 lines, which means the good solvers highlighted an additional 24 lines that were distinct from the additional 81 lines highlighted by the poor solvers. The overlapping highlighted lines tended to be ones that the chapter already emphasized, such as by italics. The fact that the poor solvers did highlight a greater number of nonoverlapping lines suggest that at least they were paying attention to the task of studying the chapters.

An alternative way to analyze the declarative encoding of the text part of chapter 5 (reported in Chi et al., 1989) is to see how accurately they recited the definition of Newton's principles, as assessed by the definitional questions given at the end of the text-reading phase of chapter 5 (see Figure 2, Phase II-D). Accordingly, each of Newton's principles was decomposed into several components (e.g., Chi, Glaser, & Rees, 1982, Study 6) and the presence of these components in the students' definitions of the principles was scored. For instance, Newton's second law can be decomposed into four propositions: (a) It applies to one body; (b) it involves all the forces on the body; (c) the net force refers to the vector sum of all the forces acting on the body; (d) $F = ma$ or the magnitude of F equals mass times the acceleration, and the direction of the acceleration is the same as F . Newton's three laws can thus be decomposed to a total of 12 components (3 for the first law and 5 for the third law, see Table 6 presented later). Using such an analysis, there was no difference whatsoever between the good and the poor

TABLE 3
Measures of How Well Students Have Encoded From Test

	Number of Times Re-read: Chapters 1-3	Number of Times Re-read: Chapter 5	Number of Nonoverlapping Highlighted Lines: Chapter 5 (64 overlapping)	Number of Components of Newton's Laws Recited (12 components)
Good solvers	1.3	1.1	24	5.5
Poor solvers	1.5	1.3	81	5.5

solvers in the number of components of Newton's principles encoded: Both groups recited a mean of 5.5 components out of 12³ (see Table 3, column 4). One could interpret this analysis to mean that both the good and the poor solvers had partial understanding of the Newton's laws initially, prior to studying the examples. Although explicit recitation of the principles (as in a definition) is a strict criterion for assessing understanding (Greeno & Riley, 1987), it is possible that this analysis may not be sensitive to a possible difference in the good and poor solvers' implicit understanding of principles (Gelman & Greeno, 1989). However, further analysis to be reported next support our conclusion that both the good and the poor solvers did have incomplete understanding of the principles after reading the text but prior to studying the examples.

Learning From Studying the Examples

The previous sections basically show that there were no apparent differences between the good and poor solvers in their entering abilities or in how well they have encoded the text (either in terms of the frequency of re-reading or in how well they could recite Newton's principles), and yet the students ultimately had differential problem-solving abilities, so they must have learned differently from studying the worked-out examples, especially because they generated a significantly different amount of self-explanations. Thus, the critical locus of difference lies in how students studied the examples. The remaining parts of this article focus on understanding the content of self-explanations. We focus on how such content can be generated from initial partial understanding of the principles by determining the source of the knowledge from which self-explanations could possibly be generated. We begin by first reviewing and extending two results reported in Chi et al. (1989), one concerned with the form of self-explanations, and the other with their principle-related content. A new analysis is then presented focusing on the content of the entire set of self-explanations.

Form of Self-Explanations

The first of the two results from (Chi et al., 1989) concerns the form or structure of self-explanation (as opposed to their content, which is discussed later). Before presenting the analysis, let us review the basic data collection and analyses procedures. The example-studying protocols of each student were initially coded into three broad categories: (a) self-explanations, (b)

³Note that this result is taken from Chi et al. (1989), therefore the contrast excludes student S102.

monitoring statements, and (c) miscellaneous (which included paraphrases and mathematical manipulations). Figure 7 shows a sample of a segment of protocols and their respective categories. Self-explanations are any comments that pertain to physics content but are not paraphrases. Thus, by definition, self-explanations infer some additional pieces of information, regardless of how minute they are. Each self-explanation is counted as a unit if it refers to the same idea, regardless of how many lines of protocol it took to express the idea. (Details of unitization of the protocols are described in Chi et al., 1989.) As shown in Chi et al. (1989), good solvers generated a significantly greater number of self-explanations (15.3 per example) than poor solvers (2.8 per example). (Note that if we exclude the data of SP1 in this current analyses and replace them by the data of student S102, as explained in footnote 2, then the number of self-explanations generated by the good solvers decreases to 12.8, because SP1 is a better problem solver, thereby a better self-explainer, than S102.) The correlation, for instance, between the number of problems solved and the number of explanations generated was 0.87 (across nine students). There were no significant differences in the number of monitoring statements made between the two groups or in the number of miscellaneous statements (which included paraphrases and mathematical manipulations).

Self-explanations took one of four forms: They either (a) combined a set of action steps into a subgoal, or inferred strategies or plans of the solution; (b) expanded or refined the preconditions of an action; (c) explicated additional consequences of an action statement; or (d) re-expressed mathematical statements in terms of meaningful physics interpretations. Table 4 shows how many cases of each kind of self-explanation were given by the good and poor solvers (excluding the data of SP1). Expanding on the preconditions and explicating the consequences of actions (which can be generally viewed as providing the justifications of actions)

FIGURE 7 Transcript of a segment of an example-studying protocols and their codings. READ indicates the student is reading an example line. The protocol lines were coded either as a self-explanation (EXPL), monitoring statement (MONIT), or miscellaneous statements (MISC) such as paraphrases, mathematical manipulations, and other uncodable ones. (Figure 5-9b mentioned in the second READ statement, can be found in Halliday and Resnick, 1981, p. 72.)

READ	<u>if the acceleration (okay) if the acceleration of m₁ (the little body) is a, then the acceleration of m₂ must be a.</u>
EXPL:	"Okay, but in that statement they're saying that the acceleration has to be equal. Which would mean that nothing is moving."
MONIT:	"I don't understand that. But..."
READ:	<u>The forces acting on m₁ and m₂ are shown in Fig. 5-9b. In which T represents the tension in the string.</u>
MONIT:	"Okay, so here, this is interesting."
EXPL:	"In both cases, the tension force is directed upward from the body."
EXPL:	"And the only other force is umm, mass times gravity in both cases."
MONIT:	"Okay."
:	:
MISC:	"So now I'll combine the gravities on this side m ₂ ... minus m ₁ g equals... so now I will combine the accelerations on this side... m ₂ minus m ₁ g equals..."

TABLE 4
The Number of Self-Explanations of Each Kind

	Good	Poor	Total
Strategic, plan like, or goal oriented	9	4	13
Expand or refine preconditions	46	24	70
Explicate consequences of actions	78	19	97
Give meaning to quantitative expressions	16	2	18
Uncodable	5	1	6

constitute the largest categories (accounting for 81% to 86% of the total self-explanations generated), for both the good and the poor solvers. The fact that the structure of self-explanations is the same for both the good and poor solvers confirms our belief that all self-explanations take the same form, regardless of who generated them. Thus, at a global level, if we view the provision of justifications as a form of teleological understanding, then we could take this qualitative characterization of self-explanations to mean that the good solvers understood the examples better, merely because they exhibited and generated a greater number of self-explanations, and not because their self-explanations took a deeper or more semantic-based characteristic. However, the question remains for us to identify in what ways these self-explanations reflect greater understanding.

Content of Self-Explanations: Components of Newton's Laws

This section first reviews a second analysis from Chi et al. (1989) and then extends it. It is the only analysis carried out previously in the literature on the content of self-explanations. As just mentioned, about a quarter of the self-explanations were related to the principles stated in the text (31% for the good and 25% for the poor solvers). Using the same analysis of the components of Newton's laws that the definitions referred to (as shown in Table 3, column 4), the self-explanations of the good solvers were found to contain 8.5 components of the principles whereas the poor solvers' self-explanations referred to 5.75 components (see Table 5, column 1). Comparing this result to the number of components of principles recited prior to studying the examples (column 2 of Table 5 or column 4 of Table 3), as assessed in the definitional questions administered at the end of reading chapter 5 (Figure 2, Phase II-D), this represented a gain of 3 additional components for the good solvers while self-explaining, whereas the poor solvers gained a mere one quarter of a component (see column 3 of Table 5). Note that these gains are distinct additional components, irrespective of how many times they were mentioned.

TABLE 5
Number of Components of Newton's Laws Referring to Self-Explanations (Out of 12)

	Example Studying	Text Studying	Gains
Good solvers	8.5	5.5	3.0
Poor solvers	5.75	5.5	0.25

Note. Contrasts in this table exclude the data of S102.

There are two possible interpretations for the source of this gain in the number of additional components articulated by the good solvers in the self-explanation. The *on-line* interpretation assumes that the new components were constructed while studying the examples, based on information presented in the examples themselves, coupled perhaps with common sense knowledge. Thus, the on-line interpretation assumes that self-explanations were not generated by using prior knowledge that was encoded from reading the text. This is the interpretation that we believe is correct, and it is consistent with the finding of defining only 5.5 components of the principles initially.

The alternative *access* interpretation (Brown, 1982; Chi, 1988), on the other hand, assumes that self-explanations are deduced from prior knowledge that was encoded from the text. This interpretation thus holds that the content of the example statements elicited what was previously encoded but not properly accessed by the definitional task. This access interpretation assumes that the good solvers had complete understanding of the principles after reading the text, but this understanding was not displayed by the definitional task. This may be because explicit recitation of Newton's principles, as required by the definitional task, was too stringent and did not assess implicit understanding of principles (Gelman & Greeno, 1989). Thus, the good solvers could have had complete implicit understanding of the principles after all, even though the definitional assessment task only showed them to possess 5.5 components of the principles. Reading the example statements somehow triggered this understanding and allowed the good solvers to generate principle-based self-explanations by accessing and applying this knowledge to the specific situation described in the example.

There is one piece of relevant evidence that can partially refute this access interpretation. If access indeed were the bottleneck to the good solvers' mentioning of the components during the definitional assessment, then one would expect a more-or-less random distribution of the components accessed during the definition recitation because of the relatively neutral retrieval context, whereas the distribution of components accessed during example studying should be nonuniform and centered around the compo-

nents addressed by the examples. Table 6 shows that the former did not occur and the latter did. Table 6 depicts the 12 components of Newton's three laws and whether the students mentioned each component while reciting them (column 1) or studying examples (column 2). Column 1 fails to show the uniform distribution of component retrievals during definitional assessment that would be predicted by a random access interpretation. Instead, all eight students tended to recite correctly the syntactic aspects of the laws. For instance, they all defined the second law in terms of the algebraic formula ($F = ma$), and the implications of it, namely that the magnitude and direction of F are proportional to those of a (Component 7 in Table 6, column 1). None of the eight students initially articulated the notion of net force (the sixth component) or the idea that it involves all the forces on the body and not the forces the body exerts on other objects in the environment (Component 5). A similar nonuniform distribution can be seen for the components of the first and the third laws. However, the good

solvers' gains of three components (Table 5, column 3) reflected their subsequent articulation of these deeper components (deeper simply means that the meaning of these components is not well explained by the exposition of the laws in the text of the chapter—see the following). Thus, what the good solvers gained is the understanding of the deeper concepts embedded in Newton's first and second law; in particular, it is the net force acting on the body that determines the body's acceleration (Components 2, 5, and 6, see column 2 of Table 6). This suggests that both groups of solvers learned the same superficial components from the text initially, but subsequently the good solvers, through the act of explaining the examples, learned the other deeper components.

Not only can the access interpretation be refuted by the foregoing argument, but moreover the on-line interpretation is further supported by the fact that the examples put greater emphasis on certain components than the text does and that these components are the ones gained by the good solvers. For instance, one component of Newton's second law that is gained by good solvers is that the forces mentioned in the law include all and only the forces acting on the body. This is only implicit in the text's definition of the law, which is: "In this equation [referring to $F = ma$] F is the (vector) sum of all the forces acting on the body, m is the mass of the body, and a is its acceleration" (Halliday & Resnick, 1981, p. 62). This text statement does not emphasize that one should sum all and only the forces acting on the body. Although later references to the law always use the phrase "forces on the body," they never emphasize the component in question, and students interpret *on* merely as a preposition. In contrast, this emphasis is present in Example 6, which says,

Because we wish to analyze the motion of the block, we choose **ALL** the forces acting ON the block. Note that the block exerts forces on other bodies in its environment (the string, the earth, the surface of the incline) in accordance with the action-reaction principle: These forces, however, are not needed to determine the motion of the block because they do not act on the block. (emphasis in original, Halliday & Resnick, 1981, p. 71)

Students who did not encode this component of the law while reading the text might well do so as they were studying the examples while exhaustively self-explaining each line.

The on-line interpretation requires that the worked-out examples mention all the components gained by the good solvers, albeit sometimes in implicit or example-specific form. Table 6 shows that this requirement is met, as most of the components gained by the good solvers are mentioned in the examples. (Column 3 shows how many times those components are mentioned in the three examples and column 2 shows the number of

TABLE 6

A Comparison of Components Articulated by the Students Versus Those Mentioned by the Example Statements

Components Mentioned	By 8 Students During Definitional Assessment	By 8 Students During Example Studying	By Lines in Examples
First law			
1. Applies to one body	1	0	0
2. No net Force	3	5	1
3. Uniform motion at rest unaccelerated inertia	8	5	4
Second law			
4. Applies to one body	2	0	0
5. Involves ALL forces ON the body	0	3	3
6. Net force is the vector sum	0	3	2
7. $F = ma$ magnitude of $F = ma$ direction of $F =$ direction of a	8	6	2
Third law			
8. Reaction opposite in direction	8	1	0
9. Reaction equal in magnitude	7	0	0
10. Two general bodies	3	1	1
11. Forces exerted on each body by the other	3	3	1
12. Action-reaction is along a straight line	0	0	0

Note. This table excludes the data of S102.

students who mentioned a given component of the laws in the self-explanations, irrespective of how many times it was mentioned per student. This requirement is shown in the greater correspondence between columns 3 and 2 than there is between columns 3 and 1.) The best way to illustrate this greater correspondence of columns 3 and 2 than columns 2 and 1 is to compare Components 8 and 9 to 5 and 6. Neither of Components 8 and 9 were mentioned explicitly in the example statements (see column 3). Yet, these were recited during definitional assessment (column 1) obviously because the definitional assessment procedure explicitly requested an explanation of the third law. However, their infrequent occurrences in the self-explanations attest to the notion that students tend to explain what the example lines mention. Conversely, because the example lines mentioned Components 5 and 6, these were gained by the students while self-explaining (column 2) but not articulated in the definitional assessment (column 1). Thus, self-explanations tend to reflect new knowledge constructed on-line from the example statements.

If we accept the on-line interpretation, then the next question obviously is: What is the source of knowledge from which self-explanations are generated? We present the argument next that examples contain new information that is not presented in the text, and thereby students often can generate specific inferences on the basis of what is encoded from the example line, coupled with either common sense knowledge or what knowledge is gained from prior lines within the same example or prior examples. Two cases illustrating direct inference from the example line coupled with common sense prior knowledge are presented:

1. Making a direct inference on the basis of what is encoded from the example line: The student (SP2) reads:

Consider the knot at the junction of the three strings to be the body,⁴

The student explains:

"Why should this be the body? I thought W (the block hanging from the three strings, see Figure 1) was the body . . . So they refer to the point as the body and now this—what was W before is a force C."

The student never knew that a point can be a body. Her normal definition of a body requires solid objects such as a block. The example line therefore presented new information that was not discussed in the text, from which she inferred that bodies do not necessarily have to be solid objects.

2. Making an inference on the example line using common sense knowledge: Student (S101) reads:

⁴Example lines are underscored, self-explanations are in quotes.

Consider two unequal masses connected by a massless string which passes over a frictionless and massless pulley, as shown in Figure 5-9a. (See Figure 3b for the diagram referred to here.)

The student explains:

"I was just thinking, if that was like this, this would just be at the bottom and that'd be stuck up there somewhere, 'cause it's a greater mass."

What the student has done is run his mental model of the situation, which predicts that the heavier object will pull the lighter object all the way up until the lighter object gets stuck at the top of the pulley. Thus, the self-explanation uses the common sense knowledge that heavier object will pull the lighter object continuously until a physical obstruction (the pulley) is in the way. So he does not understand why the diagram depicts the two masses as dangling in mid-air.

In order to gain deeper insight into how self-explanations might be generated (by determining the source of the knowledge from which they can be generated), the next section presents a more comprehensive analysis of their content, covering the entire set of self-explanations, whereas the analysis presented in this section examined only the subset which pertains to Newton's laws. Because our goal in this analysis was to see the extent to which principle-related self-explanations contained the same or different components of Newton's laws from the definitional assessments of the principles prior to studying examples, the analysis took a template approach. The next analysis is based solely on what the students said as they studied the examples, without determining in advance of the analysis which aspects of the content (e.g., the 12 components of the Newton's laws) to look for. Thus, the next analysis represents a bottom-up approach.

Content of Self-Explanations: Constituent Analysis

In order to understand what is contained in all of the self-explanations (not just the subset related to principles), the content of each self-explanation is captured by characterizing it in terms of *constituent knowledge pieces*. Constituent knowledge pieces are propositions that are abstracted from each self-explanation. Sometimes a self-explanation can contain two or more pieces of constituent knowledge. As an analysis tool, characterizing self-explanations in terms of constituent knowledge pieces allows us to see more clearly what their content is. *Knowledge* may be a bit of a misnomer, because some constituent knowledge pieces are incorrect. However, the students think the pieces are correct. The pieces are their knowledge even though scientifically some are false beliefs.

There are several additional pragmatic advantages for recoding self-

explanations into constituent knowledge pieces. First and most important, it gets rid of any contaminations of the basic result (of good solvers generating more self-explanations) due to the good solvers possibly being more verbose. By recoding self-explanations into constituent knowledge, we are basically treating each piece of constituent knowledge as a distinct piece of knowledge, irrespective of how many separate self-explanations generated it. Second, it normalizes the self-explanations into a standard form, so that linguistic variations among and within students can be overlooked. Third, it provides a format by which we can later trace the extent to which constituent knowledge is used and acquired over problem solving protocols. Finally, analyzing self-explanations in the form of constituent knowledge offers the possibility of converting them into instructable rules.

For the 9 students in this study, there were a total of 258 self-explanations. Of these, 173 (around 67%) were codable into constituent knowledge. Uncodable ones were either nonsensical (i.e., we could not understand what the student was trying to say), mathematical deductions, or had no specific constituent knowledge content (e.g., "oh, that's the x-axis"). From such an analyses, we obtained a composite list of 110 distinct pieces of constituent knowledge, across nine students. The reduction of 258 self-explanations to 110 pieces of constituent knowledge suggests that our original criterion of identifying any comment that pertained to the domain of physics as a self-explanation was liberal, whereas the constituent analysis is more stringent in that it weeds out much of those self-explanations which had incomplete or no meaningful physics content.

Several interesting properties about the content of self-explanations became apparent from such a constituent analysis. First, a taxonomy emerged. Basically, constituent knowledge can be categorized into four types, reflecting (a) knowledge of systems (e.g., the Atwood's machine, consisting of two masses hanging from a pulley), (b) technical procedures (e.g., summing vectors), (c) principle-related constituent knowledge (having to do with the balance of forces as it relates to motion), and (d) knowledge of concepts (e.g., weight, forces). (Note that technical procedures exclude comments that pertain primarily to algebraic manipulations. Those were not classified in the original study as self-explanations, thus would not have been included in this analysis.) Figure 8 provides a sample of the four kinds of constituent knowledge and the self-explanations associated with each.

Because constituent analysis rules out any concerns that one might have about the verbosity of the good solvers, we can now reexamine the data to see whether the good solvers generated a greater number of distinct pieces of constituent knowledge. Table 7 shows the mean number of constituent knowledge pieces the good and poor solvers generated for each category. Not only did the good solvers generate a significantly greater total number

1) Systems:
Self-Expl. "... since they're connected by a string that doesn't stretch, ummm, their accelerations are going to be opposite but equal."
Constit. K. *If the string on an Atwood's machine doesn't stretch, then the acceleration of the masses will be equal and opposite.*

2) Technical procedures:
Self-Expl. "It's because it's going in a negative direction it points, they give it a negative value it's below the ... X axis."
Constit. K. *If the projection of a force (or any other vector) onto an axis points in the negative direction of that axis, then the magnitude of that component will be negative.*

3) Principle-related:
Self-Expl. "Okay, so they got that by, they said that these are all the forces acting on the block and it's not accelerating and their forces must equal zero."
Constit. K. *If a body is not accelerating, then the sum of all the forces acting on the body must equal zero.*

4) Concepts:
Self-Expl. "The frictional force moving in the direction opposite to the movement."
Constit. K. *If there is a force of friction, then it will act in the opposite direction of movement.*

FIGURE 8 Four kinds of constituent knowledge and the self-explanations associated with each.

TABLE 7
Mean Number of Distinct Pieces of Constituent Knowledge
Generated By the Good and Poor Solvers for Each Category

	<i>Systems</i>	<i>Technical Procedures</i>	<i>Principles</i>	<i>Concepts</i>	<i>Total</i>
Good solvers	3.50	9.00	6.00	6.25	24.75*
Poor solvers	1.00	4.74	0.75	2.00	8.49

*Main effect significant at the .05 level.

of distinct constituent knowledge pieces than the poor solvers (24.75 vs. 8.49), but the good solvers generated significantly more constituent knowledge pieces uniformly across every category ($p < .05$). Thus, the good solvers have simply acquired more pieces of meaningful physics-related knowledge in the process of self-explaining. Note that these results—contrasting the constituent knowledge pieces—are quite pronounced even though a "weaker" good solver is used in this analysis (S102) than in the Chi et al. (1989) analyses (SP1).

A second general property about constituent knowledge is that none of them are quantitative. When mathematical propositions appear, they are qualitative in that they mention only equality, relative magnitude and equality with zero (cf. de Kleer & Bobrow, 1984). The textbook and examples, on the other hand, tend to use standard quantitative mathematics even when qualitative statements would do. This invites the conjecture that self-explanations involve translating precise quantitative mathematical assertions that are difficult to think with into more easily used qualitative

mathematical assertions (recall that precise quantitative statements in the protocols were not classified originally as self-explanations).

A third property of constituent knowledge is that some of them (those concerning systems and concepts) seem to be knowledge that can be gained primarily from the content of example lines (evidence in support of this conjecture is presented next). This may explain why learning from examples are particularly facilitative to problem-solving successes. That is, the general finding in the literature shows that students prefer and rely on examples to learn to solve problems. For example, Anderson, Greeno, Kline, and Neves (1981) claimed that students, in learning to program in Lisp, spend a considerable amount of time studying examples and they often commit knowledge of worked-out examples to memory. Pirolli and Anderson (1985) found that 18 of their 19 novice subjects relied heavily on analogies to examples in the early stages of learning to program recursion. All the nine students in Chi et al. (1989) referred to the examples while solving problems, although there were significant differences both qualitatively as well as quantitatively in the way good and poor solvers used the examples. Reder, Charney, and Morgan (1986) also found that the most effective manual for instructing students how to use a personal computer are those that contain examples. LeFevre and Dixon (1986) actually found subjects to ignore the written instruction and use only the examples when learning a procedural task. Similar reliance on examples over text is also found in the classroom. VanLehn (1986) showed that 85% of the systematic errors collected from several thousand arithmetic students could be explained as deriving from some type of learning from examples. Thus, clearly some greater understanding can be gained from studying examples in general, perhaps because they explicate systems and concepts.

In some cases, studying examples can even replace text and instructor's presentation. Zhu and Simon (1987) showed a clear advantage (a 3-year course can be reduced to a 2-year math curriculum) if students are given only examples and problems to work on, as opposed to the standard instruction with a text and instructor's presentations. We hypothesize that this is possible because the examples supply the much needed technical procedures for solving problems. Notice in Table 7 that technical procedures constitute the largest category of constituent knowledge acquired by both the good and the poor solvers.

This result, which shows the acquisition of a large number of technical procedural knowledge, also explains why it is that the success at solving problems correlated so highly with the success at solving subprocedure problems (see Figures 5 and 6 again). The transfer from subprocedure problems to actual problems is determined by the amount of overlap in the technical subskills required for both kinds of problems.

We turn now to a discussion of the specifics of each category of

constituent knowledge. Our discussion centers on the source from which a self-explanation is generated, which fuels our later speculations on what mechanism(s) might generate the constituent knowledge of each category. We are ultimately interested in whether self-explanations are constructed on-line using primarily information given in the example statement or deduced from information presented earlier in the prose parts of the text.

Systems. The first category of knowledge to be discussed concerns *systems*. Systems refer to constituent knowledge that conveys a mental model of an entire configuration (e.g., a block on an inclined plane) or components of a configuration. That is, it requires references to relations between or among components (e.g., two masses connected by a string). There were 20 pieces of constituent knowledge in this class. Three examples of self-explanations which were coded as constituent knowledge about systems are shown in Figure 9. In Figure 9, the first example shows three self-explanations generated by Students S109, S102, and S101 after reading the same line from Example 8 about pulleys that were all recoded into the same piece of constituent knowledge, as shown.⁵

Because the examples contain the first mention of system configurations such as inclined planes and pulleys, it is likely that the constituent knowledge about systems was not known prior to studying examples. In order to estimate the proportion of the constituent knowledge pieces that could possibly be inferred during the studying of the examples, we coded each student's self-explanation that yielded a piece of systems constituent knowledge individually, as to whether the self-explanation was probably inferred (a) from the example line, (b) from common sense knowledge, (c) from knowledge acquired from previous examples or previous lines within the same example, or (d) from text materials. The first three categories are considered nontext sources. This coding was intentionally a conservative estimate of nontext sources because we coded any self-explanation as being a deduction from text if there was any mention of that information in the text, ignoring possible failures of memory access, failures of encoding from the text in the first place, or failures to understand what was presented in the text. Using this analysis, of all the self-explanations that were classified as system related, 95% were coded as inference from a nontext source versus 5% as plausible deduction from textbook. This suggests that at most 5% of the constituent knowledge learned about systems could have been learned a priori from knowledge gained from the text.

Technical procedures. The next category to discuss contains 32 pieces of constituent knowledge about the more technical subskills of

⁵Constituent knowledge are presented in italics.

physics problem solving, such as choosing a reference frame and projecting force vectors onto the axes. (See Figure 10 for three examples. Notice that the third self-explanation was recoded into four pieces of constituent knowledge, which included those recoded from the first and second self-explanations.) As in the case of the systems knowledge, these topics simply did not come up in the textbook chapter (although vector addition outside the context of forces was discussed in chapter 1). Because the examples were the first place to mention them, we can assume that most of them were learned while studying the examples.

Because constituent knowledge about technical procedures are instantiations of general knowledge about vectors, we cannot rule out the possibility that they were deduced from specializing general knowledge about vectors (acquired from chapter 1), even though it seems obvious that they were sometimes constructed from information presented in the example lines. See for example, the first self-explanations in Figure 10 in which Student SP1 did not initially know that vectors can be decomposed. However, in most cases (including the other two shown in Figure 10), both deductions and constructions are plausible, so the protocols unfortunately do not allow us to discriminate between these two processes. Hence, this category of constituent knowledge was not traced to the source of knowledge responsible for generating the self-explanations.

Principle related. This category of 25 pieces of constituent knowledge consists mostly of fragments and qualitative versions of Newton's laws.

- 1) READ In the special case where $m_1 = m_2$ we obtain $a = 0$. (pulley example, see Fig. 3b)
- Self-Expl. "Cause it wouldn't go anywhere." (S108)
- Self-Expl. "Whenever the tension equals the mass, both masses, then the umm... there isn't any movement." (S102)
- Self-Expl. "They'll just all there." (S101)
- Constr. K. If two equal masses are on an Atwood's machine, then they will be at rest.
- 2) READ The pull of the string on the block will be removed. And the resultant force will no longer be zero. (inclined plane example, see Fig. 3a)
- Self-Expl. "That this block will be at rest even without the rope, before? And we said that there would be, but it's impossible. Most probably because there is no friction here. OK, so it has to move." (SP2)
- Constr. K. If a block is on a frictionless inclined plane and nothing is holding it, then it has to move.
- 3) READ If the acceleration of m_1 is a , the acceleration of m_2 must be $-a$. (pulley example, see Fig. 3b)
- Self-Expl. "Umm, this would make sense, because, since they're connected by a string that doesn't stretch, umm, their accelerations are going to be opposite but equal." (S110)
- Constr. K. If the string on an Atwood's machine doesn't stretch, then the acceleration of the masses will be equal and opposite.

FIGURE 9 Constituent knowledge for systems and the corresponding quotes of self-explanations in response to example lines read. The student who uttered each self-explanation is identified in the parenthesis following each self-explanation.

Figure 11 lists a couple of examples of self-explanations and their associated constituent knowledge codings. Our criterion for deciding whether a self-explanation is principle related is based on whether it refers to motion and the balance of forces. The first example in Figure 11 illustrates the responses of two students (SP1 and SP2) after reading the same example line. These two self-explanations were recoded into two separate pieces of constituent knowledge, both related to Newton's first law, as shown. Note that when the premise of the proposition is unclear, we enclose a generic premise—given the current situation—in parentheses. The second example, on the other hand, shows responses given by two students which were coded into the same constituent knowledge. Note also that the sample of principle-related constituent knowledge illustrated were all taken from self-explanations of the good solvers (see Figure 6 for identification of the good solvers); this is because the poor solvers acquired very few principle-related constituent knowledge pieces (see Table 7, column 3 again).

As one might expect, consistent with the data presented in Table 6, the majority of the constituent knowledge pieces in this category are variants of Newton's first law, such as:

- If all the forces are equal to zero, then the body will not move.*
When a body does not move, all the forces acting on it have to be equal to zero.
If a body does not move in a dimension, then the sum of forces are

- 1) READ Choosing the x and y axes as shown, we can write this vector equation as three scalar equations. (string example, see Fig. 1)
- Self-Expl. "Ummm, I guess always before when I thought of force vectors I just thought of them as going in a particular direction, I forgot about them having x components and y components and being, ummm, broken down into them." (SP1)
- Constr. K. Forces can be expressed as the sum of their components.
- 2) READ With this choice of coordinates, only one force must be resolved into components in solving the problem. (inclined plane example, see Fig. 3a)
- Self-Expl. "I see that because it's the only one that wouldn't be on one of those axes." (S103)
- Constr. K. If a force lies entirely on an axis, then it does not have to be resolved into components.
- READ It is convenient to choose the x -axis of our reference frame to be along the incline and the y -axis to be normal to the incline. (inclined plane example, see Fig. 3a)
- Self-Expl. "Basically it looks like they are going to split up these three forces into their respective components, and it's very, ummm, wise to choose a, a reference frame that's parallel to the incline, parallel and normal to the incline, because that way you'll only have to split up one, the other forces are already, component vectors for you." (S110)
- Constr. K.
- 1) Forces can be expressed as the sum of their components.
 - 2) Choose a reference frame such that the least number of forces need to be resolved.
 - 3) If a force does not lie entirely on an axis, then resolve it into its scalar components.
 - 4) If a force lies entirely on an axis, then it does not have to be resolved into components.

FIGURE 10 Constituent knowledge for technical procedures and the corresponding quotes of self-explanations in response to example lines read.

equal to zero in that dimension.

If the body remains at rest, then the forces acting on it must cancel out.

When the acceleration of a body is zero, the forces acting on it balances.

or variants of Newton's second law:

If the forces acting on a body do not sum to zero, then the body will move.

If a body is accelerating, then its net force must not be zero.

When a body has acceleration, then it must experience a net force.

If a body has a net force, then it is accelerating.

If a body is not at rest, then the net force will not equal zero (incorrect).

What is the source of these self-explanations? Our previous analyses, based on the components of Newton's laws, suggested that the self-explanations could not have been deduced from prior understanding of the text. This conclusion was based on two findings: First, that the principle-related components gained while self-explaining seemed to be constructed from the information presented in the example statements themselves (because of the correspondence between columns 3 and 2 in Table 6); second, that the students seemed to have incomplete understanding of the principles after reading the text (because of the systematic distribution of components articulated during definitional assessment, see column 1 of Table 6). On the other hand, we cannot rule out the plausibility that most of these propositions can in principle be simply deduced from knowing the syntax of the formula $F = ma$, and knowledge of the relationship between quantitative mathematics. For instance, the constituent knowledge piece that states *If a body is accelerating, then the net force on that body is not zero* can be deduced from the formula, $F = ma$, by first applying the following inference schema:

If $V_1 = S \cdot V_2$, where V_1 and V_2 are vectors, and S is a nonzero scalar, then whenever V_2 is nonzero, V_1 is nonzero.

Then you can apply the common sense knowledge that when an object is accelerating, its acceleration is nonzero. Because most of the constituent knowledge pieces in this category can be easily deduced from the syntactic version of the principles, it is very difficult to determine from the protocols whether these self-explanations were generated from knowledge gained prior to studying the examples (even if that knowledge is incomplete) or from the example line or a little of both. However, just because principle-

related self-explanations could in principle be deduced from information presented in the text, this does not necessarily preclude the fact that they could in fact be constructed. Nevertheless, to maintain objectivity, we refrained from coding this category of constituent knowledge for determining the source from which each self-explanation was generated and thus cannot say anything with certainty about how this category of constituent knowledge was acquired.

Aside from the issue of determining whether principle-related constituent knowledge pieces were in fact deduced from prior knowledge or constructed from information presented in the example statements, there are several interesting properties about them that may shed light on how encoded principles can become usable through self-explanations. One interesting property of principle-related constituent knowledge pieces is that they are very minor variations of each other. For instance, one piece of constituent knowledge states that *When a body does not move, all the forces acting on it have to be equal to zero*, and a separate one states that "If the forces acting on a body do not sum to zero, then the body will move." This aspect of constituent knowledge is not different from the separate independent deductions that are possible from logical applications of qualitative inference schemas, such as the one just mentioned. Nonetheless, we think that articulating these minor variations are important for learning in two ways. First, articulating them may be equivalent to creating multiple distinct rules, each usable with clearly specified condition. Second, once articulated, the knowledge no longer remains in tacit form.

A second important property of principle-related constituent knowledge is that many of them relate Newton's principles to motion such as *When a body is accelerating, then it is moving, or when a body does not move, all the forces acting on it has to be equal to zero*. The notion of *moving* is not explicitly stated in Newton's laws, so it must come from students' naive notions about acceleration and motion. That is, they typically equate a lack of motion with a lack of acceleration. Thus they are doomed to be confused about constant velocity (having motion and yet no acceleration). However, one way that students seem to attempt to understand the meaning of technical principles is to incorporate them with their naive intuitions. The extent to which this incorporation is necessary for proper usage is intriguing and needs to be further explored.

Finally, because many of the constituent knowledge pieces can take the form of condition-action rules, this suggests that self-explanations also serve the purpose of converting equivalent relations (e.g., $F = ma$), which are difficult to apply, into conditional forms, so that the conditions of their applicability are spelled out.

Concepts. The remaining category of 33 pieces of constituent knowledge—the largest to be discussed—concerns physics concepts such as body,

tension, weight, acceleration due to gravity, normal force, and so forth. Figure 12 lists five examples of self-explanations corresponding to their translated constituent knowledge. The first example in Figure 12 illustrates a case in which the constituent knowledge is coded after Student SP2 read the explained several lines. The fourth example shows an occasion when a self-explanation is generated after the student has been thinking about it for awhile, so that the utterances are not tied specifically to the previously read example line. This represents a case of inferencing either from previous self-explanations or previous example lines or both. The fifth example illustrates our coding of the last sentence of the self-explanation. The previous comments were included for context, but they were recoded into other pieces of constituent knowledge.

The fact that this is the largest category suggests that there are numerous concepts to be learned. Three of the concepts (body, normal force, and friction) are not defined in the text at all. Three others (tension, opposing forces, and static systems) are briefly defined. Three others (weight, acceleration, and acceleration due to gravity) are amply defined and discussed in the text. We coded each student's self-explanations which pertained to concepts with respect to their source (in the same way we coded the system-related self-explanations) and found that 42% of the concept-related self-explanations could be constructed from example lines, which means that at most 58% of them could be deduced from text presentations. This means about half of the concepts' meanings become explicit only when they are discussed in the context of the examples.

Summary. The analyses presented in this section coded 173 self-explanations into constituent knowledge pieces. Figures 9-12 illustrate a variety of the ways that they were coded and classified into 4 categories of constituent knowledge: systems, technical procedures, principle-related, and concepts. Of the 173 self-explanations, 61 of them were judged to be

- 1) Read The body remains at rest under the action of the three forces shown in Fig. 3-5b. (stating example Figure 1)
- Self-Expl. "So that means they would have to cancel out..." (SP1)
- Constit. K. If (given the current situation), then the forces acting on the body must cancel out.
- Self-Expl. "So the sum of forces should be zero." (SP2)
- Constit. K. If (given the current situation), then the sum of all the forces acting on the body must equal zero.

- 2) Read Since the block is unaccelerated we obtain $F_N \sin \theta = 0$.
- Self-Expl. "Okay, so they got that by, they said that these are all the forces acting on the block and it's not accelerating and their forces must equal zero." (SP1)
- Self-Expl. "And there seems to be the three forces involved also they all sum to zero somehow." (S110)
- Constit. K. If the body is not accelerating, then the sum of all forces acting on it must equal zero.

FIGURE 11 Constituent knowledge for principles and the corresponding quotes of self-explanations in response to example lines read. Note that when the premise of a proposition is unclear, it is put in parenthesis. All the constituent knowledge pieces illustrate fragments of Newton's first law. (Figure 5-6b mentioned in the first READ statement can be found in Halliday and Resnick, 1981, p. 70.)

- 1) Read Consider the force at the junction of the three strings at the "top body". (stating example, see Fig. 1)
- Self-Expl. "Why this should be the body? I thought it was the body." (SP2)
- Read The body remains at rest under the action of the three forces.
- Self-Expl. "I see. So the W will be the force and not the body OK."
- Read ...at the three forces shown in Fig. 3-5b.
- Self-Expl. "Uh huh, so... So they refer to the point as the body."
- Constit. K. A point may be considered a body.

- 2) Read ...at the three forces shown in Fig. 3-5b.
- Self-Expl. "Then it's minus W, minus W. W would weigh object weight. Weight is a force, OK? OK." (SP2)
- Constit. K. Weight is a force.

- 3) Read Find the relation in the signs and the acceleration of the masses. (stating example, see Fig. 2b)
- Self-Expl. "And this is going to have more negative mass it has greater mass." (S101)
- Constit. K. Objects with greater mass will have greater weight.

- 4) Read In the special case where $\theta = 0$, we obtain $F_N \sin \theta = 0$ and $F_N \cos \theta = mg$ which in the above should be corrected.
- Self-Expl. "...I've finally figured out why tension makes a difference. Uh, because that's going to be the force pulling it down. That's going to be how the masses are going to be carried over." (S101)
- Constit. K. Tension is a way of having a force act at a distance.

- 5) Read Now that these relations obtain in the special case, let us consider cases of tension and pull angles. (Qualified phrase example, see Figure 3a)
- Self-Expl. "And when the angle is 90 degrees, there is no normal force? Is that what it means? Cause call that be zero. Cause will be zero and there will just be the tension on the rope and there will be no, no normal force!"
- Constit. K. If a block does not push the surface, then there will be no normal force.

FIGURE 12 Constituent knowledge for concepts and the corresponding quotes of self-explanations in response to example lines read. (Figure 5-6b mentioned in the third READ statement can be found in Halliday & Resnick, 1981, p. 70.)

either concept related or system related. These are the only two categories of self-explanations for which we can objectively trace their source so that we can determine whether these self-explanations were deduced from prior text information or constructed via (a) direct inference from the example statement, (b) inference from previously generated self-explanation or encoded example line, and (c) inference from commonsense knowledge. Of these 61 self-explanations, 68.5% ($M = 95\%$ for systems and 42% for concepts) of them could have been derived from nontext sources and 31.5% ($M = 5\%$ and 58%, respectively) were possibly deduced from text. This proportion was the same for both the good and the poor solvers, suggesting that all students were capable of making on-line construction. This analysis suggests that over half of the self-explanations about concepts and systems were extrapolated from information provided in the example statements in conjunction with using common sense knowledge the students already have. We suspect that over half of the self-explanations about principles and technical procedures are also acquired by extrapolating information provided in the example line. The best evidence to support this conjecture is that a substantial portion of the principle-related constituent knowledge are incorrect (6 of 25 were incorrect); whereas logical deductions, using primarily the syntactic formula and a qualitative inference schema, would not produce incorrect self-explanations, unless common sense knowledge was used and it was incorrect or the initial knowledge of the principle was

incomplete. Thus, we conjecture that principle-related self-explanations were generated probably by a combination of knowledge encoded from the example lines, prior text knowledge and common sense knowledge.

We attributed a piece of constituent knowledge source to the declarative text only if the text had discussed that piece of knowledge. Thus, we were overly liberal in attributing deductions from text. It is equally plausible that even when the declarative text did present the material, students have not necessarily encoded them, so that more self-explanations could be considered to be on-line constructions. To verify this conjecture, for each student's constituent knowledge piece for which we had coded as having a source in the declarative text, we went back to the text to see if that particular student had highlighted that part of the text. The interesting result reveals that for the four good solvers, only 1 out of the 17 places in the text from which self-explanations could have been inferred were actually highlighted. In contrast, for the four poor solvers, six of the seven self-explanations for which there were text presentation were highlighted. This suggests that a majority of the good solvers' self-explanations might have been extrapolated or constructed on-line from the example statements, even though theoretically the information could have been inferred from the text. Poor solvers, on the other hand, could only explain during example studying what they had explicitly encoded from the text.

There are several additional properties to note about the nature of self-explanations in general, as reflected in this constituent knowledge analyses. First, because a great proportion of constituent knowledge are explanations about concepts that are embodied in the principles, they provide qualifications about a concept and make explicit its properties, its variants, and basically what constitutes an instance of that concept. For example, consider what can constitute a body: Self-explanations point out that a point and a block both can be considered to be a body. This piece of knowledge cannot be gained from studying the chapter 5 text, because that fact is not mentioned at all. We speculate that self-explaining these kind of principle-relevant concepts enabled the students to gain greater understanding of the principles after studying examples. We are thus implicitly assuming that understanding a principle consists primarily of understanding the concepts underlying a principle, more so than understanding the multiplicative equivalence relations governing the syntax of a principle. Second, self-explanations articulate numerous technical problem-solving procedures that are simply not available from the text, such as when a dimension can be ignored, when a negative answer can be obtained, and so forth. These technical pieces of constituent knowledge must play a major role in enabling students to solve problems correctly, as supported by the fact that it is the largest category of constituent knowledge acquired by both the good and poor solvers (see Table 7 again). Acquisition of this technical

knowledge also accounts for the high correlation (0.82, mentioned earlier) between successes at solving problems and solving the subprocedure problems. Finally, because both conceptual and systems knowledge are acquired largely by construction during example studying, this acquisition may explain the empirical findings in the literature of preference, reliance, and greater gains of learning from examples than from text.

CONCLUSIONS

This research on learning from examples has uncovered several interesting phenomena. Most prominently, it has found that students who are successful at solving problems are those who learned the materials in a different way than those students who were less successful at solving problems. More specifically, the good solvers' learning from examples was characterized by the generation of a greater amount of self-explanations than the poor solvers. Furthermore, the good solvers' self-explanations also embodied a greater number of distinct pieces of constituent knowledge than the poor solvers' (see Table 7). Self-explanations had the characteristic of providing primarily justifications to the actions of the example solutions. The justifications consist of expanding the conditions and explicating the consequences of actions (see Table 4).

This article has four specific goals. The first one showed more conclusively that both the good and the poor solvers initially had only partial understanding of the physics principles introduced in the text. This was done by noting that if the good students indeed had understood all the components of the Newton's laws, then their recitations of them would display a random pattern of retrieval, because a definition task provides a relatively neutral retrieval context. But in fact this was not the case. All the students tended to define the Newton's principles according to the syntax; little of it embodied the deeper or nonobvious concepts such as the body or net force (see column 1, Table 6). Furthermore, there was a mapping between those components of the laws that were actually mentioned in the examples with those that were actually articulated in the self-explanations (column 3 and 2, Table 6). These two results, taken together, strongly suggest that students did have incomplete understanding of the principles introduced in the text, and subsequently gained greater understanding from generating self-explanations while studying examples.

The second goal of this article was to characterize the content of self-explanations. We found, by translating all the self-explanations into distinct constituent knowledge pieces, that self-explanations contain four categories of knowledge. They were about systems, technical procedures, principles, and concepts. Although such recoding attempted to preserve as

much of the original meaning of the self-explanations as possible, it nevertheless reduced the number of self-explanations generated by all the students (258) down to a composite list of around 110 distinct pieces of constituent knowledge. The constituent knowledge represented the actual propositions that students have gained from self-explaining. Even though a constituent analysis only counted the number of distinct propositions that a student has gained, the good solvers nevertheless gained a significantly greater number of constituent knowledge pieces from self-explaining than the poor solvers (despite the exclusion of SP1, a strong self-explainer, from the set of good solvers).

Analysis of the content of self-explanations also informed us about what is learned from examples that enable students to solve problems. Primarily, students learned a great deal about technical procedures that are needed directly for solving problems. Technical procedures constituted the largest category of constituent knowledge acquired by both groups of students (see Table 7). The use of these technical procedures explains why students who were more successful at solving problems were also more successful at solving subprocedure problems, in which no principles were involved.

The third goal of this article was to determine from what knowledge are self-explanations generated, given that students apparently only have partial understanding of the domain principles. One class of plausible learning mechanisms are those implemented in the first generation of explanation-based learning (EBL) models. The key assumption in these EBL models is that the knowledge that is necessary to perform an explanation is already there, which means that there is actually no learning "at the knowledge level" (Dietterich, 1986). These EBL techniques consist of drawing out the deductive consequences of what is already known in order to make it more easily used. On the other hand, our empirical data suggest that students are capable of generating new knowledge that is neither explicitly known nor implicitly known as a reformulation or deductive implication of explicit knowledge, because about half of the information presented in the examples (about systems and concepts) were definitely not mentioned in the text. Thus, inferencing based directly on information presented in the example line must occur in order to generate a majority of the self-explanations. This inference process based on information presented in the example lines, which we have called *construction*, seems to embody several subprocesses, such as natural language inferencing using common sense knowledge, inferencing based directly on new information presented in the example line, and perhaps a kind of generalization after several self-explanations have been generated. Although we cannot discriminate unequivocally which process (construction or deduction) is entirely responsible for generating self-explanations, we are confident that the data clearly show that at minimum over half of the self-explanations are

generated by some kind of constructive process. Our conclusion is based on the fact that students obviously had incomplete understanding of the domain principles prior to studying examples. Moreover, for two categories of constituent knowledge (systems and concepts) for which we could explicitly identify the source of information from which explanations could have been generated, the examples (vs. the text) contained about half or more of the information source.

The final goal of this article was to speculate on how self-explanations can enhance a student's original incomplete understanding of the principles. Because this enhancement of the original understanding is not explicitly captured in the protocols, we can only speculate on the process. Three processes can foster this enhancement. When students have syntactic knowledge of principles, presumably they can generate all the principle-based self-explanations via some kind of deductive process. However, such deductions often do not embody meanings of concepts (e.g., what is a body) that are inextricable aspects of understanding principles. Thus, in order to understand the key concepts underlying a principle, as we stated in the constituent analysis, some further type of understanding process besides deduction is necessary. Another process that can contribute to greater understanding of principles is generalization. That is, even if the principle-related self-explanations could be generated by deductions (so that a student acquires various condition-action rules related to a principle, as shown in the section on constituent analysis of principle-related self-explanations), the meaning of a principle may not be understood unless a student generalizes across several of these individual and independent constituent rules. For example, in the constituent knowledge pieces listed earlier which showed variations of Newton's first law (see Figure 11, the first example), if a single student possessed both the constituent knowledge pieces about the forces acting on the body cancelling out as well as the sum of forces should be zero, then presumably a more generalized version covering both constituent knowledge pieces can be formed. Finally, the most direct way to explain why self-explanations enhance greater understanding of principles is simply to speculate that the act of self-explaining may make the tacit knowledge (if deduced from syntax of the principles) more explicit and available for use.

There are important implications to be derived from this research. More generally, in terms of theories of learning from examples, our data suggest that students can learn, with understanding, from a single or a few examples, contrary to other available empirical evidence (i.e., students can learn more than just syntactic rules and can transfer what they have learned to dissimilar problems). However, only those students who provide adequate explanations during studying are able to see the degree to which they can generalize their problem-solving skills.

Our results, however, have further explicit implications for empirical

laboratory findings, theoretical modeling efforts, and instructional techniques. First, in the empirical literature, our results resolve some of the major discrepant findings about learning from examples. The discrepancy in the literature is that on the one hand, students prefer, rely, and learn successfully from examples (Anderson et al., 1981; LeFevre & Dixon, 1986; Reder et al., 1986; VanLehn, 1986; Zhu & Simon, 1987), but on the other hand, training studies that empirically manipulate learning from examples show minimal transfer (Eylon & Helfman, 1982; Reed, Dempster, & Ettinger, 1985; Sweller & Cooper, 1985; also see discussion in Chi & Bassok, 1989). Our results suggest that unless the student self-explains from examples, there is little opportunity for transfer.

Theoretically, our results also point to limitations in the assumptions of the existing theories of learning from examples. For instance, current theories hold that explanations serve the purpose of justifying an example as an instance of a principle, assuming that the student has complete knowledge about the principle. Our results suggest, by contrast, that not only can explanations be generated without complete understanding of the domain principles, but the generation of self-explanations can serve the additional important function of enhancing and completing a student's understanding of the principles.

It is suggested, moreover, that the processes of generating self-explanations is construction via straightforward simple inferences from the information presented in the example statements themselves, sometimes combined with common sense knowledge, and sometimes combined with prior knowledge gained from the text. Errorful self-explanations can sometimes occur, either because the common sense knowledge is naive and incorrect, or the physics knowledge gained from the text is incomplete. This is to be contrasted, however, with logical deductions from knowledge of the principles; such logical deductions do occur but they are not the predominant characteristic of self-explanations. To elaborate, according to the way the self-explanations are analyzed with respect to their sources, stringent criteria were used to determine whether construction did or did not occur. Construction was hypothesized to occur if the sources of the inference were not based on information provided in the text, but were provided instead either in the example line, common sense knowledge, or previous self-explanations or example lines. Only when we could clearly determine what the source of the inference was did we attribute that self-explanation to construction. On the other hand, we liberally attributed any self-explanation to having been generated by the process of deduction from text if there was any mention of the principles, concepts, or technical procedures that may have their sources in the text, irrespective of whether such text presentations were actually encoded, stored, understood, or retrieved. Thus, we should only conclude that in theory, self-explanations

could be generated by deductions from knowledge gained in the text, much like an EBL-deductive mechanism, but in practice, the majority of the self-explanations were probably generated by on-line construction from information provided in the example statements themselves. A test case of our conclusion would be a study in which only examples are given and no text information is provided, except perhaps for the syntactic presentation of the key principles, because we know that this knowledge was easily acquired by all the students. It would be interesting to see whether self-explanations generated without text information were similar to the ones generated here. Another test case is to see how students acquire the skill of problem solving if they were not given any examples at all.

There are also instructional implications of this research. Taking our results at face value and interpreting it concretely would suggest that a promising approach to learning would be to teach students to construct self-explanations. Such a training approach would be more direct, more efficient, and more generalizable than an alternative instructional approach of improving and crafting textbook writers' presentation of examples (making them more complete, incorporating explanations that a good solver would naturally supply, etc.). The latter approach, however, would result in redundant information in the examples for the good solvers (who can supply their own inferences). Although there is no doubt that example presentations in textbooks could profit in general from improvements, the former instructional approach of training students to construct self-explanations seems more productive and feasible, in terms of it being a transferrable skill. Because we hypothesize that self-explanations are primarily generated by a constructive process using little more than simple everyday inference processes, there is no reason why training such a learning skill as self-explaining would not be facilitative. Two reasons postulated earlier for why self-explaining might succeed were that (a) they clarify and specify the conditions and consequences of actions and (b) they explicate tacit knowledge. VanLehn (personal communication) developed a model of self-explanation based on the assumption that students explain the lines of an example by attempting to re-derive them. Re-derivation sometimes leads to impasses, which indicate an area of missing knowledge and invite an active focused search for clarification. Thus, re-derivation tends to uncover tacit constituent knowledge.

ACKNOWLEDGMENTS

We are grateful to the Office of Naval Research, Personnel, and Training Research Programs, Psychological Sciences Division, for support of this work under Contract Numbers N00014-84-0542 and N00014-85-C-0688.

Preparation of this article was sponsored in part by the Center for the Study of Learning (CSL) at the Learning Research and Development Center of the University of Pittsburgh. CSL is funded by the Office of Educational Research and Improvement of the U.S. Department of Education. Codings and analyses by Adam Fineman are appreciated.

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