The Acquisition of Qualitative Physics Knowledge during Textbook-Based Physics Training

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Abstract

Several earlier investigations found that teaching standard textbook physics causes only moderate change in qualitative understanding. Many investigations have tried to explain why teaching textbook physics results in so little learning of qualitative physics. In contrast, we examined cases where learning did occur and tried to understand them, hoping that this might help us to understand how to support such learning. We developed computerized simulation models of both qualitative, conceptual problem solving and quantitative problem solving and used them to assess changes in students' qualitative knowledge as they learned textbook physics. In many cases, qualitative knowledge has been acquired on the basis of information explicitly presented in the textbook. However, we also found cases where learning of qualitative physics took place on the basis of information only implicitly addressed in the instruction. Even more important, in various cases this newly acquired qualitative knowledge led to a less frequent use of incorrect qualitative pre-knowledge. This suggests that successfull students did not only learn what has been explicitly presented in the instruction. Rather, they did also learn by deriving and constructing information left implicit in the instruction, relating this information to their pre-knowledge and possibly refining and modifying their pre-knowledge in those cases where conflicts became aware.

1 Introduction

One important and widely replicated finding in physics education is that students do not enter physics courses without any pre-knowledge about the domain. Instead, they bring along conceptualizations that are inconsistent with the principles taught in physics textbooks (e.g., Halloun &

Hestenes, 1985a; McCloskey, Caramazza & Green, 1980). These so-called misconceptions show up mostly when students are asked to solve problems that do not require the use of algebraic equations, such as "Which forces act on a coin that has been tossed straight up into the air?". We will denote these problems as "conceptual problems". We will use the term "qualitative physics knowledge" to refer to the knowledge addressed by conceptual problems. Qualitative physics knowledge might encode information about the conditions under which physics concepts can legitimately be applied, the attributes possessed by physics concepts as well as the values these attributes might have, for example.

We will use the term "quantitative problems" to denote problems that are frequently posed in physics textbooks and that ask for a precise quantitative solution, such as "A block is being held in place on a inclined, frictionless surface by a massless spring. The surface is inclined at an angle of 40 degrees. If the mass of the block is 10 kg, what force must the spring be exerting on the block?". In many cases, quantitative problems require both the use of qualitative physics knowledge and "quantitative physics knowledge" (e.g., Chi, Feltovich & Glaser, 1981; Ploetzner, 1995). Quantitative physics knowledge relies on mathematical formalisms and describes definitions of and functional relationships between physics concepts by means of algebraic and vector-algebraic equations. Of course, qualitative and quantitative physics knowledge cannot be separated at a clear-cut boundary. Rather, this distinction refers to the ends of a continuum with a considerable body of knowledge between them (e.g., Ploetzner, Spada, Stumpf & Opwis, 1990; Van Joolingen, 1994; White & Frederiksen, 1990).

Instead of being quickly discarded, students frequently hold onto misconceptions even after receiving many hours of physics instruction (e.g., McCloskey, Caramazza & Green, 1980; Halloun & Hestenes, 1985b). Several explanations have been offered. Misconceptions in physics may be resistant to change because they are rooted in countless everyday experiences (e.g., diSessa, 1988, 1993) or because they are part of a coherent world view similar to those held by medieval scientists (e.g., McCloskey, 1983; Wiser, 1987) or because they are materialistic and causal whereas scientific physics requires an acausal, constraint-based, nonmaterialistic ontology (e.g., Carey, 1991; Chi, 1992; Chi, Slotta & de Leeuw, 1994).

We examined what happens when students do learn qualitative physics from textbook physics training and then tried to find out what prevented the other students from learning. Several earlier investigations found that teaching textbook physics results in moderate changes in qualitative physics knowledge. For instance, McCloskey, Caramazza and Green (1980) found a particular misconception that was evident in the answers of 49% of their subjects who had no physics training, 34% who had one high school physics course and 14% who had completed one or more college physics courses. Although these are cross-sectional results, they suggest that textbook physics training can improve conceptual problem solving to some degree. Halloun and Hestenes (1985b) administered a test of qualitative physics knowledge before and after various standard physics courses (2 high school, 3 college, and 4 university courses). They found that the posttest scores were on average about one standard deviation higher than the pretest scores.

If we would understand the learning of the students who did acquire qualitative physics knowledge during textbook physics training, then we could perhaps better support other students in doing the same learning. As a first step towards reaching this understanding, we developed computerized simulation models of both conceptual problem solving and quantitative problem solving. Within these models, qualitative as well as quantitative physics knowledge is represented by means of if-then rules.[1] Both models were developed on the basis of data that were taken from a study conducted by Chi, Bassok, Lewis, Reimann and Glaser (1989). This study included assessments of students' qualitative physics knowledge before and after they were given instruction in classical mechanics on the basis of a standard textbook (Halliday & Resnick, 1981).

As a second step, we took advantage of these models to analyze changes in students' qualitative knowledge as they learned textbook physics. We based our analyses on the notion of *transfer* (cf. Singley & Anderson, 1989): In which ways does textbook physics training and quantitative problem solving promote transfer to conceptual problem solving? Given the transfer framework, one would expect to find three kinds of learning qualitative physics:

1. *Applying a new piece of knowledge.* The student's pre-knowledge lacked some piece of qualitative physics knowledge, which caused errors on the pretest. During the instruction, new qualitative knowledge is learned on the basis of information explicitly presented in the textbook which leads to fewer errors on the posttest. For instance, untrained students rarely believe that when an object rests on a surface, the surface exerts a force upon it.

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[1]

During the instruction students could learn that such a force exists.

2. *Not applying an already available piece of knowledge.* The student's pre-knowledge includes an incorrect piece of qualitative phyiscs knowledge which is explicitly addressed by the instruction, thus causing the student not to apply the knowledge anymore in the context in question. For instance, untrained students often believe that when an object moves in a circle, a centrifugal force acts on it, pushing it away from the center of the circle. The physics training could explicitly discuss centrifugal force, showing that it is a misinterpretation due to viewing the situation from an accelerating reference frame and thus convincing students that it is not a real force.

3. *Applying a new and not applying a conflicting piece of knowledge.* The student's pre-knowledge includes an incorrect piece of qualitative phyiscs knowledge, which caused errors on the pretest. During the instruction, new qualitative physics knowledge is learned on the basis of information explicitly presented in the textbook. This newly acquired knowledge is in conflict with the incorrect pre-knowledge. That is, both the new knowledge and the pre-knowledge apply to the same context, but they draw different conclusions. Faced with this conflict, the student decides not to apply the pre-knowledge to this context anymore. For instance, students often believe that the gravitational force acting on a projectile gets stronger the longer the projectile is up in the air and thus the projectile falls more and more rapidly toward the ground. Assuming only small changes in altitude, physics textbooks teach that the gravitational force on a projectile is constant. Since the gravitational force on a projectile cannot be both constant and increasing, the new and old information are in direct conflict. Transfer occurs when the student decides to abandon the belief that the gravitational force changes in the context of projectile motion.

If these are the only kinds of transfer that can occur, then any change in qualitative physics knowledge must be due to one of them. For instance, if one expects students to stop believing in a particular kind of force, then the instruction must either explicitly teach that the force in question is inappropriate or explicitly teach some knowledge that conflicts with the force.

This could explain why some misconceptions survive standard physics instruction. For instance, a notorious misconception is the belief that motion implies a force (e.g., Halloun &

Hestenes, 1985a; McCloskey, 1983). That is, when an object is moving, even if it is moving with a constant speed, then many students believe there must be a force in the direction of motion that propels it along. In order to change this belief, the instruction must either explicitly address it or teach some procedure that yields only correct forces such as the one advocated by Reif and Heller (1982). Since many textbooks neither discuss this belief nor present an exhaustive method for enumerating forces, it is unsurprising that students continue to hold the belief despite extensive physics instruction. In short, adopting the transfer framework suggests how the desired learning could take place, which in turn suggests why it sometimes might not be taking place.

The main goal of this paper is to test the proposed transfer mechanisms empirically. The main result is that the transfer framework explains most of qualitative physics learning, but not all. There appears to be a form of learning unpredicted by the transfer framework. In various cases, students stop applying misconceptions when they acquire new knowledge on the basis of information only implicitly presented in the instruction. This suggests that successfull students learn not only what is explicitly presented in the instruction. Rather, they also learn by deriving and constructing information left implicit in the instruction, relating this information to their pre-knowledge and possibly refining and modifying their pre-knowledge in those cases where conflicts become aware.

The rest of the paper is organized as follows. In the next section, the design of the Chi et al. (1989) study is outlined. Thereafter, the pre- and posttest to assess the students' qualitative physics knowledge before and after they were given physics instructions are described and the degree of observed transfer between the two tests is determined. The computer models of conceptual problem solving and of quantitative problem solving are delineated next. On the basis of these models two analyses are presented. In the first analysis, it is determined whether the degree of rule-overlap between the two models is consistent with the observed degree of transfer between the pre- and posttest. In the second analysis, we determine the kinds of learning that occured during the physics training between the pre- and posttest. In addition, we explore why in various cases there was no learning observable. The paper concludes with a discussion and potential pedagogical implications of our findings.

2 The Design of the Chi et al. (1989) Study

In order to understand how students acquire physics knowledge, Chi et al. (1989) investigated nine subjects as they studied a common college physics textbook (Halliday & Resnick, 1981). All of them had taken high school physics courses. None of the subjects had taken a college physics course. Within this study, the subjects completed the following steps:

1. The subjects took a pretest that included a set of conceptual problems taken from the literature on misconceptions.

2. The subjects studied the first three chapters of the textbook, which reviewed concepts such as the of units of measurement, velocity, and acceleration in the context of motion in one dimension. They received feedback on their work on each chapter until a criterion test was passed.

3. The subjects studied the first part of the textbook's chapter on Newtonian mechanics, which introduces the concepts of force and mass as well as their relation to acceleration. This part of the chapter had no worked-out examples in it. As in the previous case, this part of the textbook had to be studied until a criterion test was passed. However, the subjects received no feedback on their work.

4. The subjects studied three worked-out examples while talking aloud. Each example consisted of a quantitative problem on Newtonian mechanics and its solution. The examples made up the second part of the textbook's chapter on Newtonian mechanics.

5. The subjects solved nineteen quantitative problems on Newtonian mechanics while talking aloud. They received no feedback on their work.

6. The subjects took a posttest that included the same conceptual problems as the pretest.

Because the subjects received feedback on the criterion tests for the first three chapters of the textbook which involve motion in one dimension, there was some opportunity for learning qualitative physics knowledge that directly conflicts with incorrect pre-knowledge in kinematics such as misconceptions about velocity and acceleration. When working on the criterion test for the chapter on Newtonian mechanics, students received no feedback, so incorrect pre-knowledge in dynamics such as misconceptions about the gravitational force were not directly confronted. Furthermore, although recent editions of the Halliday and Resnick (1981) textbook address common misconceptions in physics, this edition did not. In particular, there was only one mention of the "motion implies force" concept, and it was brief, cryptic and historical. Thus, except for the criterion tests for the first three chapters, the instruction did not teach qualitative physics knowledge that directly conflicts with incorrect pre-knowledge. Therefore, the transfer framework predicts that learning due to conflicts between newly acquired qualitative knowledge and already available incorrect pre-knowledge should hardly ever be observable, especially not in the case of misconceptions about forces.

3 The Pre- and Posttest and the Observed Degree of Transfer

The pre- and posttest posed in the Chi et al. (1989) study included two different sets of conceptual physics problems. The first set consists of multiple choice questions that address various physics concepts and principles. The second set consists of sixteen free-response problems similar to those administered in many earlier studies (e.g., Clement, 1982, 1983; Halloun & Hestenes, 1985b; Kaiser, Proffitt & McCloskey, 1986; McCloskey, 1983; McCloskey, Washburn & Felch, 1983). In order to avoid modeling the guessing and foil-elimination associated with multiple choice questions, our analysis is based exclusively on the sixteen free-response problems. Of these, thirteen asked subjects to draw the path that a given object will travel under certain conditions. This set of problems includes, for instance, a walking man who drops a steel ball, a rocket which is traveling in outer space and fires its engine for a short period of time, a ball projected from various cliffs, and a ball shot through curved tubes. Thus, these problems addressed the concepts of distance, velocity and acceleration as well as the physics principles that relate them.

The remaining three free-response problems provided the correct path an object will travel when thrown vertically or obliquely (see Figure 1a and 1b) or when swung from a string (see Figure 1c). With respect to these problems, subjects had to draw arrows indicating the forces which act on the object at different points in its path as well as its acceleration. Thus, these problems address the concepts of force and acceleration as well as the physics principles that relate them.

In addition to answering the free-response problems, the subjects were requested to write a brief justification for each. On the basis of these justifications, it was found that subjects often applied misconceptions, even when giving correct answers. The most important and frequently occurring misconception was the belief that motion implies a force (e.g., Clement, 1982, 1983; Halloun & Hestenes, 1985a; McCloskey, 1983; Viennot, 1979). Subjects often assumed that continuing motion is due to a continuing force in the same direction as the motion and that the force dies out or builds up in order to account for changes in an object's motion.

Insert Figure 1 about here.

We determined the degree of transfer that occured between the pre- and posttest using one of the standard techniques discussed by Singley and Anderson (1989). They suggest the following formula:

$$\frac{\text{posttest score} - \text{pretest score}}{\text{pretest score}} \times 100$$

The mean scores increased significantly from the pretest (46% correct answers) to the posttest (65% correct answers; t-test, $p < .01$). Thus, the degree of transfer that occured between them was 41%. This is comparable to other studies of transfer from standard physics training to qualitative understanding. For instance, Halloun and Hestenes (1985b) found that their university students' scores increased significantly from 50% to 64% correct answers which represents 28% transfer from their physics courses to qualitative problem solving. Thus, the tests administered in the Chi et al. (1989) study indicated a positive shift in qualitative physics knowledge whose size was consistent with the moderate improvement found by other studies on the effect of standard physics training on qualitative understanding. Because in this study the instruction was laboratory-based and well controlled, it seemed likely that we could locate and understand the learning events that caused the observed improvement.

4 Modeling Conceptual and Quantitative Physics Problem Solving

As our model of quantitative physics problem solving, we adopted CASCADE (VanLehn, Jones & Chi, 1992), which was constructed to model how the subjects in the Chi et al. (1989) study acquire new physics knowledge while explaining the worked-out examples and solving the nineteen quantitative physics problems. CASCADE is implemented in Prolog, a rule-based language, to represent qualitative and quantitative physics knowledge. Its knowledge base includes 62 physics rules which are sufficient for solving all but two of the nineteen quantitative physics problems posed in the Chi et al. (1989) study.

In order to model conceptual physics problem solving, we used the same representation language as in CASCADE. In this case, however, the ontology underlying this representation was taken from research on qualitative reasoning about physical systems (e.g., de Kleer & Brown, 1984; Forbus, 1984; for a collection of papers see Weld & de Kleer, 1990). For instance, to represent the path an object travels, we adopted a modified version of the representation of trajectories as used in de Kleer's (1977) NEWTON system (see also Davis, 1990). Trajectories are described by a sequence of trajectory elements which might be either segments or points. A trajectory segment is characterized by the qualitative direction of its tangent and inward normal as well as by its qualitative curvature. The qualitative curvature of a trajectory segment is positive if it turns to the right, negative if it turns to the left, and zero if it is straight. The curvature of the trajectory of the pendulum in Figure 1c swinging to the right is negative, for example.

In a first step, we constructed a knowledge base made up of 39 rules that correctly solves all the free-response problems of the pre- and posttest in the Chi et al. (1989) study. In a second step, we also included 85 rules that generate incorrect answers as provided by the subjects in the Chi et al. (1989) study (all correct and incorrect rules are listed in the appendix). Of these 85 rules, 43 address the misconception that motion implies a force. It predicts motion in the absence of a proper force in the direction of motion, or in the face of a proper force which obviously opposes motion. Since this concept resembles in many ways the concept of impetus as it was discussed during the 6th century by Philoponus and others (for a discussion see Franklin, 1976), we refer to it as the "impetus force".

In many cases, students operated with an impetus force as they did with a proper physics force. With respect to the posed problems, they seemed to treat an impetus force as a vector quantity with the properties of magnitude and direction, and combined it vectorially with other forces in order to determine the net force. Hence, the model's representation of an impetus

Table 1: Derivation that the acceleration of an object in free fall is constant

	Proposition	Justification
1	The object is a body.	Given
2	The object is not the earth.	Given
3	The object is not massless.	Assumed
4	Gravity should not be ignored on this problem.	Assumed
5	There exists a gravitational force on the object due to the earth.	Rule 1 applied to 1, 2, 3 and 4
6	The object is falling.	Given
7	The gravitational force on the object causes the object's acceleration.	Rule 89 applied to 5 and 6
8	The magnitude of the object's acceleration is proportionally related to the magnitude of the gravitational force on the object.	Rule 100 applied to 7
9	The magnitude of the gravitational force on the object is constant.	Rule 2 applied to 5
10	The magnitude of the object's acceleration is constant.	Qual. reasoning on 8 and 9

makes it a kind of force, just like a tension force or a gravitational force.

Most of the sixteen problems we analyzed require between 2 and 5 rules to be applied for determining the correct solution. For example, one question with respect to the problem presented in Figure 1a was how the ball's acceleration will change while it travels along the provided path. The derivation shown in Table 1 leads to the correct solution of the problem, namely, that the acceleration is constant (cf. the appendix for the rules which have been applied). This solution is derived by our model by establishing that the acceleration of a falling object is caused by the gravitational force which is constant.

The derivation shown in Table 2 results in the frequently observed incorrect solution that the acceleration of an object changes in the same way as the object's velocity (cf. the appendix for the rules which have been applied). This solution is due to a misconception which does not differentiate between velocity and acceleration.

If a subject's response to a problem can be derived by the model, then that response is considered as reconstructible by the model.[2] Overall, 78% of the 516 encoded responses to the

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[2] If more than one derivation is possible and one of these derivations shares more rules with derivations of other responses from the same subject, then that derivation is accepted as the most likely one for this response. Thus, we allowed the model of a particular student's knowledge to contain inconsistencies. Furthermore, we did not include highly specific rules into the model just to reconstruct a single student's response to a single problem.

Table 2: Derivation that the acceleration of an object in free fall changes in the same way as its velocity

	Proposition	Justification
1	The object is a body.	Given
2	The object moves along a trajectory with a directed tangent pointing downwards.	Given
3	The object has velocity.	Rule 80 applied to 1 and 2
4	The object is accelerated.	Rule 88 applied to 3
5	The magnitude of the object's acceleration is proportionally related to the magnitude of the object's velocity.	Rule 90 applied to 3 and 4
6	The vertical direction of the object's velocity equals the vertical direction of the trajectory's tangent.	Rule 87 applied to 2 and 3
7	The magnitude of the object's velocity increases.	Rule 81/82 applied to 2, 3 and 6
8	The magnitude of the object's acceleration increases.	Qual. reasoning on 5 and 7

pretest and 89% of the 481 encoded responses to the posttest are reconstructible by the model.[3] However, 46% of the responses to the pretest and 65% of the responses to the posttest were correct, and as all correct responses on both tests were derivable from the knowledge base, the more telling figure is the percentage of incorrect responses that are reconstructible. The model reconstructed 59% of the incorrect responses to the pretest and 78% of the incorrect responses to the posttest.[4]

To further assess the empirical adequacy of the model, two people who were not involved in the development of the model's knowledge base were asked to determine whether a rule can be aligned with some written justification provided by a subject. The judges agreed in 89% of the cases. Disagreements were settled by a third judge. Of the 39 correct rules in the knowledge base, 21 (54%) were found to be supported by subject justifications. Most of the unsupported correct rules specify the direction of some vector quantity. Subjects rarely mentioned directions in their justifications. However, these directions were always clearly indicated by the arrows the subjects drew, so despite the lack of verbal support, the evidence for these rules is still strong.

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[3] For the sixteen free-response problems, the nine subjects produced a total of about 500 responses to the pre- and posttest, because various problems required multiple responses. However, the subjects occasionally omitted some responses, so the number of responses encoded was not the same on both tests.

[4] The model could be evaluated more rigorously if we had hidden some of each subject's responses from ourselves before developing the model. These hidden responses could then be used to evaluate the model of that subject. Given the small number of responses per subject, we could not afford this degree of rigor.

The remaining unsupported correct rules mainly specify magnitudes of vector quantities as well as their changes over situations. These rules are also supported by graphic data, namely, the length of arrows drawn by the subject and their successive variation.

Of the 85 incorrect rules, the judges found that 32 (38%) were supported by subjects' justifications. As in the case of correct rules, the majority of the unsupported incorrect rules specified either the directions of vector quantities or changes of their magnitudes. For instance, a set of nine unsupported incorrect rules implement a misconception first noticed by McCloskey (1983). He discovered that individuals differ in their beliefs about the interaction of an impetus and the gravitational force. Whereas some subjects assume that the gravitational force affects an object flying off a cliff immediately regardless how much impetus it has, some subjects believe that gravity does not affect the object until its impetus has dissipated below some critical threshold. Although the rules formalizing this misconception provided accurate explanations of several subjects' answers, none of the subjects explicitly justified their answers in these terms. Nonetheless, despite the lack of verbal justification, the support of the graphic data makes us confident that these rules accurately model the subjects' beliefs.

During the last ten years, research on students' alternative frameworks has extensively documented the content of (incorrect) qualitative pre-knowledge utilized by students. Since the problems we analyzed are comparable to those problems used in several earlier studies which investigated the misconceptions held by students, there should be overlap in the misconceptions reported by these investigations and those embedded in our model. Table 3 lists the misconceptions represented in our model along with a selection of studies which have previously reported these misconceptions. The list of mentioned studies is not intended to be complete. There appear to be several misconceptions which might not have been noticed until now (it is hardly possible to review all several hundred papers about students' alternative frameworks which exist meanwhile for the domain of physics alone; for a bibliography see Pfundt & Duit, 1994). A brief discussion of these misconceptions can be found in the appendix.

In summary, there are three reasons to have some confidence in the analysis offered here. (1) The model's knowledge base accounts for all the correct responses and for most of the incorrect responses of the subjects. (2) Two independent judges found ample agreement between our analysis of the subjects' answers and their written justifications. (3) The misconceptions

Table 3: The misconceptions represented in the model

Gravitational force:

- The gravitational force increases over time.

Tension force:

- The magnitude of the tension force (in a pendulum) is constant.
- The magnitude of the tension force (in a pendulum) equals the magnitude of the gravitational force.

Impetus:

- The impetus acts in the direction of motion (Clement, 1982; 1983; Halloun & Hestenes, 1985a; McCloskey, 1983; McCloskey, Washburn & Felch, 1983; Viennot, 1979).
- The impetus acts in horizontal direction.
- The impetus acts in vertical direction (McCloskey, 1983).
- The impetus acts as an opposer (diSessa, 1983; 1988).
- The impetus acts as a mover.
- The impetus (in a pendulum) acts in forward direction (Clement, 1982; 1983).
- The impetus (in a pendulum) acts in backward direction.
- The impetus (in a pendulum) is curvilinear (Kaiser, Proffitt & McCloskey, 1986; McCloskey, 1983).
- An impetus keeps an object moving for some time in the way it was set going (Kaiser, Proffitt & McCloskey, 1986; McCloskey, 1983; McCloskey, Washburn & Felch, 1983).
- An impetus increases or decreases over time (Clement, 1982, 1983; diSessa, 1983; 1988; Halloun & Hestenes, 1985a; Kaiser, Proffitt & McCloskey, 1986; McCloskey, 1983; McCloskey, Washburn & Felch, 1983).

Force and velocity:

- Velocity is in the direction of an applied (net) force, and proportional to the applied (net) force (Champagne, Klopfer & Anderson, 1980; Clement, 1982; 1983; diSessa, 1982, 1983, 1988; Halloun & Hestenes, 1985a).

Acceleration and velocity:

- Acceleration and velocity are not differentiated (Halloun & Hestenes, 1985a).

identified in our model are similar to those found by other investigators.

5 Does Rule-Overlap Account for the Observed Degree of Transfer?

Contemporary theories predict that transfer from one skill to another is proportional to the overlap in the rules underlying the two skills. Following Singley and Anderson (1989), we counted how many of the rules in the model of conceptual problem solving were also found in the model of quantitative problem solving, and determined if the degree of rule-overlap was comparable to the observed degree of transfer between the pre- and posttest.

The overlap was measured in two ways. Of the 39 correct rules in our model of conceptual problem solving, 6 (15%) are also in CASCADE's knowledge base. This count includes only rules that are *exactly* the same in both knowledge bases. Another way to measure overlap is to also include rules in the model of conceptual problem solving that can be derived from CASCADE's rules by means of qualitative interpretation or deduction. For instance, consider the following two physics rules: (1) an object's weight is defined to be the gravitational force acting on that object and (2) $w = m \times g$ where w is the magnitude of an object's weight, g is the (constant) gravitational acceleration on earth and m is the object's (constant) mass. These two rules imply that the gravitational force on an object is constant. According to this measurement, 14 (36%)[5] of the 39 correct rules in the model of conceptual problem solving overlap with CASCADE's knowledge base. These rules address gravitational force (5 rules), net force (2 rules), tension force (3 rules), and acceleration (4 rules).

The surprising result is that there are 25 correct physics rules in the model of conceptual problem solving that do not correspond to physics rules in CASCADE's knowledge base. These rules concern the following topics:

- 13 rules describe properties of curved trajectories of projectiles.
- 10 rules describe forces that were not covered in the physics instruction, including rocket thrust, centripetal force, and certain types of reaction forces.

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[5] These 14 rules are comprised of the 6 rules of the first measurement, plus 2 rules which are qualitative interpretations of rules in CASCADE's knowledge base and 6 rules that are implied by two or more rules in CASCADE's knowledge base.

- 2 rules describe properties of tension force that were not covered in the instruction.

If the instruction lasted a whole semester or year, then perhaps some of these topics would have been covered, thus increasing the rule overlap.

Because the degree of rule-overlap (36%) is close to the degree of transfer (41%), this analysis supports the transfer framework. However, the analysis does not shed any light on the question of how the learning during the instruction actually took place. To answer this question, a more detailled analysis is needed.

6 Does the Transfer Framework Account for the Observed Learning?

The transfer framework predicts that only rules will be acquired that correspond to some information explicitly presented in the instruction. Furthermore, if some of the newly acquired rules conflicts with already existing rules, then students may stop applying the existing rules. Incorrect pre-knowledge that was explicitly discussed and discredited by the instruction might also disappear, but in the Chi et al. (1989) study the instruction on dynamics did not explicitly address any misconceptions.

Now that we have an assessment of which rules were used by the subjects to solve the posed problems, we can determine which rules were used on each test, and thus gain some insight into what the students learned. In this analysis we excluded one of the nine subjects who participated in the Chi et al. (1989) study because of lost posttest data. Futhermore, we analyzed just the free-response problems that provided the path an object will travel. Although we did construct student models for the other free-response problems, we did not analyze them, because the students received feedback on kinematics during the criterion testing at the end of studying each of the first three chapters of the Halliday and Resnick (1981) textbook (cf. Section 2). Thus, there was some opportunity for learning qualitative phsyics knowledge that directly conflicts with incorrect pre-knowledge in kinematics.

Although only three problems were analyzed, the subjects had to make several responses to each problem, so sufficient data were available for constraining the analysis. The first problem (cf. Figure 1a) asked the subjects to draw the forces acting on a vertically thrown object at each of the four positions indicated. The problem also asked the subjects whether the acceleration at point A was greater than, equal to or less than the acceleration at point C and whether the acceleration at point A was in the same direction as the acceleration at point D, or in the opposite direction. In the following, we refer to this first problem as "Problem VT" (for vertical throw). The second problem (cf. Figure 1b) asked subjects to draw the forces acting on the obliquely thrown object at each of four positions indicated by dots along the trajectory of the object. At additional dots to the right of the trajectory, connected by dotted lines, the students were asked to draw the acceleration of the object. We refer to the second problem as "Problem OT" (for oblique throw). The third problem asked subjects to draw the forces acting on a pendulum which is swinging from left to right at each of four positions. One position is shown in Figure 1c. We refer to the third problem as "Problem PF" (for pendulum forces).

Although we conducted the analysis in terms of rules, we have summarized the results as shown in Table 4. The first column identifies the subject. The second column identifies the problem. The remaining columns correspond to rules that were assessed on the pre- and posttest. The rules were grouped into categories according to the concept they address. For instance, the column labelled "Gravity" refers to rules that describe whether a gravitational force is present, and if so, what its direction and relative magnitude are. The cells indicate the content of the rules diagnosed with respect to a certain subject on the pretest (before the slash) and posttest (after the slash). The codes in the columns have the following meanings:

- *Gravity:* The code "none" means that the subject left off the gravitational force on this problem. The code "change" means that the subject assigned a gravitational force, but the magnitude changes over time. The code "const" means that the subject assigned a gravitational force that was constant across time, which is correct. The "?" code means that we cannot tell what the subject believed on this problem, in most of the cases because the subject skipped answering the problem. The "?" code has this meaning for all columns.

- *Tension:* Tension forces are relevant only on the pendulum problem (Problem PF), so the cells in other rows are left blank. The code "none" means that the subject did not include a tension force. The code "pres" means that the subject did include a tension force, which was correct.

- *Acceleration:* Only problems VT and OT asked the subject to draw an acceleration, so

Table 4: Rule usage by each subject on each problem (pretest/posttest)

Subj	Prob	Gravity	Tension	Acceleration	Impetus	Net force
P1	VT	const/const		velo/(grav)	none/none	neither/neither
P1	OT	change/const		?/centripetal	pres/pres	?/neither
P1	PF	const/const	none/pres		pres/pres	neither/velo
P2	VT	change/none		velo/velo	pres/pres	velo/velo
P2	OT	change/none		velo/velo	pres/pres	velo/velo
P2	PF	const/const	pres/pres		pres/none	velo/accel
101	VT	const/const		grav/grav	none/none	accel/accel
101	OT	const/?		grav/grav	pres/?	velo/?
101	PF	const/const	none/pres		pres/none	velo/accel
102	VT	change/change		velo/grav	pres/pres	neither/velo
102	OT	const/const		velo/grav	pres/pres	neither/velo
102	PF	const/const	pres/pres		none/pres	accel/(velo)
103	VT	const/const		(velo)/grav	pres/none	neither/accel
103	OT	change/const		(velo)/grav	pres/pres	neither/neither
103	PF	const/const	none/none		pres/pres	neither/neither
105	VT	?/change		velo/(grav)	?/pres	?/velo
105	OT	none/none		velo/(grav)	pres/pres	velo/velo
105	PF	none/const	pres/pres		pres/none	velo/(accel)
109	VT	const/const		?/velo	?/?	?/?
109	OT	?/const		?/(velo)	?/?	?/?
109	PF	const/const	none/none		pres/?	neither/neither
110	VT	const/const		grav/grav	none/none	accel/accel
110	OT	const/const		grav/grav	none/none	accel/accel
110	PF	const/const	none/none		none/none	neither/neither

we have left the cells for Problem PF blank. The typical incorrect answer was to draw acceleration so that it had the same direction and relative magnitude as the object's velocity (code "velo"). The correct answer was to draw the acceleration straight down and constant, because for projectile problems, the acceleration is entirely due to gravity (code "grav"). When the subject's acceleration arrows are only partially consistent with velocity or gravity, the code is enclosed in parentheses. The code "centripetal" is used for the one subject who drew accelerations for the oblique throw problem that pointed toward the "center" of the curved trajectory.

- *Impetus:* If the subject used any impetus forces on the problem, the code "pres" was entered, and "none" otherwise.
- *Net force:* The interpretation in this column requires more inference, because the subjects were not explicitly asked to indicate the net force on an object. However, if the sum of the forces that they drew is consistent with the object's velocity, then the code "velo" is entered. If the sum is consistent with the object's acceleration, then the code "accel" is entered. If the sum of forces is consistent with neither the velocity nor the acceleration, then "neither" is the code entered.

The cells where a change has occurred from pre- to posttest are boxed. There are 27 such changes.[6] The purpose of isolating these changes in knowledge is to categorize them according to the types of learning processes involved and whether those learning processes are predicted by the transfer framework.

6.1 What Kind of Qualitative Physics Learning Occured?

Learning a new force. There were 3 cases where a subject learned a new force. Subjects P1 and 101 added a tension force to their solution of Problem PF, which is the only place where it can appear. Subject 105 added a gravitational force to Problem PF, although not to Problem OT. Although he omitted the critical responses on the pretest of Problem VT, he did use a gravitational force on the posttest of Problem VT. Thus, we have good evidence that subjects

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[6] In 20 cases, changes are from an incorrect setting to a more correct one ($McNemar - Chi - Square = 6.26, df = 1, p < 0.05$). In only 7 cases are there changes from a (partially) correct setting to an incorrect setting. This is another measure showing that subjects learned qualitative physics knowledge from the physics training in the Chi et al. (1989) study.

P1 and 101 learned a new force (tension) and some evidence that Subject 105 partially learned a new force (gravity).

These two forces occur frequently in the instructional material, so it is no surprise that they were learned. For instance, tension and gravity appear on all three examples from the chapter on Newtonian mechanics. Of the nineteen quantitative problems on Newtonian mechanics, tension occurs on ten problems and gravity on sixteen.

Revising the properties of forces and acceleration. There were 13 cases where subjects changed their predictions about properties of either a force or acceleration. As shown in the column labelled "Acceleration", subjects P1, 102, 103, and 105 learned that projectile acceleration does not always have the same magnitude and direction as velocity, but is always straight down and constant. Although this revision of beliefs may be specific to projectile motion, it is a step towards removing the notorious confusion of acceleration with velocity (cf. Reif, 1987).

The column labelled "Gravity" indicates that subjects P1 and 103 abandoned their prediction that the gravitational force gets stronger as the impetus wears off during the flight of a projectile. They adopted the correct belief that gravity is constant.

The column labelled "Net force" indicates that subjects P2, 103 and 105 made some progress toward predicting that the net force on an object is proportional to its acceleration. In all cases, this required abandoning the prediction that net force was based partially or totally on velocity.

Gravitational force, net force and acceleration appear not only in the chapters of the textbook studied, but also on almost every example and problem from the chapter on Newtonian mechanics, so it is not surprising that subjects learned their properties. For instance, while studying the textbook, subjects repeatedly received explicitly the information that gravitational acceleration on earth is always straight down and equal in magnitude to 9.8 m/s^2.

Moreover, learning the new properties must have created conflicts with the pre-existing beliefs. On Problem OT, for instance, a subject cannot draw an acceleration arrow that is both vertical and aligned with the velocity. This kind of conflict must have been noticed by subjects. It probably motivated them not to apply their old beliefs about gravity to the posttest.

Dropping an impetus force. There were 4 cases where subjects dropped an impetus force. On their pretests, subjects P2, 101, and 105 included an impetus force acting on the pendulum bob of Problem PF. On their posttests, they included only the correct forces, gravity and tension.

Two possible explanations for the subjects' learning seem plausible. One explanation turns on the fact that 3 of the 4 cases occurred on the pendulum problem where an object is suspended by a string. During the phsyics training, the subjects studied two examples and solved five quantitative problems in which objects were suspended by strings. In all these problems, the object had exactly two forces on it, a tension force and a gravitational force. Since the pendulum problem is quite similar to these cases, the subjects may have drawn an analogy to them. However, this explanation does not explain why Subject 103 stopped to apply impetus forces to Problem VT, where an object is thrown vertically.

The second explanation is more complex but covers all the cases. Notice that in all 4 cases where an impetus force was dropped, the subjects also acquired qualitative knowledge about Newton's second law. Moreover, in 9 cases where students could have dropped an impetus and did not, they also did not learn qualitative aspects of Newton's law. There is little chance that this configuration could be a coincidence ($Chi - Square, p < .001$).

Newton's second law mandates that the net force on an object should be consistent with the object's acceleration. For instance, if all the forces are vertical, then acceleration must be vertical as well, and vice-versa. If the subjects had qualitative knowledge about Newton's second law, and they knew the object's acceleration, then they might add or drop forces in order to make the net force qualitatively consistent with the acceleration.

For instance, Subject 103 learned that the acceleration on Problem VT was straight down and constant. Since the subject also believed that gravity is straight down and constant, there is no way to squeeze an impetus force in without upsetting the balance of forces. Thus, the subject might not only have learned and repeatedly applied the definition of Newton's second law as it has been explicitly presented in the instruction. Rather, the subject might also have derived and constructed knowledge about qualitative aspects of Newton's law that have been left implicit in the instruction. This additionally constructed knowledge might in turn have lead to conflicts with already existing incorrect pre-knowledge and might have caused the subject to stop using an impetus force.

This explanation can also explain why the other three subjects dropped their impetus forces. However, the explanation requires assuming that the subjects can determine the acceleration of the object without using the net force. This is a plausible assumption in the case of projectiles, because their acceleration is always straight down and constant. However, these three subjects also changed their beliefs about impetus force on Problem PF, where the object is part of a pendulum. One must have a good understanding of the pendulum's motion in order to mentally envision its acceleration. Subjects 101 and 105 seem to have grasped the difference between acceleration and velocity, so this assumption is plausible for them, but Subject P2 seems to still confuse acceleration and velocity on the posttest, and thus it is doubtful that she can "see" the pendulum's acceleration with sufficient certainty that she would feel compelled to stop using an impetus force.

In short, the analogy-based explanation works well for three of the four cases, and so does the explanation based on a qualitative interpretation of Newton's second law. It could be that subjects drew some confidence from both lines of reasoning, albeit in different degrees, when deciding not to use an impetus force.

Adding an impetus. Subject 102 adds an impetus force on Problem PF, and makes several changes to her predictions about net force on all three problems. These 4 cases can be explained by extending the Newton's second law explanation used in the preceding section.

This subject seems to adopt the belief that net force should be consistent with *velocity.* Since subjects often confuse acceleration with velocity, it is plausible that the subject did learn this partially correct version of Newton's law from the physics training. On Problem PF, which was answered correctly on the pretest, the subject adds an impetus force, probably because this makes the net force consistent with the pendulum's velocity. Although not shown in Table 4, the subject also changes the direction and magnitude of some of her impetus forces on problems VT and OT, probably in order to make their sum consistent with the object's velocity. We suspect that the subject changed her impetus beliefs because the old versions were in conflict with her new (incorrect) belief about Newton's second law.

Table 5: Summary of changes in the use of rules across tests

Applying a new piece of knowledge	
3	Acquiring new rules.
Applying a new piece of knowledge on the basis of explicitly presented information and not applying a conflicting piece of knowledge	
13	Acquiring new correct rules causes suppression of the old rules.
3	Acquiring new incorrect rules causes suppression of the old rules.
Applying a new piece of knowledge on the basis of only implicitly presented information and not applying a conflicting piece of knowledge	
4	Acquiring new correct rules causes suppression of old incorrect rules.
1	Acquiring new incorrect rules causes addition of new incorrect rules.
Inexplicable	
2	Dropping the gravitational force.
1	Making the net force consistent with velocity.

Miscellaneous changes in belief. The foregoing categories explained 24 of the 27 changes in rule usage shown in Table 4. The remaining 3 changes (one change of Subject P1 with respect to the use of the net force and two changes of Subject P2 with respect to gravity) do not seem to fall into any pattern.

Summary of results. Table 5 summarizes the results of our analyses of how rule usage changed across the administered tests. There were 3 cases where subjects learned new knowledge about forces that complemented their qualitative physics knowledge. There were 13 cases where subjects revised their beliefs about properties of forces, and an additional 3 cases where the subject mis-learned that net force is proportional to velocity. In the first 16 cases, the new rules were explicitly addressed in the instructional material and were in conflict with already available ones. These conflicts seem to have caused suppression of the old rules on the posttest. Thus, these two kinds of learning are consistent with the transfer framework.

The most interesting category includes the 5 cases where students added or dropped impetus forces apparently in order to bring their predictions into conformance with their qualitative interpretations of Newton's second law. Since such qualitative interpretations of Newton's law were only implicitly addressed in the instruction, this finding is not consistent with the transfer framework as it stands. Although there is another, analogical explanation for 4 of these 5 cases, the Newton's second law explanation suggests that the transfer framework needs to be extended such that it accounts not only for learning on the basis of explicitly, but also for learning on the basis of only implicitly presented information.

Lastly, there were three cases that are not consistent with the transfer framework, even if we extend it to allow learning on the basis of only implicitly presented information. We are particularly at a loss to explain why Subject P2 stopped using gravitational force, which is perhaps the most familiar of all forces.

6.2 Why was There so Little Learning?

Although these findings elucidate the cases where transfer did occur, it is also worth trying to explain why transfer often did *not* occur. For instance, on Problem VT of both the pretest and the posttest, Subject 102 thought that the gravitational force varied in magnitude as the projectile rose and fell. The instruction did cover gravitational forces, and the student knew that a gravitational force's magnitude was 9.8 m/s^2, which is a constant. Why didn't the subject abandon his belief in variable gravitational forces? The nature of our data makes it difficult to answer a question like this directly. However, we sought hints in our data for explanations of failures to transfer.

As stated above, three kinds of learning were observed: (1) learning new knowledge on the basis of information that is explicitly presented in the instruction, (2) learning new knowledge on the basis of information that is explicitly addressed in the instruction and that conflicts with some incorrect pre-knowledge and (3) learning new knowledge on the basis of information that is only implicitly addressed in the instruction and that conflicts with some incorrect pre-knowledge. Thus, when transfer fails, it probably fails because some rule was not learned that should have been learned, or a conflict was not noticed that should have been noticed. We consider each possibility in turn.

There are three possible reasons why students could fail to learn a new rule: (1) the relevant information was not at all included in the instruction, (2) the relevant information was explicitly included in the instruction but the student nonetheless failed to learn it well enough or (3) the relevant information was implicitly included in the instruction but the student failed to derive it.

In Section 4, we pointed out that 39 correct qualitative physics rules are sufficient to answer

Table 6: Correlation of learning with descriptions in the instruction

	Gravity	Tension	Acceleration	Impetus	Net force
Transfer	3	2	7	4	7
No transfer	2	3	2	9	8
Frequency	.60	.40	.78	.31	.47
Descriptions	23	14	25	1	17

all the problems on the tests, and that 25 of these rules are not at all covered in the instruction. Thus, it should often be the case that students fail to learn because the rules they need to learn are just not taught.

However, for the three problems that were given a fine-grained analysis, it turns out that all the information required to answer them correctly is either implicitly or explicitly included in the instruction. These cases deserve closer examination, for these are the ones that have frustrated educators the most and sparked the curiousity of theorists such as diSessa (1988, 1993), McCloskey (1983) and Chi (1992).

Perhaps the simplest explanation for such failures to transfer is that the requisite rules presented in the instruction were not learned well enough. In order to transfer the presented rules, one must notice that they are relevant to the conceptual problems in question. If the rules acquired during the instruction were not practiced adequately, the context of conceptual problems may not suffice to trigger retrieval of the appropriate rules.

In order to test this simple explanation, we performed a simple correlational analysis comparing the frequency of transfer with the amount of instruction. In particular, we counted the number of times each concept was described in the instructional materials. We also counted the number of transfers from the phsyics training to qualitative physics for each concept, and the number of times there could have been a transfer, but none occurred. The results are shown in Table 6. The categories are the same as the ones used in Table 4. The frequency of transfer (labelled "Frequency") was calculated with the formula $T/(T+NT)$ where T is the number of cases of transfer for that concept (labelled "Transfer"), and NT is the number of cases in Table 4 where transfer could have occurred, because the pretest performance was incorrect, and yet the posttest performance showed no change (labelled "No transfer"). The correlation between the number of descriptions of a concept in the instructional material (labelled "Descriptions") and the frequency of transfer for that concept was .896 ($R^2 = .803$), which supports the hypothesis that failure to transfer was caused by inadequate practice with the taught knowledge.

It could also be that a subject did learn the appropriate rule, either because it has explicitly been taught or because it has been derived. However, the subject might have failed to notice that it conflicts with an already available incorrect rule and thus continued to apply the incorrect rule. Transfer is a potentially complex cognitive process, so there could well be other factors blocking transfer as well. For instance, while many of the 27 cases of observed learning involved acquiring qualitative aspects of quantitative rules, it could be that some students are unskilled at inferring qualitative aspects of quantitative rules.

Several students may not have even considered applying quantitative knowledge to the conceptual problems posed in the pre- and posttest. Whereas well-trained physicists have their qualitative and quantitative knowledge tightly related and are skilled at employing both in a coordinated way to solve conceptual as well as quantitative problems (cf. Ploetzner, 1995), some students may believe that there is no way to take advantage of $w = m \times g$ and other quantitative rules to solve conceptual problems, so they disregard them, either intentionally or unintentionally.

Still another possibility is that there is something intrinsically different about the misconceptions themselves that makes mere conflict inadequate for belief revision. For instance, Chi, Slotta and de Leeuw (1994; see also Carey, 1985, 1991; Chi, 1992) hypothesize that many mechanics misconceptions are rooted in a miscategorization of force as a materialistic property when it is really a constraint-based interaction. Because constraint-based interaction is an ontological category that is unfamiliar to students, they cannot correct their misconceptions by simply recategorizing "force" in the same way that they can correct their misconceptions about "whale" by recategorizing whales as mammals (a familiar ontological category) instead of fish.

7 Discussion

The research problem addressed here is explaining what kind of learning takes place when students learn qualitative physics during standard textbook-based physics training. Our approach was to use a standard framework for transfer. We developed and implemented computerized rule-based simulation models of both conceptual physics problem solving and quantitative physics problem solving. On the basis of these models two analyses were conducted. In the first analysis, it was determined whether the degree of rule-overlap between the two models is consistent with the observed degree of transfer between the pre- and posttest on qualitative physics. In the second analysis, we determined the kinds of learning that occured during the physics training between the pre- and posttest.

We found that 41% of the rules in the model on quantitative problem solving had corresponding rules in the model on conceptual problem solving, which compared favorably to the 36% transfer we found from the pre- to the posttest. Although the rule overlap supported the transfer framework, it painted the wrong picture of how transfer actually occurs. It suggested that all transfer was due to learning missing correct rules in the students' qualitative physics knowledge. The missing knowledge caused errors before the physics training, but the rules acquired during the instruction filled the gaps and prevented those errors on the posttest. A finer-grained analysis of the students' conceptual problem solving revealed that this was only one of three kinds of observed transfer, and it was the least influential of the three. The following lists the three kinds of transfer that were found:

- On the pretest, students made errors because they lacked some qualitative physics knowledge. The instruction explicitly provided information from which that knowledge was learned. On the posttest, the students applied the newly acquired knowledge and ceased making the errors they made on the pretest. We observed 3 cases of this kind of transfer, all involving the learning of a force.

- On the pretest, students made errors because they applied an incorrect piece of knowledge instead of a correct one. The instruction explicitly provided information from which the correct knowledge was learned. The newly acquired correct knowledge conflicted with the incorrect pre-knowledge in that both were are applicable to the same situations but drew different conclusions. The students noticed the conflict between the correct and incorrect knowledge, and stopped applying the incorrect knowledge. We observed 13 cases of this kind of transfer, and an additional 3 cases that were similar, except that the acquired knowledge was only partially correct.

- On the pretest, students made errors because they applied an incorrect piece of knowledge instead of a correct one. The correct knowledge was learned by deriving and constructing information only implicitly presented in the instruction. The newly acquired correct knowledge conflicted with the incorrect pre-knowledge in that both were are applicable to the same situations but drew different conclusions. The students noticed the conflict between the correct and incorrect knowledge, and stopped applying the incorrect knowledge. We observed 4 cases where students stopped applying incorrect knowledge when it conflicted indirectly with information presented during the physics training. In particular, the students stopped applying impetus forces, probably because they conflicted with newly acquired qualitative knowledge about Newton's second law. In one additional case, a student learned an incorrect version of Newton's second law, which seemed to have caused the student to add an impetus force.

Of the 27 cases of learning that we observed, the first 19 cases listed above were consistent with the transfer framework as it stands. Though this finding supports the framework fairly strong, there appears to be an important form of learning unpredicted by the transfer framework. In particular, as a consequence of such learning students stopped applying misconceptions. However, the instruction did not explicitly address the inappropriateness of these misconceptions nor did it provide explicitly information that conflicted with these misconcpetions. This suggests that in these cases of learning students were able to interpret quantitative physics knowledge such as Newton's second law qualitatively and thus to construct new qualitative physics knowledge that conflicted with their incorrect pre-knwoledge.

Thus, successful students learned not only by memorizing what has explicitly been stated in the instruction. Rather, they also learned by constructing information left implicit in the instruction, relating this information to their pre-knowledge and possibly refining and modifying their pre-knowledge in those cases where conflicts became aware making their knowledge internally at least in part consistent. However, what could have been the basis of these constructive cognitive activities?

Chi et al. (1989) found that all of the subjects in their study produced a number of self-explanations while studying the three worked-out examples (cf. Section 2). During self-explaining, a subject explains the different solution steps provided in the worked-out example to oneself. In particular, Chi et al. (1989) denoted those comments of a subject as self-explanations which referred to the content of physics, as opposed to paraphrases, monitoring statements or other statements. In a further analysis, Chi and VanLehn (1991) investigated the content of self-explanations. Among others, they found that subjects frequently identified and articulated various qualitative aspects of Newton's laws while self-explaining. Thus, one important source for the constructive, knowledge-building activities described above might have been the subjects' self-explanations.

Our findings provide evidence that students may fail to learn the knowledge required for successful conceptual problem solving, because of three reasons: (1) the corresponding information is not included in the instruction, (2) the newly acquired knowledge is not practiced enough or (3) students do not sufficiently engage in constructive cognitive activities to refine, modify and appropriately extend their qualitative physics knowledge. With respect to the first two reasons, it is to hope that physics textbooks with an increased emphasis on qualitative physics help to overcome these shortcommings (e.g., Hewitt, 1981; Reif, 1995a, 1995b).

With respect to the third reason, a pedagogically important question is how students can be encouraged to engage in construcitve cognitive activities. The elicitation of self-explanations (Chi, de Leeuw, Chiu & LaVancher, 1994) is just one possibility to support the appropriate modification of already existing and the construction of new knowledge. Drawing diagrams, explaining to others, arguing, and critiquing are examples of further activities which may lead to the construction of new knowledge. As Ohlsson (1995) points out, in the past quite elaborate models of cognitive skill acquisition have been developed. What we need next are models of epistemic, knowledge constructing activities. We need to understand what precisely constitutes these activities, under which conditions they can fruitfully be taken advantage of, how they can complement each other as well as processes of skill acquisition and how they can be initiated and supported in instructional settings.

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A Appendix

This appendix briefly discusses some misconceptions found in the students responses that may not have appeared yet in the misconceptions literature. The rest of the appendix is a list of the rules used to model the student's correct and incorrect solutions to the pre- and posttest problems.

A.1 Misconceptions that may not have been Observed Previously

Three subjects responded to the problems shown in Figures 1a and 1b with a gravitational force that was increasing across the provided trajectory. On the one hand, Driver (1981) reports one subject's false belief that gravity has a greater effect on higher objects. The subject justified this belief with the observation that objects which fall from higher distances cause greater damage. On the other hand, the belief that gravity pulls harder when an object is closer to earth was found by several authors to be prevalent among students in introductory physics courses (e.g., Champagne, Klopfer & Anderson, 1980; Halloun & Hestenes, 1985a; McDermott, 1984). However, none of these studies reported the misconception we discovered during our analysis. We hypothesize that the line of reasoning that leads to the belief that the gravitational force is increasing over time can be considered complimentary to the reasoning that leads to the belief that an object's impetus is decreasing over time. Typically, an impetus is believed to be imparted to an object by some action of rather short duration such as a push or a kick. The imparted impetus accounts for subsequent motion of the object, but it is believed to be continuously decreasing since nothing supplies it any more and the object's motion consumes it. On the other hand, the pull of the earth, which is responsible for the gravitational force on an object, acts continuously on the object, and hence builds up the gravitational force over time.

The belief that the tension force on a pendulum bob is constant or that its magnitude equals the magnitude of the gravitational force was expressed by two subjects in response to a sequence of four pendulum situations, one of which is shown in Figure 1c. Both subjects believed that an impetus makes the pendulum swing in the specified direction. Hence, for these subjects there may have been no need to account for the pendulum's curved path by means of a changing tension force. However, not every subject who considered an impetus as responsible for the pendulum's motion assumed a constant tension force.

Three different conditions under which subjects invented an impetus force might not have been differentiated elsewhere in the literature: an impetus which is always in a horizontal direction, an impetus which makes an object move in the direction it will subsequently travel, and an impetus in an oscillating system such as a pendulum which makes the system swing backwards. These could simply be a result of our analysis, which, like most computational analyses, is more detailed than those conducted by hand.

A.2 The Rules Used in Modeling the Subjects' Responses

An overall of 124 rules were used in modeling the subjects' responses to the pre- and posttest of the Chi et al. (1989) study. Preceeding each rule is a two-letter code. The first letter is "C" if the rule is correct and "W" if it is not. The second letter is "E" if the rule is equal to some CASCADE rule, "Q" if it is a derived version of a CASCADE rule, "I" if it is implied by two or more CASCADE rules, and "N" if it does not correspond to any CASCADE rules.

Gravitational force

1 CE If there is a body which is not the earth, the body is not massless, gravity is not to be ignored, then there is a gravitational force on the body due to the earth.

2 CQ If there is a gravitational force on a body due to the earth, then the magnitude of the gravitational force is constant.

3 CQ If there is a gravitational force on a body due to the earth, then the magnitude of the gravitational force is larger than zero.

4 CE If there is a gravitational force on a body due to the earth, then the vertical direction of the gravitational force equals down.

5 CE If there is a gravitational force on a body due to the earth, then the horizontal direction of the gravitational force equals none.

Tension force

6 CE If there is a body which is tied to a pulley, then there is a tension force on the body due to the pulley.

7 WN If there is a tension force on a body due to a pulley and the magnitude of the body's velocity is zero, then the magnitude of the tension force is zero.

8 CN If there is a tension force on a body due to a pulley, then the magnitude of the tension force is inversely related to the magnitude of the difference between the inclination of the pulley and 90 degrees.

9 WN If there is a tension force on a body due to a pulley, then the magnitude of the tension force is constant.

10 WN If there is a tension force on a body due to a pulley and there is a gravitational force on the body due to the earth, then the magnitude of the tension force is proportionally related to the magnitude of the gravitational force.

11 CN If there is a gravitational force on a body due to the earth, there is a tension force on a body due to a pulley, the magnitude of the body's velocity is larger than zero and the inclination of the pulley equals 90 degrees, then the magnitude of the tension force is larger than the magnitude of the gravitational force.

12 WN If there is a gravitational force on a body due to the earth and there is a tension force on a body due to a pulley, then the magnitude of the tension force equals the magnitude of the gravitational force.

13 CE If there is a tension force on a body due to a pulley, then the horizontal direction of the tension force equals the relative horizontal position of the pulley with respect to the body.

14 CE If there is a tension force on a body due to a pulley, then the vertical direction of the tension force equals the relative vertical position of the pulley with respect to the body.

Reaction force

15 CN If there is an action force on a body, then there is also a reaction force on the body.

16 CN If there is an action force on a body and there is a reaction force on the body, then the horizontal direction of the reaction force is opposite to the horizontal direction of the

action force.

17 CN If there is an action force on a body and there is a reaction force on the body, then the vertical direction of the reaction force is opposite to the vertical direction of the action force.

Thrust force

18 CN If there is a rocket which fires its engine, then there is a thrust force on the rocket due to the engine.

19 CN If the is a thrust force on a rocket due to its firing engine, then the magnitude of the thrust force is constant.

20 CN If there is a thrust force on a rocket due to its firing engine, then the horizontal direction of the thrust force equals the horizontal direction in which the engine fires.

21 CN If there is a thrust force on a rocket due to its firing engine, then the vertical direction of the thrust force is opposite to the vertical direction in which the engine fires.

Centripetal force

22 CN If there is a body and some object causes curvilinear motion of the body, then there is a centripetal force on the body due to the object.

23 CN If there is a curved tube which contains a moving body, then the tube causes curvilinear motion of the body.

24 CN If there is a rotating string with a body tied to it, then the string causes curvilinear motion of the body.

Net force

25 CI If there is at least one force on a body, then there is a net force on the body.

26 WN If there is a net force on a body and the body has velocity, then the sign of the magnitude of the net force equals the sign of the magnitude of the velocity.

27 WN If there is a net force on a body and the body has velocity, then the magnitude of the net force is proportionally related to the magnitude of the velocity.

28 WN If there is a net force on a body and the body has velocity, then the horizontal direction of the net force equals the horizontal direction of the velocity.

29 WN If there is a net force on a body and the body has velocity, then the vertical direction of the net force equals the vertical direction of the velocity.

All forces

30 WN If there is a force on a body, the body has velocity and the magnitude of the velocity equals zero, then the magnitude of the force equals zero.

31 WN If there are exactly two forces on a body, then the horizontal direction of the larger force equals the horizontal direction of the net force.

32 WN If there are exactly two forces on a body, then the vertical direction of the larger force equals the vertical direction of the net force.

33 CI If there are exactly two forces on a body and the magnitude of the net force equals zero, then the two forces have equal magnitudes.

34 WN If there is a force on a body and the force is continuously kept up by some object, then the magnitude of the force increases over time.

35 WN If there is a force on a body and the force is not continuously kept up by some object, then the magnitude of the force decreases over time.

Impetus

36 WN If a body has velocity and there is no force on the body whose direction equals the direction of the velocity, then the body has an impetus.

37 WN If a body has velocity and there is no force on the body whose horizontal direction equals the horizontal direction of the velocity or whose vertical direction equals the vertical direction of the velocity, then the body has an impetus.

38 WN If a body has velocity and there is no force on the body whose horizontal direction equals the horizontal direction of the velocity, then the body has an impetus.

39 WN If a body has velocity and there is no force on the body whose vertical direction equals the vertical direction of the velocity, then the body has an impetus.

40 WN If a force is impressed into a body by some agent, then the body has an impetus.

41 WN If a body moves along a trajectory in outer space, then the body has an impetus.

42 WN If a body has velocity, the magnitude of the velocity equals zero and there is a force on the body, then the body has an impetus.

43 WN If a body starts moving along a trajectory and there is no force on the body whose horizontal and vertical direction is along the trajectory, then the body has an impetus.

44 WN If a body is swinging back and forth as a pendulum and there is no force on the body whose horizontal and vertical direction equals the horizontal and vertical direction in which the object is moving, then the body has an impetus.

45 WN If a body is swinging back and forth as a pendulum and there is no force on the body whose horizontal and vertical direction equals the horizontal and vertical direction in which the object will move backward, then the body has an impetus.

46 WN If a body has an impetus, there is exactly one force on the body and the magnitude of the 'net force' equals zero, then the sign of the magnitude of the impetus equals the sign of the magnitude of the force.

47 WN If a body has an impetus and the body has velocity, then the magnitude of the impetus is proportionally related to the magnitude of the velocity.

48 WN If a body has an impetus, the body has velocity and the magnitude of the velocity equals zero, then the magnitude of the impetus equals zero.

49 WN If a body has an impetus and the impetus is continuously impressed by some agent, then the magnitude of the impetus increases over time.

50 WN If a body has an impetus and the impetus is not continuously impressed by some agent, then the magnitude of the impetus decreases over time.

51 WN If a body has an impetus and the body has velocity, then the horizontal direction of the impetus equals the horizontal direction of the velocity.

52 WN If a body has an impetus and the body has velocity, then the vertical direction of the impetus equals the vertical direction of the velocity.

53 WN If a body has an impetus, then the horizontal direction of the impetus equals none.

54 WN If a body has an impetus, then the vertical direction of the impetus equals none.

55 WN If a body has an impetus, the body has velocity and there is exactly one force on the body, then the horizontal direction of the impetus is chosen in a way that the horizontal direction of the 'net force' equals the horizontal direction of the velocity.

56 WN If a body has an impetus, the body has velocity and there is exactly one force on the body, then the vertical direction of the impetus is chosen in a way that the vertical direction of the 'net force' equals the vertical direction of the velocity.

57 WN If a body has an impetus, there is exactly one force on the body and the magnitude of the 'net force' equals zero, then the horizontal direction of the impetus is opposite to the horizontal direction of the force.

58 WN If a body has an impetus, there is exactly one force on the body and the magnitude of the 'net force' equals zero, then the vertical direction of the impetus is opposite to the vertical direction of the force.

59 WN If a body has an impetus which has been impressed by some agent, then the horizontal direction of the impetus equals the horizontal direction of the action which impressed the impetus.

60 WN If a body has an impetus which has been impressed by some agent, then the vertical direction of the impetus equals the vertical direction of the action which impressed the impetus.

61 WN If a body has an impetus and the body moves along a trajectory, then the curvature of the impetus equals the curvature of the trajectory.

62 WN If a body has an impetus and there is exactly one force on body, then the sign of the magnitude of the impetus equals the sign of the magnitude of the force.

63 WN If a body has an impetus and there is exactly one force on the body, then the horizontal direction of the impetus is opposite to the horizontal direction of the force.

64 WN If a body has an impetus and there is exactly one force on the body, then the vertical direction of the impetus is opposite to the vertical direction of the force.

65 WN If a body has an impetus and the body starts moving along a trajectory, then the horizontal direction of the impetus equals the horizontal ‘direction’ of the tangent of the trajectory.

66 WN If a body has an impetus and the body starts moving along a trajectory, then the vertical direction of the impetus equals the vertical ‘direction’ of the tangent of the trajectory.

67 WN If a body has an impetus and the body moves along a trajectory, then the horizontal direction of the impetus equals the horizontal ‘direction’ of the tangent of the trajectory.

68 WN If a body has an impetus and the body moves along a trajectory, then the vertical direction of the impetus equals the vertical ‘direction’ of the tangent of the trajectory.

69 WN If a body has an impetus and the body has velocity, then the magnitude of the impetus is inversely related to the magnitude of the velocity.

70 WN If a body has an impetus and the body is swinging back and forth as a pendulum, then the horizontal direction of the impetus equals the horizontal ‘direction’ of the tangent of the trajectory and the body will move backward.

71 WN If a body has an impetus and the body is swinging back and forth as a pendulum, then the vertical direction of the impetus equals the vertical ‘direction’ of the tangent of the trajectory the body will move backward.

72 WN If a body is swinging back and forth as a pendulum, the body has velocity, the magnitude of the velocity is larger than zero, the body moves in a certain horizontal and vertical direction, the body has a first impetus in the horizontal and vertical direction it moves and the body has a second impetus in the horizontal and vertical direction it will move backward, then the magnitude of the first impetus is larger than the magnitude of the second impetus.

73 WN If a body is swinging back and forth as a pendulum, the body has velocity, the magnitude of the velocity equals zero, the body moves in a certain horizontal and vertical direction, the body has a first impetus in the horizontal and vertical direction it moves and the body has a second impetus in the horizontal and vertical direction it will move backward, then the magnitude of the first impetus equals the magnitude of the second impetus.

74 WN If a body has an impetus, there is exactly one force on the body, the body has velocity and the magnitude of the velocity changes from increasing to decreasing or vice versa, then the magnitude of the impetus equals the magnitude of the force.

75 WN If a body has an impetus, there is exactly one force on the body, the body has velocity and the horizontal direction of the force equals the horizontal direction of the velocity or the vertical direction of the force equals the vertical direction of the velocity, then the magnitude of the impetus is smaller than the magnitude of the force.

76 WN If a body has an impetus, there is exactly one force on the body, the body has velocity and neither the horizontal direction of the force equals the horizontal direction of the velocity nor the vertical direction of the force equals the vertical direction of the velocity, then the magnitude of the impetus is larger than the magnitude of the force.

77 WN If a body moves along a trajectory in outer space and the body has impetus, then the horizontal direction of the impetus equals the horizontal 'direction' of the tangent of the trajectory.

78 WN If a body moves along a trajectory in outer space and the body has impetus, then the vertical direction of the impetus equals the vertical 'direction' of the tangent of the

trajectory.

Velocity

79 CN If the distance covered by a body increases, then the body has velocity.

80 CN If a body moves along a trajectory, then the body has velocity.

81 WN If a body has velocity and the vertical direction of the velocity equals up, then the magnitude of the velocity decreases.

82 WN If a body has velocity and the vertical direction of the velocity equals down, then the magnitude of the velocity increases.

83 CN If a body has velocity and the magnitude of the velocity changes from decreasing to increasing or vice versa at a certain time, then the magnitude of the velocity is zero at that time.

84 CN If a body has velocity, the horizontal direction of the velocity changes from left to right or vice versa at a certain time and the vertical direction of the velocity changes from up to down or vice versa at a certain time, then the magnitude of the velocity is zero at that time.

85 WN If a body has velocity, then the sign of the magnitude of the velocity equals plus.

86 CN If a body has velocity and the body moves along a trajectory, then the horizontal direction of the velocity equals the horizontal 'direction' of the trajectory's tangent.

87 CN If a body has velocity and the body moves along a trajectory, then the vertical direction of the velocity equals the vertical 'direction' of the trajectory's tangent.

Acceleration

88 WN If a body has velocity, then the body is accelerated.

89 CN If there is a gravitational force on a body due to the earth and the body is falling, then the gravitational force causes the body to be accelerated.

90 WN If a body is accelerated and the body has velocity, then the magnitude of the acceleration is proportionally related to the magnitude of the velocity.

91 WN If a body is accelerated and the body has velocity, then the sign of the magnitude of the acceleration equals the sign of the magnitude of the velocity.

92 WN If a body is accelerated in two situations, in the first situation there is a force on the body whose direction is opposite to the direction of the acceleration and in the second situation there is no such force, then the magnitude of the acceleration in the first situation is smaller than the magnitude of the acceleration in the second situation.

93 WN If a body is accelerated, then the magnitude of the acceleration is constant.

94 WN If a body is accelerated and the body has velocity, then the horizontal direction of the acceleration equals the horizontal direction of the velocity.

95 WN If a body is accelerated and the body has velocity, then the vertical direction of the acceleration equals the vertical direction of the velocity.

96 WN If a body is accelerated, then the vertical direction of the acceleration equals none.

97 WN If a body is accelerated and the body moves along a trajectory which has a nonzero curvature, then the horizontal direction of the acceleration points toward the center of the curvature.

98 WN If a body is accelerated and the body moves along a trajectory which has a nonzero curvature, then the vertical direction of the acceleration points toward the center of the curvature.

99 CI If a single force causes a body to be accelerated, then the sign of the magnitude of the acceleration equals the sign of the magnitude of the force.

100 CI If a single force causes a body to be accelerated, then the magnitude of the acceleration is proportionally related to the magnitude of the force.

101 CI If a single force causes a body to be accelerated, then the horizontal direction of the acceleration equals the horizontal direction of the force.

102 CI If a single force causes a body to be accelerated, then the vertical direction of the acceleration equals the vertical direction of the force.

103 WN If a body is accelerated, then the horizontal direction of the acceleration equals none.

104 WN If a body is accelerated, then the vertical direction of the acceleration equals down.

A.3 Distance

105 CN If a body moves along a trajectory, then the distance covered by the object increases.

Trajectories

106 CN If a body is released from a carrier, the carrier moves along a trajectory and the carrier does not act on the body, then the body moves in that point along the same trajectory.

107 WN If a body is released from a carrier, the carrier moves along a trajectory and the carrier acts on the body in a certain horizontal direction, then the body moves in that horizontal direction.

108 WN If a body is released from a carrier, the carrier moves along a trajectory and the carrier acts on the body in a certain vertical direction, then the body moves in that vertical direction.

109 CN If a body moves horizontally and some agent acts vertically on the body, then the body moves along a trajectory with a curvature.

110 WN If a body moves horizontally and some agent acts vertically on the body, then the body moves in a straight line in the vertical direction.

111 WN If a body moves along a trajectory and some agent acts on the body in a certain direction, then the body moves in a straight line in this direction.

112 CN If a body moves upward to the right, there is exactly one force on the body and the direction of the force is down, then the body moves along an upward trajectory with a negative curvature.

113 CN If a body moves downward to the right, there is exactly one force on the body and the direction of the force is down, then the body moves along a downward trajectory with a negative curvature.

114 CN If a body is released from a carrier, the body has no velocity and there is exactly one force on the body, then the body moves in a straight line in the horizontal and vertical direction of the force.

115 WN If a body moves horizontally, the body has an impetus, there is a gravitational force on the body due to the earth, the magnitude of the impetus is smaller than a critical threshold and the magnitude of the impetus is smaller than the magnitude of the gravitational force, then the body moves along a downward trajectory with a curvature.

116 WN If a body moves along a trajectory, the body has an impetus, the body moved initially in a certain horizontal and vertical direction and the impetus is completely dissipated, then the body resumes moving in the initial horizontal and vertical direction.

117 WN If a body moves horizontally, the body has an impetus, there is a gravitational force on the body due to the earth, the magnitude of the impetus is smaller than a critical threshold and the magnitude of the impetus is larger than the magnitude of the gravitational force, then the body moves along a downward trajectory with a curvature.

118 WN If a body moves upward to the right, the body has an impetus, there is a gravitational force on the body due to the earth, the magnitude of the impetus is smaller than a critical threshold and the magnitude of the impetus is larger than the magnitude of the gravitational force, then the body moves along a upward trajectory with a negative curvature.

119 WN If a body moves downward, the body has an impetus, there is a gravitational force on the body due to the earth and the magnitude of the impetus equals zero, then the body moves in a straight line in the horizontal and vertical direction of the gravitational force.

120 WN If a body is released into projectile motion, the body moves in a certain horizontal and vertical direction, the body has an impetus and the magnitude of the impetus is larger

than a critical threshold, then the body keeps moving in the same horizontal and vertical direction.

121 WN If a body is released into projectile motion, the body has no impetus and there is a gravitational force on the body due to the earth, then the body moves in a straight line in the horizontal and vertical direction of the gravitational force.

122 WN If a body is released from an object, the object caused curvilinear motion of the object and the body has an impetus with a certain curvature, then the body moves along a trajectory with the same curvature.

123 WN If a body is released from an object, the object caused curvilinear motion of the object and the body has an impetus, then the body moves along a trajectory whose curvature is proportionally related to the magnitude of the impetus.

124 WN If a body moves downward, the body has an impetus and the direction of the impetus is to the right, then the body moves along a trajectory with a positive curvature.