

Table of  $z$  values and probabilities for the standard normal distribution.  $z$  is the first column plus the top row. Each cell shows  $P(X \leq z)$ . For example  $P(X \leq 1.04) = .8508$ . For  $z < 0$  subtract the value from 1, e.g.,  $P(X \leq -1.04) = 1 - .8508 = .1492$ .

$z$	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Table of  $t$  values and right tail probabilities. Degrees of freedom are in the first column (df). **Right tail probabilities** are in the first row. For example for  $d.f. = 7$  and  $\alpha = .05$  the critical  $t$  value for a two-tail test is 2.365 and for  $d.f. = 10$  and  $\alpha = .1$  the critical  $t$  value for a one-tail test is 1.372.

df	.1	.05	.025	.01	.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
35	1.306	1.690	2.030	2.438	2.724
40	1.303	1.684	2.021	2.423	2.704
45	1.301	1.679	2.014	2.412	2.690
50	1.299	1.676	2.009	2.403	2.678
75	1.293	1.665	1.992	2.377	2.643
100	1.290	1.660	1.984	2.364	2.626

Table of  $F$  values for right tail probabilities of .05. Degrees of freedom for denominator are in the first column and degrees of freedom for the numerator are in the top row.

denom.	numerator df									
df	1	2	3	4	5	7	10	15	50	100
1	161.45	199.5	215.71	224.58	230.16	236.77	241.88	245.95	251.77	253.04
2	18.51	19	19.16	19.25	19.3	19.35	19.4	19.43	19.48	19.49
3	10.13	9.55	9.28	9.12	9.01	8.89	8.79	8.7	8.58	8.55
4	7.71	6.94	6.59	6.39	6.26	6.09	5.96	5.86	5.7	5.66
5	6.61	5.79	5.41	5.19	5.05	4.88	4.74	4.62	4.44	4.41
6	5.99	5.14	4.76	4.53	4.39	4.21	4.06	3.94	3.75	3.71
7	5.59	4.74	4.35	4.12	3.97	3.79	3.64	3.51	3.32	3.27
8	5.32	4.46	4.07	3.84	3.69	3.5	3.35	3.22	3.02	2.97
9	5.12	4.26	3.86	3.63	3.48	3.29	3.14	3.01	2.8	2.76
10	4.96	4.1	3.71	3.48	3.33	3.14	2.98	2.85	2.64	2.59
11	4.84	3.98	3.59	3.36	3.2	3.01	2.85	2.72	2.51	2.46
12	4.75	3.89	3.49	3.26	3.11	2.91	2.75	2.62	2.4	2.35
13	4.67	3.81	3.41	3.18	3.03	2.83	2.67	2.53	2.31	2.26
14	4.6	3.74	3.34	3.11	2.96	2.76	2.6	2.46	2.24	2.19
15	4.54	3.68	3.29	3.06	2.9	2.71	2.54	2.4	2.18	2.12
16	4.49	3.63	3.24	3.01	2.85	2.66	2.49	2.35	2.12	2.07
17	4.45	3.59	3.2	2.96	2.81	2.61	2.45	2.31	2.08	2.02
18	4.41	3.55	3.16	2.93	2.77	2.58	2.41	2.27	2.04	1.98
19	4.38	3.52	3.13	2.9	2.74	2.54	2.38	2.23	2	1.94
20	4.35	3.49	3.1	2.87	2.71	2.51	2.35	2.2	1.97	1.91
21	4.32	3.47	3.07	2.84	2.68	2.49	2.32	2.18	1.94	1.88
22	4.3	3.44	3.05	2.82	2.66	2.46	2.3	2.15	1.91	1.85
23	4.28	3.42	3.03	2.8	2.64	2.44	2.27	2.13	1.88	1.82
24	4.26	3.4	3.01	2.78	2.62	2.42	2.25	2.11	1.86	1.8
25	4.24	3.39	2.99	2.76	2.6	2.4	2.24	2.09	1.84	1.78
26	4.23	3.37	2.98	2.74	2.59	2.39	2.22	2.07	1.82	1.76
27	4.21	3.35	2.96	2.73	2.57	2.37	2.2	2.06	1.81	1.74
28	4.2	3.34	2.95	2.71	2.56	2.36	2.19	2.04	1.79	1.73
29	4.18	3.33	2.93	2.7	2.55	2.35	2.18	2.03	1.77	1.71
30	4.17	3.32	2.92	2.69	2.53	2.33	2.16	2.01	1.76	1.7
40	4.08	3.23	2.84	2.61	2.45	2.25	2.08	1.92	1.66	1.59
60	4	3.15	2.76	2.53	2.37	2.17	1.99	1.84	1.56	1.48
100	3.94	3.09	2.7	2.46	2.31	2.1	1.93	1.77	1.48	1.39
1000	3.85	3	2.61	2.38	2.22	2.02	1.84	1.68	1.36	1.26

<p>Chapter 1: no key formulas.                  Chapter 2: Relative Frequency=freq. of the class / n.                  Approx. Class Width:                  =(largest value-smallest value) / number of classes.                  Chapter 3: sample and population means</p> $\bar{x} = \sum x_i/n \text{ and } \mu = \sum x_i/N$ <p>Weighted mean and geometric mean</p> $\bar{x} = \sum w_i x_i / w_i \text{ and } \bar{x}_g = [(x_1)(x_2) \dots (x_n)]^{1/n}.$ <p>Interquartile Range: IQR = <math>Q_3 - Q_1</math>.                  Population and sample variance</p> $\sigma^2 = \frac{\sum (x_i - \mu)^2}{N} \text{ and } s^2 = \frac{\sum (x_i - \bar{x})^2}{n - 1}$ <p>Population and sample standard deviation</p> $\sigma = \sqrt{\sigma^2} \text{ and } s = \sqrt{s^2}.$ <p>Coefficient of Variation</p> $\left( \frac{\text{Standard deviation}}{\text{Mean}} \times 100 \right) \%$ <p>z-Score: <math>z_i = \frac{x_i - \bar{x}}{s}</math>.                  Population and Sample Covariance</p> $\sigma_{xy} = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{N} \text{ and } s_{xy} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n - 1}$ <p>Population and Sample Pearson Correlation</p> $\rho_{xy} = \sigma_{xy} / (\sigma_x \sigma_y) \text{ and } r_{xy} = s_{xy} / (s_x s_y).$ <p>Chapter 4: Counting Rule for Combinations</p> $C_n^N = \binom{N}{n} = \frac{N!}{n!(N-n)!}.$ <p>Counting Rule for Permutations</p> $P_n^N = n! \binom{N}{n} = \frac{N!}{(N-n)!}.$ <p>Probability Rules: <math>P(A) = 1 - P(A^c)</math></p>	<p>Chapter 4 continued:</p> $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $P(A B) = \frac{P(A \cap B)}{P(B)}$ $P(A \cap B) = P(B)P(A B) = P(A)P(B A).$ <p>Multiplication Law for Independent Events</p> $P(A \cap B) = P(B)P(A).$ <p>Bayes' Theorem</p> $P(A_i B) = \frac{P(A_i)P(B A_i)}{P(A_1)P(B A_1) + P(A_2)P(B A_2) + \dots + P(A_n)P(B A_n)}$ <p>Chapter 5:                  Discrete Uniform Probability Mass Function: <math>f(x) = 1/n</math>.                  Expected Value of a Discrete R. V.: <math>E(x) = \mu = \sum x f(x)</math>.                  Variance of a Discrete R. V.:</p> $Var(x) = \sigma^2 = \sum (x - \mu)^2 f(x).$ <p>Number of Experimental Outcomes Providing Exactly <math>x</math> Successes in <math>n</math> Trials</p> $\binom{n}{x} = \frac{n!}{x!(n-x)!}.$ <p>Binomial Probability Mass Function</p> $P(X = x) = f(x) = \binom{n}{x} p^x (1-p)^{(n-x)}.$ <p>Expected Value for Binomial Distribution: <math>E(x) = \mu = np</math>.                  Variance for Binomial Distr.: <math>Var(x) = \sigma^2 = np(1-p)</math>.                  Poisson Probability Mass Function:</p> $P(X = x \mu) = f(x) = \frac{\mu^x e^{-\mu}}{x!}.$ <p>Hypergeometric Probability Mass Function and Expected Value:</p> $f(x) = \frac{\binom{r}{x} \binom{N-r}{n-x}}{\binom{N}{n}} \text{ and } E(x) = \mu = \frac{nr}{N}.$	<p>Chapter 5 continued: Variance for the Hypergeometric Distribution:</p> $Var(x) = \sigma^2 = n \left( \frac{r}{N} \right) \left( 1 - \frac{r}{N} \right) \left( \frac{N-n}{N-1} \right).$ <p>Chapter 6: Uniform PDF</p> $f(x) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$ <p>Normal PDF The density function is</p> $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right).$ <p>Converting to the Standard Normal rv:</p> $z = \frac{x - \mu}{\sigma}.$ <p>Exponential PDF and CDF for <math>x \geq 0</math></p> $f(x) = \mu^{-1} e^{-x/\mu} \text{ and } P(x \leq x_0) = 1 - e^{-x_0/\mu}.$ <p>Chapter 7: expected value of <math>\bar{x}</math></p> $E(\bar{x}) = \mu.$ <p>Standard Deviation of <math>\bar{x}</math> (Standard Error)</p> $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}.$ <p>Expected Value and Std Dev (Standard Error) of <math>\bar{p}</math></p> $E(\bar{p}) = p \text{ and } \sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}}$ <p>Finite Pop. Correction Factor: <math>\sqrt{(N-n)/(N-1)}</math>.                  Chapter 8: Interval Estimate of Population Mean, <math>\sigma</math> known and unknown</p> $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \text{ and } \bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$ <p>Necessary Sample Size for Interval Estimate of <math>\mu</math></p> $n = \frac{(z_{\alpha/2})^2 \sigma^2}{E^2}$
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<p>Chapter 8 continued: Interval Estimate of <math>p</math></p> $\hat{p} \pm z_{\alpha/2} \frac{p(1-p)}{\sqrt{n}}$ <p>Necessary Sample Size for Interval Estimate of <math>p</math></p> $n = \frac{(z_{\alpha/2})^2 p^*(1-p^*)}{E^2}$ <p>Chapter 9: Test Statistic for Hypothesis Tests About <math>\mu</math>, <math>\sigma</math> known and unknown</p> $z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \text{ and } t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$ <p>Test Stat for Hypothesis About <math>p</math></p> $z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$ <p>Chapter 10: Point Estimate and Standard Error for Difference in Two Population Means</p> $\bar{x}_1 - \bar{x}_2 \text{ and } \sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ <p>Interval Estimate and Test Statistic for Difference in Two Means with Known Variances</p> $\bar{x}_1 - \bar{x}_2 \pm z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \text{ and } z = \frac{\bar{x}_1 - \bar{x}_2 - D_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$ <p>Interval Estimate and Test Statistic for Difference in Two Means with Unknown Variances</p> $\bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \text{ and } t = \frac{\bar{x}_1 - \bar{x}_2 - D_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$ <p>Degrees of Freedom for <math>t</math>, Two Independent Random Samples</p> $df = \frac{1}{\frac{1}{n_1-1} \left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2-1} \left(\frac{s_2^2}{n_2}\right)^2} \left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)$	<p>Chapter 10 continued: Test Statistic (Matched Samples)</p> $t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}}$ <p>ANOVA Related:</p> $\bar{x}_j = \frac{\sum_{i=1}^{n_j} x_{ij}}{n_j} \quad s_j^2 = \frac{\sum_{i=1}^{n_j} (x_{ij} - \bar{x}_j)^2}{n_j - 1} \quad \bar{\bar{x}} = \frac{\sum_{j=1}^k \sum_{i=1}^{n_j} x_{ij}}{n_T}$ $MSTR = \frac{SSTR}{k-1} \quad SSTR = \sum_{j=1}^k n_j (\bar{x}_j - \bar{\bar{x}})^2 \quad MSE = \frac{SSE}{n_T - k}$ $SSE = \sum_{j=1}^k \sum_{i=1}^{n_j} (x_{ij} - \bar{x}_j)^2 \quad F = MSTR/MSE$ $SST = \sum_{j=1}^k \sum_{i=1}^{n_j} (x_{ij} - \bar{\bar{x}})^2 \quad SST = SSTR + SSE$ <p>Chapter 11: not covered in this course</p> <p>Chapter 12: <math>y = \beta_0 + \beta_1 x + \epsilon</math></p> $E(y) = \beta_0 + \beta_1 x \quad \hat{y} = b_0 + b_1 x \quad b_0 = \bar{y} - b_1 \bar{x}$ $b_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} \quad SSE = \sum (y_i - \hat{y}_i)^2$ $SST = \sum (y_i - \bar{y})^2 \quad SSR = \sum (\hat{y}_i - \bar{y})^2 \quad SST = SSR + SSE$ $r^2 = \frac{SSR}{SST} \quad r_{xy} = (\text{sign of } b_1) \sqrt{r^2} \quad s^2 = MSE = \frac{SSE}{n-2}$ <p>Standard Error of the Estimate, <math>s = \sqrt{MSE}</math>.</p> $\sigma_{b_1} = \frac{\sigma}{\sqrt{\sum (x_i - \bar{x})^2}} \quad s_{b_1} = \frac{s}{\sqrt{\sum (x_i - \bar{x})^2}} \quad t = \frac{b_1}{s_{b_1}}$ <p>For simple regression, <math>MSR = SSR</math> because there is only one independent variable.</p> $F = \frac{MSR}{MSE} \quad s_{\hat{y}^*} = s \sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{\sum (x_i - \bar{x})^2}}$ <p>Confidence Interval for <math>E(y^*)</math>: <math>\hat{y}^* \pm t_{\alpha/2} s_{\hat{y}^*}</math></p> $s_{\text{pred}} = s \sqrt{1 + \frac{1}{n} + \frac{(x^* - \bar{x})^2}{\sum (x_i - \bar{x})^2}}$	<p>Chapter 8 continued: Interval Estimate of <math>p</math></p> $\hat{p} \pm z_{\alpha/2} \frac{p(1-p)}{\sqrt{n}}$ <p>Necessary Sample Size for Interval Estimate of <math>p</math></p> $n = \frac{(z_{\alpha/2})^2 p^*(1-p^*)}{E^2}$ <p>Chapter 9: Test Statistic for Hypothesis Tests About <math>\mu</math>, <math>\sigma</math> known and unknown</p> $z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \text{ and } t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$ <p>Test Stat for Hypothesis About <math>p</math></p> $z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$ <p>Chapter 10: Point Estimate and Standard Error for Difference in Two Population Means</p> $\bar{x}_1 - \bar{x}_2 \text{ and } \sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ <p>Interval Estimate and Test Statistic for Difference in Two Means with Known Variances</p> $\bar{x}_1 - \bar{x}_2 \pm z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \text{ and } z = \frac{\bar{x}_1 - \bar{x}_2 - D_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$ <p>Interval Estimate and Test Statistic for Difference in Two Means with Unknown Variances</p> $\bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \text{ and } t = \frac{\bar{x}_1 - \bar{x}_2 - D_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$ <p>Degrees of Freedom for <math>t</math>, Two Independent Random Samples</p> $df = \frac{1}{\frac{1}{n_1-1} \left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2-1} \left(\frac{s_2^2}{n_2}\right)^2} \left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)$
<p>Chapter 10 continued: Test Statistic (Matched Samples)</p> $t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}}$ <p>ANOVA Related:</p> $\bar{x}_j = \frac{\sum_{i=1}^{n_j} x_{ij}}{n_j} \quad s_j^2 = \frac{\sum_{i=1}^{n_j} (x_{ij} - \bar{x}_j)^2}{n_j - 1} \quad \bar{\bar{x}} = \frac{\sum_{j=1}^k \sum_{i=1}^{n_j} x_{ij}}{n_T}$ $MSTR = \frac{SSTR}{k-1} \quad SSTR = \sum_{j=1}^k n_j (\bar{x}_j - \bar{\bar{x}})^2 \quad MSE = \frac{SSE}{n_T - k}$ $SSE = \sum_{j=1}^k \sum_{i=1}^{n_j} (x_{ij} - \bar{x}_j)^2 \quad F = MSTR/MSE$ $SST = \sum_{j=1}^k \sum_{i=1}^{n_j} (x_{ij} - \bar{\bar{x}})^2 \quad SST = SSTR + SSE$ <p>Chapter 11: not covered in this course</p> <p>Chapter 12: <math>y = \beta_0 + \beta_1 x + \epsilon</math></p> $E(y) = \beta_0 + \beta_1 x \quad \hat{y} = b_0 + b_1 x \quad b_0 = \bar{y} - b_1 \bar{x}$ $b_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} \quad SSE = \sum (y_i - \hat{y}_i)^2$ $SST = \sum (y_i - \bar{y})^2 \quad SSR = \sum (\hat{y}_i - \bar{y})^2 \quad SST = SSR + SSE$ $r^2 = \frac{SSR}{SST} \quad r_{xy} = (\text{sign of } b_1) \sqrt{r^2} \quad s^2 = MSE = \frac{SSE}{n-2}$ <p>Standard Error of the Estimate, <math>s = \sqrt{MSE}</math>.</p> $\sigma_{b_1} = \frac{\sigma}{\sqrt{\sum (x_i - \bar{x})^2}} \quad s_{b_1} = \frac{s}{\sqrt{\sum (x_i - \bar{x})^2}} \quad t = \frac{b_1}{s_{b_1}}$ <p>For simple regression, <math>MSR = SSR</math> because there is only one independent variable.</p> $F = \frac{MSR}{MSE} \quad s_{\hat{y}^*} = s \sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{\sum (x_i - \bar{x})^2}}$ <p>Confidence Interval for <math>E(y^*)</math>: <math>\hat{y}^* \pm t_{\alpha/2} s_{\hat{y}^*}</math></p> $s_{\text{pred}} = s \sqrt{1 + \frac{1}{n} + \frac{(x^* - \bar{x})^2}{\sum (x_i - \bar{x})^2}}$	<p>Chapter 12 continued: Prediction Interval for <math>y^*</math>:</p> $\hat{y}^* \pm t_{\alpha/2} s_{\text{pred}}$ <p>Residual for Observation <math>i</math>: <math>y_i - \hat{y}_i</math></p> <p>Chapter 13:</p> $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_p x_p + \epsilon$ $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_p x_p$ $\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + \dots + b_p x_p$ $SST = SSR + SSE \quad R^2 = \frac{SSR}{SST}$ $R_a^2 = 1 - (1 - R^2) \frac{n-1}{n-p-1}$ $MSR = \frac{SSR}{p} \quad MSE = \frac{SSE}{n-p-1} \quad F = \frac{MSR}{MSE}$ $t = \frac{b_i}{s_{b_i}}$ <p>Other Math Rule Reminders:</p> $e^x = \exp(x)$ $\ln 1 = 0 \quad \ln e = 1$ $x! = (x)(x-1)(x-2) \dots (2)(1)$ $0! = 1 \quad x^0 = 1$	<p>Chapter 12 continued: Prediction Interval for <math>y^*</math>:</p> $\hat{y}^* \pm t_{\alpha/2} s_{\text{pred}}$ <p>Residual for Observation <math>i</math>: <math>y_i - \hat{y}_i</math></p> <p>Chapter 13:</p> $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_p x_p + \epsilon$ $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_p x_p$ $\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + \dots + b_p x_p$ $SST = SSR + SSE \quad R^2 = \frac{SSR}{SST}$ $R_a^2 = 1 - (1 - R^2) \frac{n-1}{n-p-1}$ $MSR = \frac{SSR}{p} \quad MSE = \frac{SSE}{n-p-1} \quad F = \frac{MSR}{MSE}$ $t = \frac{b_i}{s_{b_i}}$ <p>Other Math Rule Reminders:</p> $e^x = \exp(x)$ $\ln 1 = 0 \quad \ln e = 1$ $x! = (x)(x-1)(x-2) \dots (2)(1)$ $0! = 1 \quad x^0 = 1$

Use the tables/output below for some of the questions.

Table 1.

ANOVA						
<i>Source of Variation</i>	<i>SS</i>	<i>df</i>	<i>MS</i>	<i>F</i>	<i>P-value</i>	<i>F crit</i>
Between Groups	40.99485	3	13.66495	4.109522	0.029617	3.410534
Within Groups	43.2275	13	3.325192			
Total	84.22235	16				

Table 2.

ANOVA						
<i>Source of Variation</i>	<i>SS</i>	<i>df</i>	<i>MS</i>	<i>F</i>	<i>P-value</i>	<i>F crit</i>
Treatments			1582.887		0.011958	3.343889
Error		14	299.0679			
Total	8935.611	17				

Table 3.

<i>Regression Statistics</i>	
Multiple R	0.983721828
R Square	0.967708634
Adjusted R Square	0.963672214
Standard Error	10.34949634
Observations	10

ANOVA						
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>	
Regression	1	25679.5034	25679.5	239.7442	3.01226E-07	
Residual	8	856.8965962	107.1121			
Total	9	26536.4				

	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>
Intercept	6.710937958	8.334869306	0.805164	0.443995	-12.5093051	25.931181
advertising	1.939363759	0.125252147	15.48368	3.01E-07	1.650531789	2.2281957

ECN221 Exam 3A Spring 2015, ASU-COX

Choose the best answer. Do not write letters in the margin or communicate with other students in any way. If you have a question note it on your exam and ask for clarification when your exam is returned. In the meantime choose the best answer. Neither the proctors nor Dr. Cox will answer questions during the exam. Dr. Cox will post a key the day after the exam or in the case of the final exam the day after all finals are given. Grades will be posted on Bb after scores are returned from the testing center.

Please check each question and possible answers thoroughly as questions at the bottom of a page sometimes run onto the next page.

Relax. You studied. You know the material. You can do great.

1. I have checked that my ID is bubbled in correctly. If it is bubbled in incorrectly I will get this question wrong.

(a) True.

(b) False.

2. The total spent on a credit card in a month is

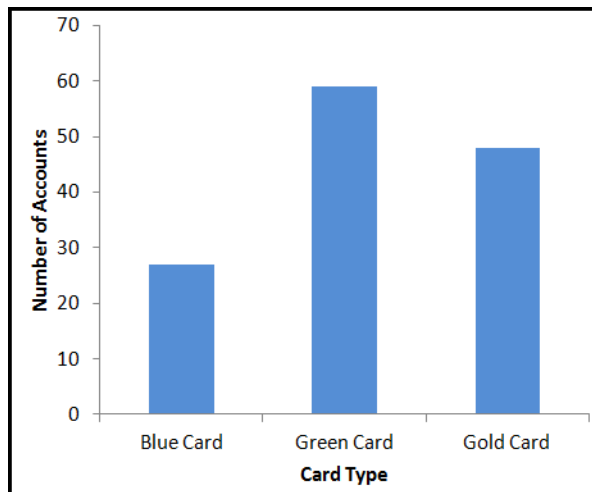
(a) interval data.

(b) ratio data.

(c) nominal data.

(d) qualitative data.

3. The display below is an example of a(n)



- (a) histogram.
  - (b) ogive.
  - (c) scatter plot.
  - (d) bar chart.
4. A set of credit card accounts have the following estimated probabilities of default: .067, .743, .042, .022, .031, .194, .001. Find the 30th percentile.
- (a) 2.1
  - (b) .031
  - (c) .036
  - (d) .022

5. A study by the Institute for Higher Education Policy found the values in the joint probability table below. The underlying data are for former college students that had taken out student loans. The table shows whether the student received a college degree versus whether they are successfully making their student loan payments. What is the probability that a former student did not complete their degree given that they are currently delinquent on their loan payments?

	holds a college degree		
loan status	yes	no	total
satisfactory	.26	.24	.50
delinquent	.16	.34	.50
total	.42	.58	1

- (a) .34.
  - (b) .68.
  - (c) .50.
  - (d) .59.
6. Suppose at ASU that 56% of students take out some form of loan to pay for their schooling. In a class of 350 students what is the expected number of students with a loan?
- (a) 210.
  - (b) 19.
  - (c) 350.
  - (d) .56.



- (e) 196.
7. Assume that the flight time for U.S. Airways flight 1267 from Philadelphia to Dallas follows the uniform probability distribution between 210 and 250 minutes. What is the probability that a randomly selected flight will take more than 225 minutes?
- (a) .425.  
(b) .625.  
(c) .50.  
(d) .375.  
(e) .525.
8. As  $n$  gets “large” the distribution of  $\bar{x}$  approaches
- (a) the normal distribution.  
(b) the distribution of  $X$ .  
(c) the distribution of  $X$  but with a smaller standard deviation.  
(d) the uniform distribution.  
(e) the centralized distribution.
9. Construct a 90% confidence interval for the mean when  $\bar{x} = 50$  and there are 31 observations. Suppose  $\sigma = 7$ . What is the confidence interval? Don’t round until the final step.
- (a) [48.24, 51.76].  
(b) [47.93, 52.07].  
(c) [47.54, 52.46].  
(d) [47.87, 52.13].  
(e) [46.54, 53.45].
10. Set the maximum acceptable probability of a Type I error at .05. Test the hypothesis that the average salary for men is at least \$1,000 more than the average salary for women;  $H_0 : \mu_M \geq \mu_W + 1,000$ . You have a sample of 40 men and 40 women and the average salaries are \$56,500 and \$53,800. The estimated standard deviations are \$1,566 and \$2,022. What is the value of the test statistic and do you reject or fail to reject  $H_0$ ? Note that there are 73 degrees of freedom.
- (a) -1.88, fail to reject

- (b) .47, fail to reject
  - (c) 2.03, reject
  - (d) 4.2, fail to reject
  - (e) 6.6, reject
11. Set the maximum acceptable probability of a Type I error at .05. Test the hypothesis that the average salary for economics majors is the same as the average salary accounting majors;  $H_0 : \mu_e = \mu_a$ . You have a sample of 32 people with economics degrees and 35 people with accounting degrees and the average salaries are \$58,700 and \$55,400. The estimated standard deviations are \$3,616 and \$2,001. What is the value of the test statistic and do you reject or fail to reject  $H_0$ ? Note that there are 47 degrees of freedom.
- (a) 2.01, reject
  - (b) 1.68, fail to reject
  - (c) 723, reject
  - (d) 4.2, fail to reject
  - (e) 4.56, reject
12. See the excel output in ANOVA Table 1. What is the test statistic?
- (a) 3.41
  - (b) .0296
  - (c) 4.109
  - (d) 3.325
  - (e) 84.222
13. See the excel output in ANOVA Table 1. How many observations were used in this study?
- (a) 17
  - (b) 16
  - (c) 13
  - (d) 3
  - (e) 20
14. See ANOVA Table 2. What is MSE? This is the table with the word "Treatments" in it. What is the test statistic?

- (a) .0119
- (b) 3.344
- (c) .189
- (d) 5.293
- (e) 1.134

15. See the excel output in ANOVA Table 1. What can you conclude?

- (a)  $\mu_1 = \mu_2$
- (b)  $\mu_1 = \mu_2 = \mu_3 = \mu_4$  is a false statement
- (c)  $\mu_1 = \mu_4$  is a false statement
- (d)  $\mu_1 = \mu_2 = \mu_3 = \mu_4$  is a true statement
- (e)  $\beta_1 \neq 0$

16. See ANOVA Table 2. What is the sum of squares due to treatments?

- (a) 1582.3
- (b) 299.1
- (c) 8936.6
- (d) 4186.9
- (e) 4748.6

17. See ANOVA Table 2. You are willing to accept a Type I error with a probability of .01. Test  $H_0$  and state your conclusion.

- (a) your decision depends on  $\alpha$ .
- (b) your decision depends on whether it is a left or right tail test or a two tail test.
- (c) you reject the null hypothesis.
- (d) you fail to reject the null hypothesis.
- (e) your decision depends on the degrees of freedom.

18. Using the least squares method for simple regression the value of  $R^2$  will be

- (a) between 0 and 1
- (b) minimized
- (c) depends on the values of  $x$  and  $y$
- (d) between -1 and 1

- (e) 0
19. Refer to the regression output in Table 3. It shows results for a regression of sales on advertising. What percent of the variation in sales can be explained by the regression?
- (a) 98.3%
  - (b) 96.7%
  - (c) 96.3%
  - (d) 10.3
  - (e) 19.39%
20. Refer to the regression output in Table 3. It shows results for a regression of sales on advertising. What is the estimated variance of the error term?
- (a) 8.33
  - (b) .967
  - (c) 10.35
  - (d) 26536
  - (e) 107.1
21. Refer to the regression output in Table 3. It shows results for a regression of sales on advertising. You want to test the hypothesis that a change in advertising has no impact on sales;  $H_0 : \beta_1 = 0$ . You
- (a) need the degrees of freedom to conduct the test; more information is needed.
  - (b) fail to reject at conventional levels of significance.
  - (c) reject  $H_0$  at conventional levels of significance.
  - (d) eat strawberry ice cream.
  - (e) decide to major in exercise science.
22. Refer to the regression output in Table 3. It shows results for a regression of sales on advertising. What is the predicted level of sales when advertising=10 ( $X = 10$ )?
- (a) 8.65.
  - (b) 13.47.
  - (c) 26.1.
  - (d) 261.
  - (e) 6.71.

23. What is  $R^2$  when total sum of squares is 144 and sum of squares due to error is 100?
- (a) .88.
  - (b) .69.
  - (c) .44.
  - (d) .305.
  - (e) .203.
24. Adjusted  $R^2$  makes an adjustment to the standard value of  $R^2$  to account for
- (a) the number of explanatory variables.
  - (b) the number of observations.
  - (c) the level of statistical significance.
  - (d) the variance of the error term.
  - (e) the correlation between the values of the explanatory variables.
25. Refer to the regression output in Table 3. It shows results for a regression of sales on advertising.  $\bar{Y} = 125.4$ , what is  $\bar{X}$ ? So average sales are 125.4 and you are finding what was the average advertising expenditure.
- (a) 34.1.
  - (b) 47.2.
  - (c) 55.4.
  - (d) 61.2.
  - (e) 71.2.
26. Refer to the regression output in Table 3. What is  $n$ ?
- (a) 1.
  - (b) 4.
  - (c) 8.
  - (d) 9.
  - (e) 10.

## Key

1. a

2. b

3. d

4. b

5. b

6. e.  $(.56)(350) = 196$ .

7. b.

$$1 - \frac{225 - 210}{250 - 210} = .625.$$

8. a.

9. b. Find the margin of error

$$z \frac{\sigma}{\sqrt{n}} = 1.645 \frac{7}{\sqrt{31}} = 2.068.$$

Then  $50 \pm 2.068$  gives approximately  $[47.93, 52.07]$ .

10. d. The standard error is

$$\sqrt{\frac{(1566)^2}{40} + \frac{(2022)^2}{40}} = 404.377$$

so that the test statistic is

$$\frac{56500 - 53800 - 1000}{404.377} = 4.20.$$

The critical value is about  $-1.665$  and  $4.2 > -1.665$  so we fail to reject the null hypothesis.

11. e.

$$t = \frac{58700 - 55400}{723.19} = 4.56$$

The critical value was about  $\pm 2.01$  so we reject the null. The standard error was found as

$$\sqrt{\frac{3616^2}{32} + \frac{2001^2}{35}} = 723.19.$$

12. c.

13. a.

14. d.

15. b. The F test statistic reported in the ANOVA table is for the hypothesis

$$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4.$$

The p-value is .0296 so given that  $\alpha$  is not stated we will assume it is .05 and reject  $H_0$  because  $.0296 < .05$ .

16. e.

17. d.

18. a.

19. b.

20. e.

21. c.

22. c. The equation is

$$\hat{Y} = 6.71 + 1.939(10) = 6.71 + 19.39 = 26.1.$$

23. d.

$$R^2 = \frac{144 - 100}{144} = \frac{44}{144} = .305.$$

24. a.

25. d. Recall that

$$b_0 = \bar{y} - b_1\bar{x}$$

which is also given in the formula sheet. The regression line goes through the mean of both variables  $(\bar{x}, \bar{y})$ . Then rearranging the equation

$$\bar{y} = 125.4 = 6.71 + 1.94(\bar{x}).$$

Then

$$\bar{x} = \frac{125.4 - 6.71}{1.94} = 61.2.$$

26. e.