

Table of  $z$  values and probabilities for the standard normal distribution.  $z$  is the first column plus the top row. Each cell shows  $P(X \leq z)$ . For example  $P(X \leq 1.04) = .8508$ . For  $z < 0$  subtract the value from 1, e.g.,  $P(X \leq -1.04) = 1 - .8508 = .1492$ .

$z$	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

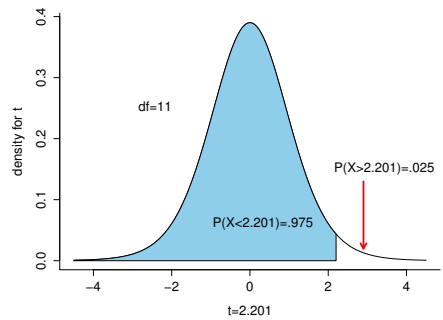


Table of  $t$  values and right tail probabilities. Degrees of freedom are in the first column (df). **Right tail probabilities** are in the first row. For example for  $d.f. = 7$  and  $\alpha = .05$  the critical  $t$  value for a two-tail test is 2.365 and for  $d.f. = 10$  and  $\alpha = .1$  the critical  $t$  value for a one-tail test is 1.372.

df	.1	.05	.025	.01	.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
40	1.303	1.684	2.021	2.423	2.704
50	1.299	1.676	2.009	2.403	2.678
75	1.293	1.665	1.992	2.377	2.643
100	1.290	1.660	1.984	2.364	2.626

Table of  $F$  values for right tail probabilities of .05. Degrees of freedom for denominator are in the first column and degrees of freedom for the numerator are in the top row.

denom.	numerator df									
df	1	2	3	4	5	7	10	15	50	100
1	161.45	199.5	215.71	224.58	230.16	236.77	241.88	245.95	251.77	253.04
2	18.51	19	19.16	19.25	19.3	19.35	19.4	19.43	19.48	19.49
3	10.13	9.55	9.28	9.12	9.01	8.89	8.79	8.7	8.58	8.55
4	7.71	6.94	6.59	6.39	6.26	6.09	5.96	5.86	5.7	5.66
5	6.61	5.79	5.41	5.19	5.05	4.88	4.74	4.62	4.44	4.41
6	5.99	5.14	4.76	4.53	4.39	4.21	4.06	3.94	3.75	3.71
7	5.59	4.74	4.35	4.12	3.97	3.79	3.64	3.51	3.32	3.27
8	5.32	4.46	4.07	3.84	3.69	3.5	3.35	3.22	3.02	2.97
9	5.12	4.26	3.86	3.63	3.48	3.29	3.14	3.01	2.8	2.76
10	4.96	4.1	3.71	3.48	3.33	3.14	2.98	2.85	2.64	2.59
11	4.84	3.98	3.59	3.36	3.2	3.01	2.85	2.72	2.51	2.46
12	4.75	3.89	3.49	3.26	3.11	2.91	2.75	2.62	2.4	2.35
13	4.67	3.81	3.41	3.18	3.03	2.83	2.67	2.53	2.31	2.26
14	4.6	3.74	3.34	3.11	2.96	2.76	2.6	2.46	2.24	2.19
15	4.54	3.68	3.29	3.06	2.9	2.71	2.54	2.4	2.18	2.12
16	4.49	3.63	3.24	3.01	2.85	2.66	2.49	2.35	2.12	2.07
17	4.45	3.59	3.2	2.96	2.81	2.61	2.45	2.31	2.08	2.02
18	4.41	3.55	3.16	2.93	2.77	2.58	2.41	2.27	2.04	1.98
19	4.38	3.52	3.13	2.9	2.74	2.54	2.38	2.23	2	1.94
20	4.35	3.49	3.1	2.87	2.71	2.51	2.35	2.2	1.97	1.91
21	4.32	3.47	3.07	2.84	2.68	2.49	2.32	2.18	1.94	1.88
22	4.3	3.44	3.05	2.82	2.66	2.46	2.3	2.15	1.91	1.85
23	4.28	3.42	3.03	2.8	2.64	2.44	2.27	2.13	1.88	1.82
24	4.26	3.4	3.01	2.78	2.62	2.42	2.25	2.11	1.86	1.8
25	4.24	3.39	2.99	2.76	2.6	2.4	2.24	2.09	1.84	1.78
26	4.23	3.37	2.98	2.74	2.59	2.39	2.22	2.07	1.82	1.76
27	4.21	3.35	2.96	2.73	2.57	2.37	2.2	2.06	1.81	1.74
28	4.2	3.34	2.95	2.71	2.56	2.36	2.19	2.04	1.79	1.73
29	4.18	3.33	2.93	2.7	2.55	2.35	2.18	2.03	1.77	1.71
30	4.17	3.32	2.92	2.69	2.53	2.33	2.16	2.01	1.76	1.7
40	4.08	3.23	2.84	2.61	2.45	2.25	2.08	1.92	1.66	1.59
60	4	3.15	2.76	2.53	2.37	2.17	1.99	1.84	1.56	1.48
100	3.94	3.09	2.7	2.46	2.31	2.1	1.93	1.77	1.48	1.39
1000	3.85	3	2.61	2.38	2.22	2.02	1.84	1.68	1.36	1.26

<p>Chapter 1: no key formulas.                  Chapter 2: Relative Frequency=freq. of the class / n.                  Approx. Class Width:                  =(largest value-smallest value) / number of classes.                  Chapter 3: sample and population means</p> $\bar{x} = \sum x_i/n \text{ and } \mu = \sum x_i/N$ <p>Weighted mean and geometric mean</p> $\bar{x} = \sum w_i x_i / w_i \text{ and } \bar{x}_g = [(x_1)(x_2) \dots (x_n)]^{1/n}.$ <p>Interquartile Range: IQR = <math>Q_3 - Q_1</math>.                  Population and sample variance</p> $\sigma^2 = \frac{\sum (x_i - \mu)^2}{N} \text{ and } s^2 = \frac{\sum (x_i - \bar{x})^2}{n - 1}$ <p>Population and sample standard deviation</p> $\sigma = \sqrt{\sigma^2} \text{ and } s = \sqrt{s^2}.$ <p>Coefficient of Variation</p> $\left( \frac{\text{Standard deviation}}{\text{Mean}} \times 100 \right) \%$ <p>z-Score: <math>z_i = \frac{x_i - \bar{x}}{s}</math>.                  Population and Sample Covariance</p> $\sigma_{xy} = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{N} \text{ and } s_{xy} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n - 1}$ <p>Population and Sample Pearson Correlation</p> $\rho_{xy} = \sigma_{xy} / (\sigma_x \sigma_y) \text{ and } r_{xy} = s_{xy} / (s_x s_y).$ <p>Chapter 4: Counting Rule for Combinations</p> $C_n^N = \binom{N}{n} = \frac{N!}{n!(N-n)!}$ <p>Counting Rule for Permutations</p> $P_n^N = n! \binom{N}{n} = \frac{N!}{(N-n)!}$ <p>Probability Rules: <math>P(A) = 1 - P(A^c)</math></p>	<p>Chapter 4 continued:</p> $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $P(A B) = \frac{P(A \cap B)}{P(B)}$ $P(A \cap B) = P(B)P(A B) = P(A)P(B A).$ <p>Multiplication Law for Independent Events</p> $P(A \cap B) = P(B)P(A).$ <p>Bayes' Theorem</p> $P(A_i B) = \frac{P(A_i)P(B A_i)}{P(A_1)P(B A_1) + P(A_2)P(B A_2) + \dots + P(A_n)P(B A_n)}$ <p>Chapter 5:                  Discrete Uniform Probability Mass Function: <math>f(x) = 1/n</math>.                  Expected Value of a Discrete R. V.: <math>E(x) = \mu = \sum x f(x)</math>.                  Variance of a Discrete R. V.:</p> $Var(x) = \sigma^2 = \sum (x - \mu)^2 f(x).$ <p>Number of Experimental Outcomes Providing Exactly <math>x</math> Successes in <math>n</math> Trials</p> $\binom{n}{x} = \frac{n!}{x!(n-x)!}$ <p>Binomial Probability Mass Function</p> $P(X = x) = f(x) = \binom{n}{x} p^x (1-p)^{(n-x)}.$ <p>Expected Value for Binomial Distribution: <math>E(x) = \mu = np</math>.                  Variance for Binomial Distr.: <math>Var(x) = \sigma^2 = np(1-p)</math>.                  Poisson Probability Mass Function:</p> $P(X = x \mu) = f(x) = \frac{\mu^x e^{-\mu}}{x!}.$ <p>Hypergeometric Probability Mass Function and Expected Value:</p> $f(x) = \frac{\binom{r}{x} \binom{N-r}{n-x}}{\binom{N}{n}} \text{ and } E(x) = \mu = \frac{nr}{N}.$	<p>Chapter 5 continued: Variance for the Hypergeometric Distribution:</p> $Var(x) = \sigma^2 = n \left( \frac{r}{N} \right) \left( 1 - \frac{r}{N} \right) \left( \frac{N-n}{N-1} \right).$ <p>Chapter 6: Uniform PDF</p> $f(x) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$ <p>Normal PDF The density function is</p> $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right).$ <p>Converting to the Standard Normal rv:</p> $z = \frac{x - \mu}{\sigma}.$ <p>Exponential PDF and CDF for <math>x \geq 0</math></p> $f(x) = \mu^{-1} e^{-x/\mu} \text{ and } P(x \leq x_0) = 1 - e^{-x_0/\mu}.$ <p>Chapter 7: expected value of <math>\bar{x}</math></p> $E(\bar{x}) = \mu.$ <p>Standard Deviation of <math>\bar{x}</math> (Standard Error)</p> $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}.$ <p>Expected Value and Std Dev (Standard Error) of <math>\bar{p}</math></p> $E(\bar{p}) = p \text{ and } \sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}}$ <p>Finite Pop. Correction Factor: <math>\sqrt{(N-n)/(N-1)}</math>.                  Chapter 8: Interval Estimate of Population Mean, <math>\sigma</math> known and unknown</p> $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \text{ and } \bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$ <p>Necessary Sample Size for Interval Estimate of <math>\mu</math></p> $n = \frac{(z_{\alpha/2})^2 \sigma^2}{E^2}$
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<p>Chapter 8 continued: Interval Estimate of <math>p</math></p> $\hat{p} \pm z_{\alpha/2} \frac{p(1-p)}{\sqrt{n}}$ <p>Necessary Sample Size for Interval Estimate of <math>p</math></p> $n = \frac{(z_{\alpha/2})^2 p^*(1-p^*)}{E^2}$ <p>Chapter 9: Test Statistic for Hypothesis Tests About <math>\mu</math>, <math>\sigma</math> known and unknown</p> $z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \text{ and } t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$ <p>Test Stat for Hypothesis About <math>p</math></p> $z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$ <p>Chapter 10: Point Estimate and Standard Error for Difference in Two Population Means</p> $\bar{x}_1 - \bar{x}_2 \text{ and } \sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ <p>Interval Estimate and Test Statistic for Difference in Two Means with Known Variances</p> $\bar{x}_1 - \bar{x}_2 \pm z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \text{ and } z = \frac{\bar{x}_1 - \bar{x}_2 - D_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$ <p>Interval Estimate and Test Statistic for Difference in Two Means with Unknown Variances</p> $\bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \text{ and } t = \frac{\bar{x}_1 - \bar{x}_2 - D_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$ <p>Degrees of Freedom for <math>t</math>, Two Independent Random Samples</p> $df = \frac{1}{\frac{1}{n_1-1} \left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2-1} \left(\frac{s_2^2}{n_2}\right)^2} \left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2$	<p>Chapter 10 continued: Test Statistic (Matched Samples)</p> $t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}}$ <p>ANOVA Related:</p> $\bar{x}_j = \frac{\sum_{i=1}^{n_j} x_{ij}}{n_j} \quad s_j^2 = \frac{\sum_{i=1}^{n_j} (x_{ij} - \bar{x}_j)^2}{n_j - 1} \quad \bar{\bar{x}} = \frac{\sum_{j=1}^k \sum_{i=1}^{n_j} x_{ij}}{n_T}$ $MSTR = \frac{SSTR}{k-1} \quad SSTR = \sum_{j=1}^k n_j (\bar{x}_j - \bar{\bar{x}})^2 \quad MSE = \frac{SSE}{n_T - k}$ $SSE = \sum_{j=1}^k \sum_{i=1}^{n_j} (x_{ij} - \bar{x}_j)^2 \quad F = MSTR/MSE$ $SST = \sum_{j=1}^k \sum_{i=1}^{n_j} (x_{ij} - \bar{\bar{x}})^2 \quad SST = SSTR + SSE$ <p>Chapter 11: not covered in this course Chapter 12: <math>y = \beta_0 + \beta_1 x + \epsilon</math></p> $E(y) = \beta_0 + \beta_1 x \quad \hat{y} = b_0 + b_1 x \quad b_0 = \bar{y} - b_1 \bar{x}$ $b_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} \quad SSE = \sum (y_i - \hat{y}_i)^2$ $SST = \sum (y_i - \bar{y})^2 \quad SSR = \sum (\hat{y}_i - \bar{y})^2 \quad SST = SSR + SSE$ $r^2 = \frac{SSR}{SST} \quad r_{xy} = (\text{sign of } b_1) \sqrt{r^2} \quad s^2 = MSE = \frac{SSE}{n-2}$ <p>Standard Error of the Estimate, <math>s = \sqrt{MSE}</math>.</p> $\sigma_{b_1} = \frac{\sigma}{\sqrt{\sum (x_i - \bar{x})^2}} \quad s_{b_1} = \frac{s}{\sqrt{\sum (x_i - \bar{x})^2}} \quad t = \frac{b_1}{s_{b_1}}$ <p>For simple regression, <math>MSR = SSR</math> because there is only one independent variable.</p> $F = \frac{MSR}{MSE} \quad s_{\hat{y}^*} = s \sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{\sum (x_i - \bar{x})^2}}$ <p>Confidence Interval for <math>E(y^*)</math>: <math>\hat{y}^* \pm t_{\alpha/2} s_{\hat{y}^*}</math></p> $s_{\text{pred}} = s \sqrt{1 + \frac{1}{n} + \frac{(x^* - \bar{x})^2}{\sum (x_i - \bar{x})^2}}$	<p>Chapter 8 continued: Interval Estimate of <math>p</math></p> $\hat{p} \pm z_{\alpha/2} \frac{p(1-p)}{\sqrt{n}}$ <p>Necessary Sample Size for Interval Estimate of <math>p</math></p> $n = \frac{(z_{\alpha/2})^2 p^*(1-p^*)}{E^2}$ <p>Chapter 9: Test Statistic for Hypothesis Tests About <math>\mu</math>, <math>\sigma</math> known and unknown</p> $z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \text{ and } t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$ <p>Test Stat for Hypothesis About <math>p</math></p> $z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$ <p>Chapter 10: Point Estimate and Standard Error for Difference in Two Population Means</p> $\bar{x}_1 - \bar{x}_2 \text{ and } \sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ <p>Interval Estimate and Test Statistic for Difference in Two Means with Known Variances</p> $\bar{x}_1 - \bar{x}_2 \pm z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \text{ and } z = \frac{\bar{x}_1 - \bar{x}_2 - D_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$ <p>Interval Estimate and Test Statistic for Difference in Two Means with Unknown Variances</p> $\bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \text{ and } t = \frac{\bar{x}_1 - \bar{x}_2 - D_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$ <p>Degrees of Freedom for <math>t</math>, Two Independent Random Samples</p> $df = \frac{1}{\frac{1}{n_1-1} \left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2-1} \left(\frac{s_2^2}{n_2}\right)^2} \left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2$
<p>Chapter 10 continued: Test Statistic (Matched Samples)</p> $t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}}$ <p>ANOVA Related:</p> $\bar{x}_j = \frac{\sum_{i=1}^{n_j} x_{ij}}{n_j} \quad s_j^2 = \frac{\sum_{i=1}^{n_j} (x_{ij} - \bar{x}_j)^2}{n_j - 1} \quad \bar{\bar{x}} = \frac{\sum_{j=1}^k \sum_{i=1}^{n_j} x_{ij}}{n_T}$ $MSTR = \frac{SSTR}{k-1} \quad SSTR = \sum_{j=1}^k n_j (\bar{x}_j - \bar{\bar{x}})^2 \quad MSE = \frac{SSE}{n_T - k}$ $SSE = \sum_{j=1}^k \sum_{i=1}^{n_j} (x_{ij} - \bar{x}_j)^2 \quad F = MSTR/MSE$ $SST = \sum_{j=1}^k \sum_{i=1}^{n_j} (x_{ij} - \bar{\bar{x}})^2 \quad SST = SSTR + SSE$ <p>Chapter 11: not covered in this course Chapter 12: <math>y = \beta_0 + \beta_1 x + \epsilon</math></p> $E(y) = \beta_0 + \beta_1 x \quad \hat{y} = b_0 + b_1 x \quad b_0 = \bar{y} - b_1 \bar{x}$ $b_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} \quad SSE = \sum (y_i - \hat{y}_i)^2$ $SST = \sum (y_i - \bar{y})^2 \quad SSR = \sum (\hat{y}_i - \bar{y})^2 \quad SST = SSR + SSE$ $r^2 = \frac{SSR}{SST} \quad r_{xy} = (\text{sign of } b_1) \sqrt{r^2} \quad s^2 = MSE = \frac{SSE}{n-2}$ <p>Standard Error of the Estimate, <math>s = \sqrt{MSE}</math>.</p> $\sigma_{b_1} = \frac{\sigma}{\sqrt{\sum (x_i - \bar{x})^2}} \quad s_{b_1} = \frac{s}{\sqrt{\sum (x_i - \bar{x})^2}} \quad t = \frac{b_1}{s_{b_1}}$ <p>For simple regression, <math>MSR = SSR</math> because there is only one independent variable.</p> $F = \frac{MSR}{MSE} \quad s_{\hat{y}^*} = s \sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{\sum (x_i - \bar{x})^2}}$ <p>Confidence Interval for <math>E(y^*)</math>: <math>\hat{y}^* \pm t_{\alpha/2} s_{\hat{y}^*}</math></p> $s_{\text{pred}} = s \sqrt{1 + \frac{1}{n} + \frac{(x^* - \bar{x})^2}{\sum (x_i - \bar{x})^2}}$	<p>Chapter 12 continued: Prediction Interval for <math>y^*</math>:</p> $\hat{y}^* \pm t_{\alpha/2} s_{\text{pred}}$ <p>Residual for Observation <math>i</math>: <math>y_i - \hat{y}_i</math></p> <p>Chapter 13:</p> $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_p x_p + \epsilon$ $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_p x_p$ $\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + \dots + b_p x_p$ $SST = SSR + SSE \quad R^2 = \frac{SSR}{SST}$ $R_a^2 = 1 - (1 - R^2) \frac{n-1}{n-p-1}$ $MSR = \frac{SSR}{p} \quad MSE = \frac{SSE}{n-p-1} \quad F = \frac{MSR}{MSE}$ $t = \frac{b_i}{s_{b_i}}$ <p>Other Math Rule Reminders:</p> $e^x = \exp(x)$ $\ln 1 = 0 \quad \ln e = 1$ $x! = (x)(x-1)(x-2) \dots (2)(1)$ $0! = 1 \quad x^0 = 1$	<p>Chapter 12 continued: Prediction Interval for <math>y^*</math>:</p> $\hat{y}^* \pm t_{\alpha/2} s_{\text{pred}}$ <p>Residual for Observation <math>i</math>: <math>y_i - \hat{y}_i</math></p> <p>Chapter 13:</p> $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_p x_p + \epsilon$ $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_p x_p$ $\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + \dots + b_p x_p$ $SST = SSR + SSE \quad R^2 = \frac{SSR}{SST}$ $R_a^2 = 1 - (1 - R^2) \frac{n-1}{n-p-1}$ $MSR = \frac{SSR}{p} \quad MSE = \frac{SSE}{n-p-1} \quad F = \frac{MSR}{MSE}$ $t = \frac{b_i}{s_{b_i}}$ <p>Other Math Rule Reminders:</p> $e^x = \exp(x)$ $\ln 1 = 0 \quad \ln e = 1$ $x! = (x)(x-1)(x-2) \dots (2)(1)$ $0! = 1 \quad x^0 = 1$

Use the output below to answer the appropriate questions in the exam. Be sure to check which table the question references.

ANOVA Table 1

	SS	DF	MS	F	p
Treatments	4.614	2	2.307	20.171	0.0008
Error	0.915	8	0.114		

Here is regression output from a model where % high school graduates is the independent variable and % below the poverty line is the dependent variable. The data are for the US for 2012.

Regression Table 2

Regression Statistics	
R Square	0.5567275
Adjusted R Square	0.5476811
Standard Error	2.3003044
Observations	51

ANOVA

	SS	DF	MS	F	p
Regression	325.641	1	325.641	61.541	0
Residual	259.279	49	5.291		

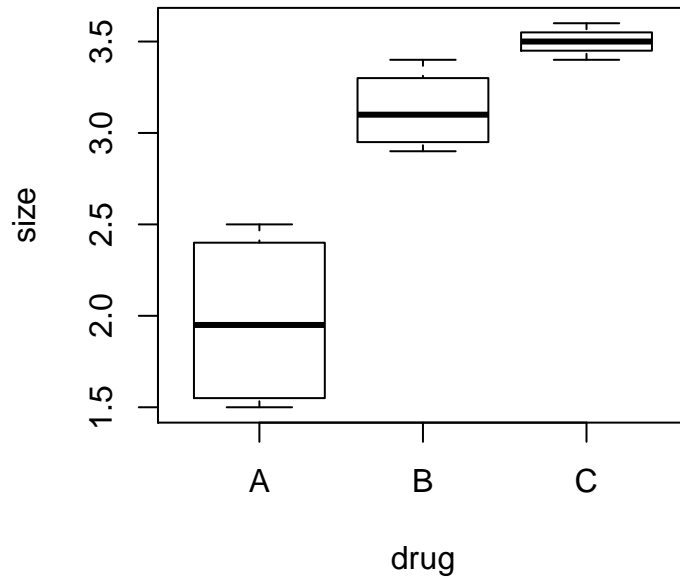
	coefficients	standard error	t stat	p-value
intercept	94.2776877	10.1995327	9.243334	0
pctHSgrads	-0.8975822	0.1144169	-7.8448375	0

ECN221 Exam 3 **VERSION A** Fall 2015 (Chapters 1-10, 12), ASU-COX

Choose the best answer. Do not write letters in the margin or communicate with other students in any way. If you have a question note it on your exam and ask for clarification when your exam is returned. In the meantime choose the best answer. Neither the proctors nor Dr. Cox will answer questions during the exam.

Please check each question and possible answers thoroughly as questions at the bottom of a page sometimes run onto the next page.

1. I have checked that my ID is bubbled in correctly. If it is bubbled in incorrectly I will get this question wrong. I also understand that questions and their possible answers may run onto the next page and so I should always check the top of the next page for possible answers. I understand that if I have a question I should simply make a note on my exam and ask Dr. Cox afterwards. I should always choose the best answer.
  - (a) False.
  - (b) I didn't read the directions.
  - (c) True.
  
2. For a given value of  $\alpha$  when you are testing the hypothesis  $H_0 : \mu_1 = \mu_2$  you
  - (a) reject  $H_0$  if the standard error is less than  $\alpha$
  - (b) reject  $H_0$  if the test statistic is less than  $\alpha$
  - (c) reject  $H_0$  if the p value is less than  $\alpha$
  - (d) reject  $H_0$  if the critical value is less than  $\alpha$
  
3. The graphs shown here are



- (a) histograms
  - (b) box and whiskers plots
  - (c) scatter plots
  - (d) stem and leaf displays
4. The probability that a  $z$  value is less than 1 is
- (a) .5
  - (b) .8413
  - (c) .3413
  - (d) .1587
5. Consult Table 1. From the table you can conclude that the total sum of squares is ?
- (a) 5.5290909
  - (b)  $-3.6990909$
  - (c) 0.915



- (d) 2.7645455
6. Consult Table 1. From the table you can conclude that the total number of observations used in this analysis/experiment was?
- (a) 9
  - (b) 2
  - (c) 8
  - (d) 11
7. Consult Table 1. From the table what can you conclude concerning the null hypothesis?
- (a) reject the null
  - (b) fail to reject
  - (c) cannot be determined
  - (d) depends on the number of observations.
8. Consult Table 1. What is the test statistic?
- (a) 0.00075
  - (b) 2.3070455
  - (c) 10.0854446
  - (d) 20.1708892.
9. Consult Table 1. What is the critical value for a test at the .05 level?
- (a) 4.46
  - (b) 2.43
  - (c) 6.94
  - (d) 19
10. Suppose that the number of times a college student changes their major follows a Poisson distribution with a mean of 2. What is the probability that a student will change their major exactly 1 time(s)?
- (a) 0.4060058
  - (b) 0.1624023

- (c) 0.7293294  
(d) 0.2706706
11. Suppose that you collect data on apartment prices in Tempe. You look at 40 different apartments and find a mean of 717 and a standard deviation of 195.3. Construct a 95% confidence interval for the mean apartment price. The interval is
- (a) [ 658.5400298, 775.4599702 ]  
(b) [ 648.5400298, 785.4599702 ]  
(c) [ 654.5400298, 779.4599702 ]  
(d) [ 589.0860268, 701.5139732 ]
12. Suppose that you collect data on apartment prices in Tempe. You look at 40 different apartments and find a mean of 717 and a standard deviation of 195.3. Test the hypothesis,  $H_0 : \mu = 624$  at the .05 level of significance.
- (a) the test statistic is 3.011693 so we reject the null  
(b) the test statistic is 3.011693 so we fail to reject  
(c) the test statistic is 2.5305237 so we reject the null  
(d) the test statistic is 3.4228623 so we fail to reject
13. Suppose you have a random variable that is uniformly distributed with a maximum of 254 and a minimum of 75. What is the expected value of this random variable?
- (a) 184.5  
(b) 149.5  
(c) 213.85  
(d) 164.5
14. Fred Sanford constructed a null hypothesis in which his son was Lamont and the alternative was that his son was Attila. After meeting Della Reese, Fred
- (a) commits a type I error  
(b) commits a type II error  
(c) correctly rejected  $H_0$   
(d) correctly failed to reject  $H_0$   
(e) none of the above

15. The F distribution
- (a) is symmetric
  - (b) is right skewed
  - (c) has mean 0
  - (d) all of the above
  - (e) none of the above
16. Consider the regression output in Table 2. What is the estimated variance of the error term?
- (a) 1.1501522
  - (b) 2.3003044
  - (c) 5.2914005
  - (d) 61.541
17. Consider the regression output in Table 2. What is the percentage of variation in poverty rates that can be explained by the variation in the percentage of high school graduates in states?
- (a)  $-54.6727463\%$
  - (b)  $66.8072956\%$
  - (c)  $54.7681085\%$
  - (d)  $55.6727463\%$
18. Consider the regression output in Table 2. What is the predicted or estimated percentage of people living in poverty for a state where 90% of the adult population are high school graduates?
- (a)  $22.4711139\%$
  - (b)  $13.4952921\%$
  - (c)  $5.4170526\%$
  - (d)  $31.4469356\%$
19. Consider the regression output in Table 2. Suppose you want to test the hypothesis that the percentage of high school graduates has no impact on the percentage of people living in poverty, i.e.  $H_0 : \beta_1 = 0$ . What is the test statistic for this hypothesis?

- (a)  $-7.8448375$
  - (b)  $0.1144169$
  - (c)  $0$
  - (d)  $9.243334$
20. Consider the regression output in Table 2. Suppose you want to test the hypothesis that the percentage of high school graduates has no impact on the percentage of people living in poverty, i.e.  $H_0 : \beta_1 = 0$ . What is your conclusion for this hypothesis test?
- (a) This is inconclusive unless we know whether it is a right tail or a left tail test.
  - (b) This cannot be determined without the appropriate df.
  - (c) reject the null
  - (d) fail to reject
21. One of the assumptions of the linear regression model is that
- (a) The parameter estimate is unbiased.
  - (b) This error term has a normal distribution.
  - (c) The intercept is 0.
  - (d) The slope is positive.
22. Unless you are out of time don't rush to answer this one; think about it and maybe even write some things down. You are testing the null hypothesis that  $\mu_1 = \mu_2$ . We observe that  $\bar{x}_1 = 10$  and  $\bar{x}_2 = 40$  and that they both have **sample** standard deviations of 2; we can assume the unknown standard deviations are equal. Use  $\alpha = .05$ . Note: the df has a lower bound of 1 and when the variances are assumed equal the df are  $n_1 + n_2 - 2$ .
- (a) We reject  $H_0$ .
  - (b) We fail to reject  $H_0$ .
  - (c) We need more information to draw a conclusion.
  - (d) We reject if it is a one tail test and fail to reject if it is a two tail test.

## Key

Most questions required your knowledge of definitions and vocabulary discussed in class and in the notes or they required you to be able to make calculations or read excel output similar to examples done in class and required in the homework. The last question is one I consider to require critical thinking. The explanation is below.

1. c
2. c
3. b
4. b
5. a
6. d
7. a
8. d
9. a
10. d
11. c
12. a
13. d
14. a
15. b
16. c
17. d
18. b
19. a
20. c
21. b

22. a, you know that the largest possible value for the standard error here is 2 because  $\sqrt{\frac{4}{2} + \frac{4}{2}} = 2$  and if  $n_1$  or  $n_2$  are larger than 2 then it will be even smaller. This means that the test statistic is at most -15. You also know that the largest possible critical value is 12.7 or 4.3 depending on how you interpreted the hint in the question. In any case  $-15 < -12.7$  so you can reject the null. Yes, I think this question was hard and required a lot of thought. If you got it right you should should feel pretty good about that.