

Table of  $z$  values and probabilities for the standard normal distribution.  $z$  is the first column plus the top row. Each cell shows  $P(X \leq z)$ . For example  $P(X \leq 1.04) = .8508$ . For  $z < 0$  subtract the value from 1, e.g.,  $P(X \leq -1.04) = 1 - .8508 = .1492$ .

$z$	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Table of  $t$  values and right tail probabilities. Degrees of freedom are in the first column (df). **Right tail probabilities** are in the first row. For example for  $d.f. = 7$  and  $\alpha = .05$  the critical  $t$  value for a two-tail test is 2.365 and for  $d.f. = 10$  and  $\alpha = .1$  the critical  $t$  value for a one-tail test is 1.372.

df	.1	.05	.025	.01	.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
35	1.306	1.690	2.030	2.438	2.724
40	1.303	1.684	2.021	2.423	2.704
45	1.301	1.679	2.014	2.412	2.690
50	1.299	1.676	2.009	2.403	2.678
75	1.293	1.665	1.992	2.377	2.643
100	1.290	1.660	1.984	2.364	2.626

Table of  $F$  values for right tail probabilities of .05. Degrees of freedom for denominator are in the first column and degrees of freedom for the numerator are in the top row.

denom.	numerator df									
df	1	2	3	4	5	7	10	15	50	100
1	161.45	199.5	215.71	224.58	230.16	236.77	241.88	245.95	251.77	253.04
2	18.51	19	19.16	19.25	19.3	19.35	19.4	19.43	19.48	19.49
3	10.13	9.55	9.28	9.12	9.01	8.89	8.79	8.7	8.58	8.55
4	7.71	6.94	6.59	6.39	6.26	6.09	5.96	5.86	5.7	5.66
5	6.61	5.79	5.41	5.19	5.05	4.88	4.74	4.62	4.44	4.41
6	5.99	5.14	4.76	4.53	4.39	4.21	4.06	3.94	3.75	3.71
7	5.59	4.74	4.35	4.12	3.97	3.79	3.64	3.51	3.32	3.27
8	5.32	4.46	4.07	3.84	3.69	3.5	3.35	3.22	3.02	2.97
9	5.12	4.26	3.86	3.63	3.48	3.29	3.14	3.01	2.8	2.76
10	4.96	4.1	3.71	3.48	3.33	3.14	2.98	2.85	2.64	2.59
11	4.84	3.98	3.59	3.36	3.2	3.01	2.85	2.72	2.51	2.46
12	4.75	3.89	3.49	3.26	3.11	2.91	2.75	2.62	2.4	2.35
13	4.67	3.81	3.41	3.18	3.03	2.83	2.67	2.53	2.31	2.26
14	4.6	3.74	3.34	3.11	2.96	2.76	2.6	2.46	2.24	2.19
15	4.54	3.68	3.29	3.06	2.9	2.71	2.54	2.4	2.18	2.12
16	4.49	3.63	3.24	3.01	2.85	2.66	2.49	2.35	2.12	2.07
17	4.45	3.59	3.2	2.96	2.81	2.61	2.45	2.31	2.08	2.02
18	4.41	3.55	3.16	2.93	2.77	2.58	2.41	2.27	2.04	1.98
19	4.38	3.52	3.13	2.9	2.74	2.54	2.38	2.23	2	1.94
20	4.35	3.49	3.1	2.87	2.71	2.51	2.35	2.2	1.97	1.91
21	4.32	3.47	3.07	2.84	2.68	2.49	2.32	2.18	1.94	1.88
22	4.3	3.44	3.05	2.82	2.66	2.46	2.3	2.15	1.91	1.85
23	4.28	3.42	3.03	2.8	2.64	2.44	2.27	2.13	1.88	1.82
24	4.26	3.4	3.01	2.78	2.62	2.42	2.25	2.11	1.86	1.8
25	4.24	3.39	2.99	2.76	2.6	2.4	2.24	2.09	1.84	1.78
26	4.23	3.37	2.98	2.74	2.59	2.39	2.22	2.07	1.82	1.76
27	4.21	3.35	2.96	2.73	2.57	2.37	2.2	2.06	1.81	1.74
28	4.2	3.34	2.95	2.71	2.56	2.36	2.19	2.04	1.79	1.73
29	4.18	3.33	2.93	2.7	2.55	2.35	2.18	2.03	1.77	1.71
30	4.17	3.32	2.92	2.69	2.53	2.33	2.16	2.01	1.76	1.7
40	4.08	3.23	2.84	2.61	2.45	2.25	2.08	1.92	1.66	1.59
60	4	3.15	2.76	2.53	2.37	2.17	1.99	1.84	1.56	1.48
100	3.94	3.09	2.7	2.46	2.31	2.1	1.93	1.77	1.48	1.39
1000	3.85	3	2.61	2.38	2.22	2.02	1.84	1.68	1.36	1.26

<p>Chapter 1: no key formulas.                  Chapter 2: Relative Frequency=freq. of the class / n.                  Approx. Class Width:                  =(largest value-smallest value) / number of classes.                  Chapter 3: sample and population means</p> $\bar{x} = \sum x_i/n \text{ and } \mu = \sum x_i/N$ <p>Weighted mean and geometric mean</p> $\bar{x} = \sum w_i x_i / w_i \text{ and } \bar{x}_g = [(x_1)(x_2) \dots (x_n)]^{1/n}.$ <p>Interquartile Range: IQR = <math>Q_3 - Q_1</math>.                  Population and sample variance</p> $\sigma^2 = \frac{\sum (x_i - \mu)^2}{N} \text{ and } s^2 = \frac{\sum (x_i - \bar{x})^2}{n - 1}$ <p>Population and sample standard deviation</p> $\sigma = \sqrt{\sigma^2} \text{ and } s = \sqrt{s^2}.$ <p>Coefficient of Variation</p> $\left( \frac{\text{Standard deviation}}{\text{Mean}} \times 100 \right) \%$ <p>z-Score: <math>z_i = \frac{x_i - \bar{x}}{s}</math>.                  Population and Sample Covariance</p> $\sigma_{xy} = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{N} \text{ and } s_{xy} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n - 1}$ <p>Population and Sample Pearson Correlation</p> $\rho_{xy} = \sigma_{xy} / (\sigma_x \sigma_y) \text{ and } r_{xy} = s_{xy} / (s_x s_y).$ <p>Chapter 4: Counting Rule for Combinations</p> $C_n^N = \binom{N}{n} = \frac{N!}{n!(N-n)!}$ <p>Counting Rule for Permutations</p> $P_n^N = n! \binom{N}{n} = \frac{N!}{(N-n)!}$ <p>Probability Rules: <math>P(A) = 1 - P(A^c)</math></p>	<p>Chapter 4 continued:</p> $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $P(A B) = \frac{P(A \cap B)}{P(B)}$ $P(A \cap B) = P(B)P(A B) = P(A)P(B A).$ <p>Multiplication Law for Independent Events</p> $P(A \cap B) = P(B)P(A).$ <p>Bayes' Theorem</p> $P(A_i B) = \frac{P(A_i)P(B A_i)}{P(A_1)P(B A_1) + P(A_2)P(B A_2) + \dots + P(A_n)P(B A_n)}$ <p>Chapter 5:                  Discrete Uniform Probability Mass Function: <math>f(x) = 1/n</math>.                  Expected Value of a Discrete R. V.: <math>E(x) = \mu = \sum x f(x)</math>.                  Variance of a Discrete R. V.:</p> $Var(x) = \sigma^2 = \sum (x - \mu)^2 f(x).$ <p>Number of Experimental Outcomes Providing Exactly <math>x</math> Successes in <math>n</math> Trials</p> $\binom{n}{x} = \frac{n!}{x!(n-x)!}$ <p>Binomial Probability Mass Function</p> $P(X = x) = f(x) = \binom{n}{x} p^x (1-p)^{(n-x)}.$ <p>Expected Value for Binomial Distribution: <math>E(x) = \mu = np</math>.                  Variance for Binomial Distr.: <math>Var(x) = \sigma^2 = np(1-p)</math>.                  Poisson Probability Mass Function:</p> $P(X = x \mu) = f(x) = \frac{\mu^x e^{-\mu}}{x!}.$ <p>Hypergeometric Probability Mass Function and Expected Value:</p> $f(x) = \frac{\binom{r}{x} \binom{N-r}{n-x}}{\binom{N}{n}} \text{ and } E(x) = \mu = \frac{nr}{N}.$	<p>Chapter 5 continued: Variance for the Hypergeometric Distribution:</p> $Var(x) = \sigma^2 = n \left( \frac{r}{N} \right) \left( 1 - \frac{r}{N} \right) \left( \frac{N-n}{N-1} \right).$ <p>Chapter 6: Uniform PDF</p> $f(x) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$ <p>Normal PDF The density function is</p> $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right).$ <p>Converting to the Standard Normal rv:</p> $z = \frac{x - \mu}{\sigma}.$ <p>Exponential PDF and CDF for <math>x \geq 0</math></p> $f(x) = \mu^{-1} e^{-x/\mu} \text{ and } P(x \leq x_0) = 1 - e^{-x_0/\mu}.$ <p>Chapter 7: expected value of <math>\bar{x}</math></p> $E(\bar{x}) = \mu.$ <p>Standard Deviation of <math>\bar{x}</math> (Standard Error)</p> $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}.$ <p>Expected Value and Std Dev (Standard Error) of <math>\bar{p}</math></p> $E(\bar{p}) = p \text{ and } \sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}}$ <p>Finite Pop. Correction Factor: <math>\sqrt{(N-n)/(N-1)}</math>.                  Chapter 8: Interval Estimate of Population Mean, <math>\sigma</math> known and unknown</p> $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \text{ and } \bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$ <p>Necessary Sample Size for Interval Estimate of <math>\mu</math></p> $n = \frac{(z_{\alpha/2})^2 \sigma^2}{E^2}$
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<p>Chapter 8 continued: Interval Estimate of <math>p</math></p> $\hat{p} \pm z_{\alpha/2} \frac{p(1-p)}{\sqrt{n}}$ <p>Necessary Sample Size for Interval Estimate of <math>p</math></p> $n = \frac{(z_{\alpha/2})^2 p^*(1-p^*)}{E^2}$ <p>Chapter 9: Test Statistic for Hypothesis Tests About <math>\mu</math>, <math>\sigma</math> known and unknown</p> $z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \text{ and } t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$ <p>Test Stat for Hypothesis About <math>p</math></p> $z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$ <p>Chapter 10: Point Estimate and Standard Error for Difference in Two Population Means</p> $\bar{x}_1 - \bar{x}_2 \text{ and } \sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ <p>Interval Estimate and Test Statistic for Difference in Two Means with Known Variances</p> $\bar{x}_1 - \bar{x}_2 \pm z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \text{ and } z = \frac{\bar{x}_1 - \bar{x}_2 - D_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$ <p>Interval Estimate and Test Statistic for Difference in Two Means with Unknown Variances</p> $\bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \text{ and } t = \frac{\bar{x}_1 - \bar{x}_2 - D_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$ <p>Degrees of Freedom for <math>t</math>, Two Independent Random Samples</p> $df = \frac{1}{\frac{1}{n_1-1} \left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2-1} \left(\frac{s_2^2}{n_2}\right)^2} \left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)$	<p>Chapter 10 continued: Test Statistic (Matched Samples)</p> $t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}}$ <p>ANOVA Related:</p> $\bar{x}_j = \frac{\sum_{i=1}^{n_j} x_{ij}}{n_j} \quad s_j^2 = \frac{\sum_{i=1}^{n_j} (x_{ij} - \bar{x}_j)^2}{n_j - 1} \quad \bar{\bar{x}} = \frac{\sum_{j=1}^k \sum_{i=1}^{n_j} x_{ij}}{n_T}$ $MSTR = \frac{SSTR}{k-1} \quad SSTR = \sum_{j=1}^k n_j (\bar{x}_j - \bar{\bar{x}})^2 \quad MSE = \frac{SSE}{n_T - k}$ $SSE = \sum_{j=1}^k \sum_{i=1}^{n_j} (x_{ij} - \bar{x}_j)^2 \quad F = MSTR/MSE$ $SST = \sum_{j=1}^k \sum_{i=1}^{n_j} (x_{ij} - \bar{\bar{x}})^2 \quad SST = SSTR + SSE$ <p>Chapter 11: not covered in this course Chapter 12: <math>y = \beta_0 + \beta_1 x + \epsilon</math></p> $E(y) = \beta_0 + \beta_1 x \quad \hat{y} = b_0 + b_1 x \quad b_0 = \bar{y} - b_1 \bar{x}$ $b_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} \quad SSE = \sum (y_i - \hat{y}_i)^2$ $SST = \sum (y_i - \bar{y})^2 \quad SSR = \sum (\hat{y}_i - \bar{y})^2 \quad SST = SSR + SSE$ $r^2 = \frac{SSR}{SST} \quad r_{xy} = (\text{sign of } b_1) \sqrt{r^2} \quad s^2 = MSE = \frac{SSE}{n-2}$ <p>Standard Error of the Estimate, <math>s = \sqrt{MSE}</math>.</p> $\sigma_{b_1} = \frac{\sigma}{\sqrt{\sum (x_i - \bar{x})^2}} \quad s_{b_1} = \frac{s}{\sqrt{\sum (x_i - \bar{x})^2}} \quad t = \frac{b_1}{s_{b_1}}$ <p>For simple regression, <math>MSR = SSR</math> because there is only one independent variable.</p> $F = \frac{MSR}{MSE} \quad s_{\hat{y}^*} = s \sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{\sum (x_i - \bar{x})^2}}$ <p>Confidence Interval for <math>E(y^*)</math>: <math>\hat{y}^* \pm t_{\alpha/2} s_{\hat{y}^*}</math></p> $s_{\text{pred}} = s \sqrt{1 + \frac{1}{n} + \frac{(x^* - \bar{x})^2}{\sum (x_i - \bar{x})^2}}$	<p>Chapter 12 continued: Prediction Interval for <math>y^*</math>:</p> $\hat{y}^* \pm t_{\alpha/2} s_{\text{pred}}$ <p>Residual for Observation <math>i</math>: <math>y_i - \hat{y}_i</math> Chapter 13:</p> $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_p x_p + \epsilon$ $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_p x_p$ $\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + \dots + b_p x_p$ $SST = SSR + SSE \quad R^2 = \frac{SSR}{SST}$ $R_a^2 = 1 - (1 - R^2) \frac{n-1}{n-p-1}$ $MSR = \frac{SSR}{p} \quad MSE = \frac{SSE}{n-p-1} \quad F = \frac{MSR}{MSE}$ $t = \frac{b_i}{s_{b_i}}$ <p>Other Math Rule Reminders:</p> $e^x = \exp(x)$ $\ln 1 = 0 \quad \ln e = 1$ $x! = (x)(x-1)(x-2) \dots (2)(1)$ $0! = 1 \quad x^0 = 1$
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Use the tables/output below for some of the questions.

Table of ANOVA results on 2013 sample of per capita GDP for different regions.

Anova: Single Factor

SUMMARY

<i>Groups</i>	<i>Count</i>	<i>Sum</i>	<i>Average</i>	<i>Variance</i>
East Asia & Pacific	14	325794.2	23271.0115	810285996
Sub-Saharan Africa	16	48516.09	3032.255844	27854886.2
Europe & Central Asia	8	291647.8	36455.9744	227640429

ANOVA

<i>Source of Variation</i>	<i>SS</i>	<i>df</i>	<i>MS</i>	<i>F</i>	<i>P-value</i>	<i>F crit</i>
Between Groups	6.69E+09	2	3344960768	9.33227585	0.000564	3.267424
Within Groups	1.25E+10	35	358429264.2			
Total	1.92E+10	37				

Table of regression results for 2013 sample of countries with per capita GDP as the dependent variable. “Latin America and Caribbean” is a dummy variable which is 1 when the country is in Latin America or the Caribbean.

SUMMARY OUTPUT

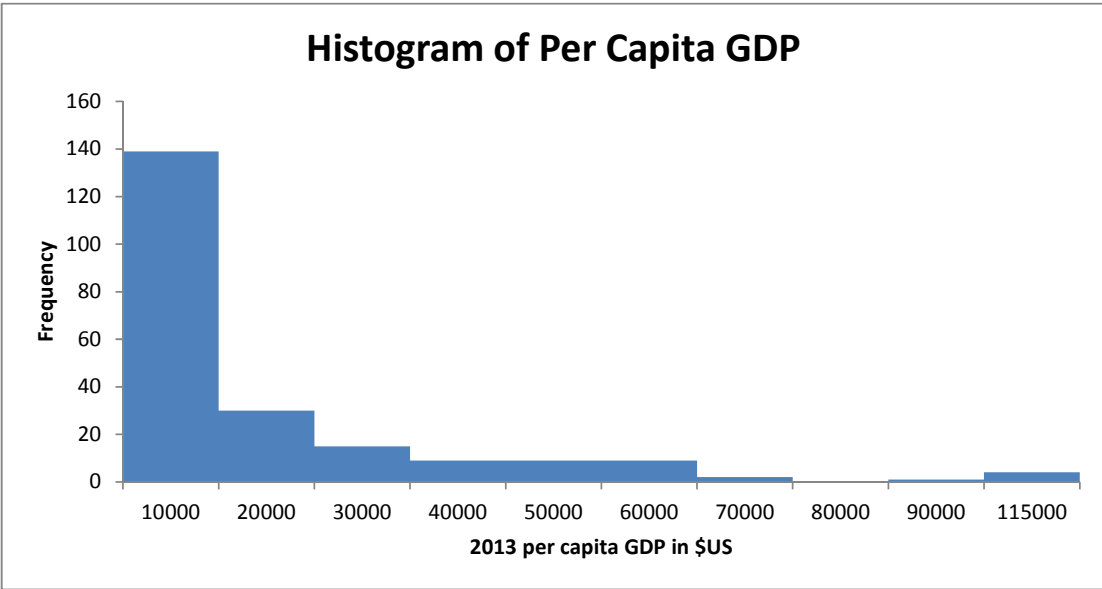
<i>Regression Statistics</i>	
Multiple R	0.556925493
R Square	0.310166005
Adjusted R Square	0.290596246
Standard Error	18128.69087
Observations	146

ANOVA

	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>
Regression	4	20835388468	5208847117	15.84925	9.78808E-11
Residual	141	46339569980	328649432.5		
Total	145	67174958449			

	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>
Intercept	13181.79656	3716.601617	3.546733797	0.00053	5834.329797	20529.2633
<i>Latin America &amp; Caribbean</i>	<i>-7045.016122</i>	<i>4068.252725</i>	<i>-1.73170562</i>	<i>0.085513</i>	<i>-15087.67304</i>	<i>997.640793</i>
2012 CPI inflation average inflation	802.0572625	581.9721562	1.378171196	0.170333	-348.4618186	1952.57634
2010-2012	-2311.026088	804.3034673	-2.87332603	0.00469	-3901.078938	-720.97324
2013 exports as % of gdp	262.3314988	48.4543909	5.413988163	2.58E-07	166.5404876	358.12251

Histogram showing distribution of per capita GDP.





ECN221 Exam Final A Spring 2015, ASU-COX

Choose the best answer. Do not write letters in the margin or communicate with other students in any way. If you have a question note it on your exam and ask for clarification when your exam is returned. In the meantime choose the best answer. Neither the proctors nor Dr. Cox will answer questions during the exam.

Please check each question and possible answers thoroughly as questions at the bottom of a page sometimes run onto the next page.

All questions below that involve data use data from the World Bank.

1. I have checked that my ID is bubbled in correctly. If it is bubbled in incorrectly I will get this question wrong. I also understand that questions and their possible answers may run onto the next page and so I should always check the top of the next page for possible answers. I understand that if I have a question I should simply make a note on my exam and ask Dr. Cox afterwards. I should always choose the best answer.
  - (a) False.
  - (b) I didn't read the directions and choose to get this question wrong.
  - (c) True.
  - (d) Please take additional points off of my score for not reading directions.
  
2. The probability of a Type I error is represented by
  - (a)  $\alpha$
  - (b)  $\beta$
  - (c) c.c.
  - (d)  $t$
  - (e)  $z$
  
3. We have the 2013 per capita GDP (in US\$) for the following countries: China 6807, Fiji 4375, Japan 38633, Mongolia 4056, Philippines 2765. For those countries what is the average per capita GDP?
  - (a) 14287
  - (b) 11327
  - (c) 1027
  - (d) 987
  - (e) 1242

4. We have the 2013 per capita GDP (in US\$) for the following countries: China 6807, Fiji 4375, Japan 38633, Mongolia 4056, Philippines 2765. What is the median per capita GDP?
- (a) 5079
  - (b) 6807
  - (c) 4375
  - (d) 4065
  - (e) 4215
5. We have the 2013 per capita GDP (in US\$) for the following countries: China 6807, Fiji 4375, Japan 38633, Mongolia 4056, Philippines 2765. For those countries what is the sample standard deviation of per capita GDP?
- (a) 14287
  - (b) 56636
  - (c) 235138406
  - (d) 15334
  - (e) 13715
6. We have the 2013 per capita GDP (in US\$) for the following countries: China 6807, Fiji 4375, Japan 38633, Mongolia 4056, Philippines 2765. For those countries what is the standard error (for per capita GDP)?
- (a) 6344.8
  - (b) 7012.1
  - (c) 6857.7
  - (d) 76331.5
  - (e) 4375
7. We have the 2013 per capita GDP (in US\$) for the following countries: China 6807, Fiji 4375, Japan 38633, Mongolia 4056, Philippines 2765. These countries are all in southeast Asia. Suppose this is a random draw of southeast Asian countries. Test the hypothesis that the average per capita GDP in southeast Asia is 10,000. What is the test statistic?
- (a) .877
  - (b) 2.63

- (c) .194
  - (d) 3.599
  - (e) 1.43
8. We have the 2013 per capita GDP (in US\$) for the following countries: China 6807, Fiji 4375, Japan 38633, Mongolia 4056, Philippines 2765. These countries are all in southeast Asia. Suppose this is a random draw of southeast Asian countries. Test the hypothesis that the average per capita GDP in southeast Asia is 10,000. What kind of test is this?
- (a) two-tail
  - (b) right tail
  - (c) left tail
  - (d) one tail
  - (e) a hard test
9. We have the 2013 per capita GDP (in US\$) for the following countries: China 6807, Fiji 4375, Japan 38633, Mongolia 4056, Philippines 2765. These countries are all in southeast Asia. Suppose this is a random draw of southeast Asian countries. Test the hypothesis that the average per capita GDP in southeast Asia is 10,000. For  $\alpha = .01$  what is the critical value for the test in absolute value?
- (a) 2.576
  - (b) 3.365
  - (c) 4.032
  - (d) 4.604
  - (e) 3.747
10. We have the 2013 per capita GDP (in US\$) for the following countries: China 6807, Fiji 4375, Japan 38633, Mongolia 4056, Philippines 2765. These countries are all in southeast Asia. Suppose this is a random draw of southeast Asian countries. Test the hypothesis that the average per capita GDP in southeast Asia is 10,000 at the .01 level of significance. What do you conclude?
- (a) reject  $H_0$ .
  - (b) fail to reject  $H_0$ .
  - (c) it cannot be determined because degrees of freedom are not provided.
  - (d) reject the left tail test but not the right tail test.

- (e) none of the above.
11. Refer to the histogram of per capita GDP. What can you say about the distribution of income in the world as it pertains to countries?
- (a) the distribution is fairly normal.
  - (b) the distribution is symmetric.
  - (c) the distribution is approximately uniform.
  - (d) the distribution is inverted.
  - (e) the distribution is skewed.
12. Refer to the histogram of per capita GDP. Which distribution might best approximate that of per capita GDP?
- (a) normal.
  - (b) standard normal.
  - (c) exponential.
  - (d) Poisson.
  - (e) uniform.
13. Refer to the ANOVA table for 3 different regions and their per capita GDPs. How many countries were used in this analysis?
- (a) 14
  - (b) 24.
  - (c) 35.
  - (d) 38.
  - (e) 40.
14. Refer to the ANOVA table for 3 different regions and their per capita GDPs. What is the mean square due to errors?
- (a) 3344960768
  - (b) 358429264.
  - (c) 9.33.
  - (d) 27854886.
  - (e) 19200000000.

15. Refer to the ANOVA table for 3 different regions and their per capita GDPs. If you test the hypothesis that the average per capita income across those regions is equal with  $\alpha = .01$  you will
- (a) reject  $H_0$
  - (b) fail to reject
  - (c) need to consult an F table because excel gives output for  $\alpha = .05$
  - (d) only reject for a one tail test, so it depends.
16. For an F test with 4 numerator df and 20 denominator df with  $\alpha = .05$  what is the critical value?
- (a) 2.67
  - (b) 5.8
  - (c) 2.87
  - (d) 4.38.
  - (e) 1.48
17. For a sample of 16 Sub-Saharan African countries the average per capita GDP in 2013 was 3032 \$US. (Imagine trying to live on that amount). The variance for that same sample was 27854886. Construct a 90% confidence interval for the mean per capita GDP for Sub-Saharan African countries.
- (a) [0, 6433]
  - (b) [445, 5618]
  - (c) [861, 5202]
  - (d) [719, 5345]
  - (e) [219, 5844]
18. Consult the regression output where per capita GDP is the dependent variable. How many countries were used in this analysis?
- (a) 8
  - (b) 4
  - (c) 145
  - (d) 150
  - (e) 146

19. Consult the regression output where per capita GDP is the dependent variable. What percentage of the variation in per capita GDP can be explained by the variables in the model?

- (a) 55.6
- (b) 31.0
- (c) 29.0
- (d) 15.8
- (e) 26.2

20. Consult the regression output where per capita GDP is the dependent variable. Because  $R^2 < .5$  we can conclude that this model does not have significant use in estimation.

- (a) true if  $\alpha \leq .05$  but not if it is larger such as  $\alpha = .1$
- (b) true but only because  $R^2$  is less than multiple R.
- (c) true
- (d) false
- (e) false but only because the p-value on the intercept term is less than .01.

21. Consult the regression output where per capita GDP is the dependent variable. You reject the null hypothesis:

$$H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$$

at the .001 level of significance.

- (a) true
- (b) false

22. Consult the regression output where per capita GDP is the dependent variable. Notice that both the 2012 inflation rate and the average inflation rate from 2010-2012 are included in the model as independent variables. One of the estimated coefficients seems to suggest that inflation has a positive impact on per capita GDP while the other seems to suggest that inflation has a negative impact on per capita GDP. What might be the cause of this apparent inconsistency?

- (a) the error terms do not have constant variance.
- (b) the error terms are correlated (meaning they are not independent).

- (c) the error terms are not normally distributed.
  - (d) the presence of multicollinearity.
  - (e) the fact that  $R^2$  is too small to make it a good model.
23. Consult the regression output where per capita GDP is the dependent variable. Use the model shown there to find a predicted value for per capita GDP for Greece. For Greece in 2013 the exports as a percent of GDP was 30.23, the 2012 inflation rate was 1.5 and the average inflation rate from 2010-2012 was 3.18. Note that Greece is in Europe. In making your calculation you can round your estimated coefficients at the second or third decimal place. The predicted per capita GDP for Greece is
- (a) 13,181.
  - (b) 21,965
  - (c) 14,963.
  - (d) 6,140.
  - (e) 7,920.
24. Consult the regression output where per capita GDP is the dependent variable. Suppose that someone has argued that every percentage point increase in exports (as a percentage of GDP) will increase per capita GDP by 100 (\$US). Use the information you have to test this hypothesis with a significance level of .01. Hint:
- $$H_0 : \beta_4 = 100.$$
- (a) reject  $H_0$  with a test statistic of 3.35.
  - (b) reject  $H_0$  with a test statistic of 5.4.
  - (c) reject  $H_0$  with a test statistic of 2.7.
  - (d) fail to reject with a test statistic of .7.
  - (e) fail to reject with a test statistic of -2.4.
25. Consult the regression output where per capita GDP is the dependent variable. Consider the variable 2013 exports as a percentage of 2013 GDP. Note that the dependent variable is 2013 per capita GDP which is GDP divided by the population. In this setting is exports as a percentage of GDP truly an independent variable?
- (a) Yes, any variable can be independent or dependent depending on the model so it might be a dependent variable in a different model but here it is independent.
  - (b) Yes, since it is used to predict per capita GDP and is statistically significant it is an independent variable.

- (c) No, because it will depend on the size of the population and so that must be taken into consideration.
  - (d) No, because its value depends directly on the value of GDP.
  - (e) It depends on the particular country.
26. Consult the regression output where per capita GDP is the dependent variable. On average what is the difference between per capita GDP in Latin American and Caribbean countries and other countries around the world that have the same inflation rates and export to GDP ratio?
- (a) 13181.
  - (b) -7045
  - (c) 6136



## Key

1. c
2. a.
3. b.
4. c.
5. d. On my casio fx260 I hit [mode][.] and then enter the data points following each one with [M+]. then I hit [shift][9]. Or you can do it “by hand.”
6. c.
$$\frac{15334}{\sqrt{5}} = 6857.67$$
7. c.
$$\frac{11327 - 10000}{6857.7} = .194$$

or something close depending on how you might have rounded.
8. a.  $H_0 : \mu_{sa} = 10,000$  so this is a two-tail test.
9. d.  $n=5$  so  $df=4$  and this is a two-tail test so on the t table we will look in the .005 column when  $\alpha = .01$ .
10. b. we fail to reject because .194 is less than the critical value of 4.6.
11. e.
12. c.
13. d.  $14+16+8=38$  or  $37+1=38$ .
14. b. Read the value off the table in the “MS” column and the “Within Groups” row.
15. a. Notice that the p-value is  $.00056 < .01$  so we reject the null hypothesis.
16. c.
17. d. the t value is 1.753 which you found in the .05 column of the t table by going down to 15 df ( $df=n-1$ ). The standard error was found by taking the square root of the variance (5277.77) and dividing by  $\sqrt{n} = \sqrt{16} = 4$ , giving you 1319.4. You then multiply the standard error by your t value to get a margin of error of 2313 and then add that to and subtract it from the average to get [719,5345]. If you rounded at different points your numbers may be slightly different but will still not come close to the other choices.

18. e.  
19. b.  
20. d.  
21. a. The p value for this test is .0000000000978 which is smaller than .001 so we reject the null hypothesis at the .001 level of significance.  
22. d.  
23. c. The estimated value is

$$\hat{Y} = 13181.79 - 7045(0) + 802.05(1.5) - 2311.03(3.18) + 262.33(30.23) = 14963$$

The precise value will depend on rounding but should leave you with an answer that is close enough to make the correct choice.

24. a. The test statistic is about

$$t = \frac{262.33 - 100}{48.45} = 3.35$$

and the critical t is around the same as for a critical z given the large number of degrees of freedom so about 2.58 (actually is is closer to 2.61 but 2.58 will work perfectly well in this situation). Then  $3.35 > 2.58$  so reject the null hypothesis.

25. d.  
26. b.