Bounds on the Reliability of Distributed Systems With Unreliable Nodes & Links

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***Index Terms*—Complexity, distributed system, probabilistic network, residual connectedness reliability.**

Summary & Conclusions

The reliability of distributed systems & computer networks in which computing nodes and/or communication links may fail with certain probabilities have been modeled by a probabilistic network. Computing the residual connectedness reliability (RCR) of probabilistic networks under the fault model with both node & link faults is very useful, but is an NP-hard problem. Up to now, there has been little research done under this fault model. There are neither accurate solutions nor heuristic algorithms for computing the RCR. In our recent research, we challenged the problem, and found efficient algorithms for the upper & lower bounds on RCR. We also demonstrated that the difference between our upper & lower bounds gradually tends to zero for large networks, and are very close to zero for small networks. These results were used in our dependable distributed system project to find a near-optimal subset of nodes to host the replicas of a critical task.

Acronyms[1]

RCR	Residual Connectedness Reliability
UB	Upper Bound
LB	Lower Bound
NEF	Node-and-Edge Fault

Notations

G	graph
$R(G)$	residual connectedness reliability of graph G
n	number of nodes in graph
q_0	node failure probability
q_1	edge failure probability
$\|A\|$	cardinality of set A
$\lfloor\cdot\rfloor$	greatest integer lower bound (floor)
$\lceil\cdot\rceil$	least integer upper bound (ceiling)
E	event
$\overline{E}$	complement event of E
$N_G(v)$	neighbor node set of node v in graph G
$\overline{R}(G)$	upper bound of $R(G)$
$\underline{R}(G)$	lower bound of $R(G)$
ΔR	difference between $\overline{R}(G)$ and $\underline{R}(G)$, i.e., $\Delta R = \overline{R}(G) - \underline{R}(G)$,
Q_p	p-dimension hypercube
$H_{k,n}$	order n Harary graph
S_n	order n Star graph
C_n	order n Circle
K_n	order n Complete graph

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[1] The singular and plural of an acronym are always spelled the same.

I. Introduction

The behavior of a distributed system can be modeled by a probabilistic network or a graph G whose nodes and/or edges may fail [1]. The ability of the communication between the residual (remaining working) nodes is measured by the RCR $R(G)$, which is the probability that the residual nodes can communicate with each other [2]–[5].

Generally, there are three kinds of fault models in a probabilistic network [1]:

- **Node fault model**: The edges of a graph are perfectly reliable, but the nodes fail independently with probability q_0.
- **Edge fault model**: The nodes of a graph are perfectly reliable, but the edges fail independently with probability q_1.
- **Node-and-edge fault model**: Edges & nodes fail independently of each other, with node & edge failure probabilities equal to q_0 & q_1, respectively.

The analysis problem is to find $R(G)$ for a given graph G. That is:

Input: Probabilistic graph G, component failure probability q_0 and/or q_1.

Output: RCR $R(G)$.

For all these three fault models, it has been shown that the analysis problems are all NP-hard [1], [5]–[7]; that is, there exists no efficient algorithms for computing $R(G)$.

To cope with this problem, various heuristic algorithms have been developed to estimate $R(G)$.

We have carefully studied all the related papers we could find. There are quite a number of papers dealing with approximation algorithms for estimating $R(G)$ under the edge fault model [8]–[12], and under the node fault model [13], [14]. Colbourn [13] proposed a polynomial algorithm of certain restricted classes of graphs, including trees, series-parallel graphs, and permutation graphs. Colbourn & Chen [14] developed efficient algorithms of arbitrary graphs, and bound expressions for estimating $R(G)$. To our best knowledge, little work has been done