C H A P T E R 2

Risk Aversion

Expected value as a criterion for making decisions makes sense provided that
the stakes at risk in the decision are small enough to \ play the long run averages."
The range of decisions for which this is true covers many situations of practical
business interest, but sometimes the stakes are high enough that this is not an
appropriate assumption.

2.1 Risk Attitude

The value of a risky alternative to the decision maker may be different than the
expected value of the alternative because of the risk that the alternative poses
of serious losses. The concept of the **certainty equivalent** is useful for such
situations, as shown in Definition 2.1.

Definition 2.1: Certainty Equivalent

> The **certainty equivalent** for an alternative is the certain amount that
> is equally preferred to the alternative. An equivalent term for certainty
> equivalent is **selling price**.

Example 2.1

Certainty equivalent. Suppose that through a previous business deal you
have come into possession of an uncertain alternative that has equal chances
of yielding a profit of \$10,000 or a loss of \$5,000. The expected value for this
alternative is $0.5 \times \$10,000 + 0.5 \times (-\$5,000) = \$2,500$. However, suppose that
you decide that you would be willing to sell this alternative for \$500 or more.
Then, your certainty equivalent for the alternative is \$500. ■

Using the concept of the certainty equivalent, it is possible to specify different
attitudes toward risk taking, as shown in Definition 2.2.

Definition 2.2: Risk Attitude

If your certainty equivalent for alternatives specified in terms of profits is less than the expected profit for an alternative, you are said to be **risk averse** with respect to this alternative. If your certainty equivalent is equal to the expected profit for the alternative, then you are said to be **risk neutral**. Finally, if your certainty equivalent is greater than the expected profit for the alternative, you are said to be **risk seeking**. These definitions are reversed for an uncertain alternative specified in terms of losses. That is, you are risk averse if your certainty equivalent is greater than the expected loss and risk seeking if your certainty equivalent is less than the expected loss.

Based on our earlier discussion in Chapter 1, if you are risk seeking with respect to the various decisions that you make, then over the long run you will probably go broke because on average you will not recover as much from the alternatives as you are willing to pay for them. This is not typical behavior in business, and therefore we will not consider risk seeking behavior further. (Note that there are situations where a risk seeking attitude may make sense in business. For example, suppose your business is in such serious trouble that it is going to go broke anyway unless you get lucky. You might as well \pray for rain" in such a situation and go against the odds. However, this is not a typical business situation.)

It is worth noting that the appropriate attitude toward risk taking can depend on the asset position of the organization taking the risk. A large Fortune 500 company may be able to play the odds and use expected value as its decision criterion in situations that would pose serious risks to a small \mom and pop" business.

2.2 Utility Functions

If certainty equivalents can be determined for the alternatives in a decision problem, then it is straightforward to determine the preferred alternative| simply select the alternative with the best certainty equivalent. This section discusses a procedure to determine certainty equivalents for the decision alternatives. The theory for how to determine certainty equivalents in a defensible manner has been developed, and we will present a practical procedure for using this theory that is appropriate for many realistic business decisions. Readers who are interested in the theory behind this approach should consult a decision analysis textbook.

Certainty equivalents can be determined using a modification of the procedure that we use to determine expected values. This modification involves introducing a new function, called the **utility function**. A typical utility function is shown in Figure 2.1. In this figure, the evaluation measure scale is shown on the horizontal axis, and the utility for each evaluation measure level is plotted on the vertical axis. The range of the evaluation measure in this example is from -500 to 2,000,

Evaluation Measure
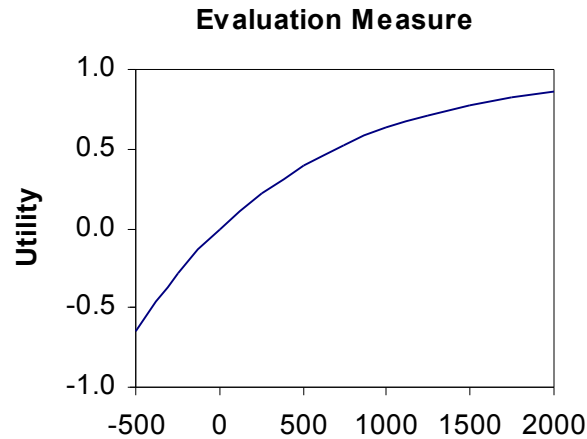
Figure 2.1 *Illustrative utility function*

and this evaluation measure might, for example, represent the net profit from a business decision in thousands of dollars. Note that the exact numbers on the vertical scale do not have specific meanings, except that greater numbers represent more preferred levels of the evaluation measure. For example, if the evaluation measure is dollars of profit, then there is greater utility for an amount of $2,000 than an amount of $1,000.

The idea underlying the approach to calculating certainty equivalents is to first convert the possible outcomes in a decision problem to utilities using the utility function, and then calculate the expected value of these utilities for each alternative *using the same procedure that was used to calculate expected values.* After determined these *expected utilities* for each alternative, then it is straightforward to determine the certainty equivalent for each alternative using a procedure discussed later in this section.

Definition 2.3: Utility Function

> A **utility function** translates outcomes into numbers such that the expected value of the **utility** numbers can be used to calculate certainty equivalents for alternatives in a manner that is consistent with a decision maker's attitude toward risk taking.

Here is an intuitive explanation of why this calculation procedure using expected utilities makes sense as a way to take risk attitude into account. Examine the utility function in Figure 2.1. Note that this function drops off rapidly as the level of the evaluation measure becomes worse (more negative), while it grows less rapidly as the value of the evaluation measure becomes better (more positive). Intuitively, this is saying that the value that we lose from each unit of decrease of

the evaluation measure becomes increasingly great as the level become more negative. Therefore, if we take an expected value of the utilities over the evaluation measure, alternatives that have a significant probability of yielding bad outcomes will be penalized more heavily in the calculation procedure than if expected value were used to evaluate the alternatives. Hence, an alternative with a significant chance of yielding bad outcomes will be downrated using a utility function from what would be true if expected value was used to evaluate alternatives.

2.3 The Exponential Utility Function

To implement the expected utility approach reviewed above, it is necessary to first determine a utility function. Both theory and practical experience have shown that it is often appropriate to use a particular form of utility function called the **exponential**. For risk averse decision makers, in decisions involving profits (more of the evaluation measure is better), this function has the form

$$u(x) = 1 - e^{-x=R}; \qquad R > 0$$

where $u(x)$ represents the utility function, x is the evaluation measure, R is a constant called the *risk tolerance*, and e represents the exponential function. (The exponential function is often designated by \exp" on a financial calculator or in a spreadsheet program.)

In situations involving costs where less of the evaluation measure is preferred, the exponential utility function has the form

$$u(x) = 1 - e^{x=R}; \qquad R > 0$$

and in this case larger values of x have lower utilities.

As noted above, the degree of risk aversion that is appropriate can depend on the asset position of the decision making entity, and R represents the degree of risk aversion. As R becomes larger, the utility function displays less risk aversion. (In fact, when R approaches infinity, the decision maker becomes risk neutral.) The utility function plotted in Figure 2.1 is an exponential utility function with $R = 1;000$.

2.4 Assessing the Risk Tolerance

The following procedure can be used to determine the approximate value of R for a particular decision maker: Ask the decision maker to consider a hypothetical alternative that has equal chances of yielding a profit of r_o or a loss of $r_o/2$. Then ask the decision maker to specify the value of r_o for which he or she would be indifferent between receiving or not receiving the alternative. (Or, put another way, ask the decision maker to adjust r_o until the certainty equivalent for this hypothetical alternative is just equal to zero.) When the decision maker has adjusted r_o in this way, then R is approximately equal to r_o. Note that the expected value for this hypothetical alternative is EV $= 0.5 \times r_o - 0.5 \times (r_o/2) = 0.25 \times r_o$, and therefore as long as r_o is greater than zero the decision maker is specifying a risk averse utility function.

We will now apply the expected utility approach to the Xanadu Traders decision.

Example 2.2

Xanadu Traders.   This is a continuation of the Xanadu Traders decision in Example 1.8. We continue to follow the conversation between Daniel Analyst and George Xanadu.

Analyst: I understand from my previous work with you that financial risks of the size involved in this deal would be uncomfortable but would not sink Xanadu Traders. If you could, you would buy some insurance against the potential loss, but you are not going to avoid the deal just because of the possible loss.

Xanadu: That's correct.

Analyst: I recall that you told me in the past that you would be just willing to accept a deal with a fifty-fifty chance of making \$2,000,000 or losing \$1,000,000. However, if the upside were \$2,100,000 and the downside were \$1,050,000, you would not take the deal.

Xanadu: That's correct.

> **Question 2.1:** Taking into account Xanadu's attitude toward risk taking, what is the preferred alternative among those shown in Figure 1.6?

To answer this question, it is first necessary to determine Xanadu's utility function. This can be done using the information in the dialog. Using the concept of the risk tolerance, $r_o =$ \$2 million when an uncertain alternative with equal chances of yielding a profit of r_o or a loss of $r_o/2$ has a certainty equivalent of 0. Hence, R is approximately equal to \$2 million. Therefore, Xanadu's utility function is

$$u(x) = 1 - e^{-x/2};$$
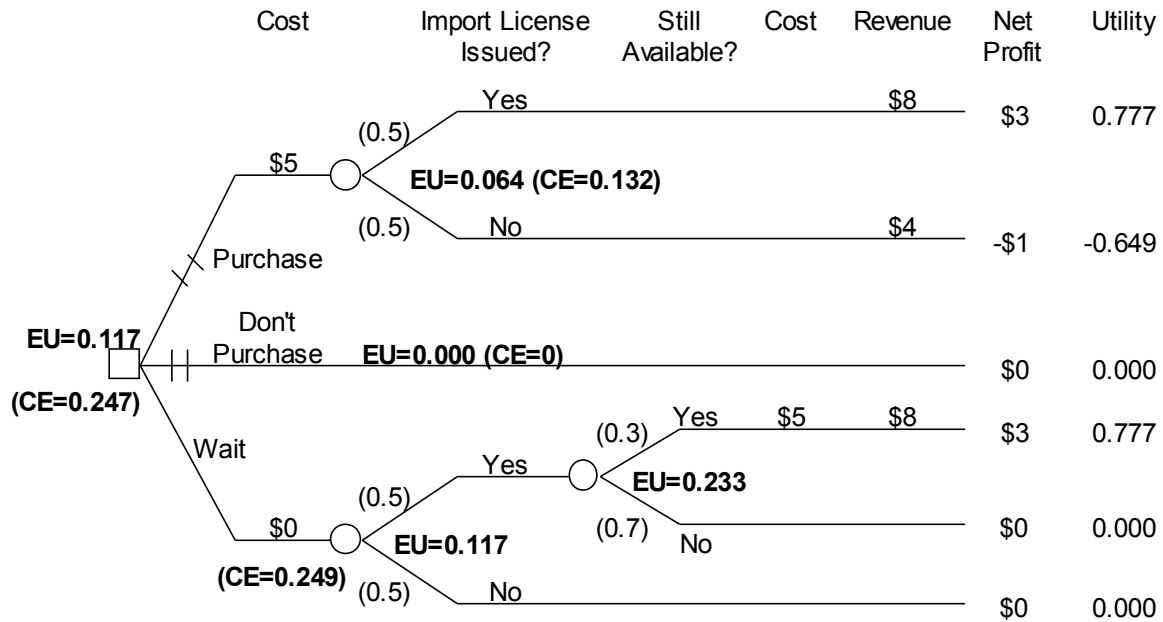
Figure 2.2   *Xanadu Traders Expected Utility Analysis*

where x is in millions of dollars. Using a spreadsheet or calculator, it is easy to find the utilities for each of the endpoint values in the Figure 1.6, and these are shown in Figure 2.2. In this figure, the utility numbers shown at the right side of the tree have been calculated using an exponential utility function with $R = \$2$ million. For example, the topmost utility number is given by $u(x) = 1 - e^{-3/2} = 0.777$.

Expected utility numbers are calculated in the same manner as expected values. For example, the expected utility for the topmost chance node is given by $EU = 0.5 \times (0.777) + 0.5 \times (-0.649) = 0.064$. This is the expected utility for the "purchase" alternative, and in a similar manner the expected utilities can be found for the "don't purchase" alternative ($EU = -1.000$) and the "wait" alternative ($EU = 0.117$). ■

2.5 Certainty Equivalent for an Exponential Utility Function

Expected utility numbers do not have a simple intuitive interpretation, but there is a specific certainty equivalent corresponding to any specified expected utility. For an exponential utility function involving profits, it can be shown that the certainty equivalent is equal to

$$CE = -R \times \ln(1 - EU);$$

where CE is the certainty equivalent, EU is the expected utility, R is the risk tolerance, and ln is the natural logarithm. Thus, the certainty equivalent for the \purchase" alternative in Figure 2.2 is given by CE $= -2 \times \ln[1 - 0.064] = \$0.132$ million. The certainty equivalents are shown for all three alternatives in Figure 2.2, and larger certainty equivalents are more preferred.

In situations involving costs, where less of an evaluation measure is preferred to more, then the certainty equivalent is equal to

$$CE = R \times \ln(1 - EU)$$

and alternatives with smaller certainty equivalents are more preferred in this case.

Since a certainty equivalent is the certain amount that is equally preferred to an alternative, the alternative with the greatest certainty equivalent is most preferred for situations where more of an evaluation measure is preferred to less. Therefore, taking Xanadu's risk attitude into account, the \purchase" alternative is no longer the preferred alternative, as it was with the expected value analysis. The \wait" alternative is now most preferred since it has a certainty equivalent of $0.249 million, and the \purchase" alternative is now the second most preferred alternative with a certainty equivalent of $0.132 million. The \don't purchase" alternative continues to be least preferred with a certainty equivalent of $0.

Note that expected utilities can be directly used to rank alternatives in a decision problem. It can be shown that the alternative with the greatest expected utility will also have the most preferable certainty equivalent. (Note that this is true regardless of whether you are dealing with costs or profits, provided that you use the appropriate utility function formula given above.) Thus the three alternatives in Figure 2.2 could have been ranked directly using the expected utilities without calculating certainty equivalents. However, it is often preferable to calculate certainty equivalents since these are easier to intuitively interpret.

Example 2.3

Xanadu Traders.   This example completes our study of Xanadu Traders. A comparison of the expected values and certainty equivalents for the three alternatives in Figure 2.2 is shown in Table 2.1. This demonstrates that the three alternatives have differing risks. There is no difference between the expected value and the certainty equivalent for the \don't purchase" alternative since there is no uncertainty with this alternative. The difference between the expected value and certainty equivalent is greatest for the \purchase" alternative indicating that it has the largest risk. This risk reduces the value of this alternative enough for Xanadu that it is no longer the most preferred alternative. The \wait" alternative also has a lower certainty equivalent than its expected value since this alternative has some risk. However, this risk is substantially lower than the risk for the \purchase" alternative, and hence this becomes the preferred alternative when Xanadu's risk attitude is taken into account.   ∎

Alternative	Expected Value	Certainty Equivalent	Difference
Purchase	1.000	0.132	0.868
Don't Purchase	0.000	0.000	0.000
Wait	0.450	0.249	0.201

Table 2.1   *Comparison of expected values and certainty equivalents*

2.6 Exercises

2.1   This is a continuation of Exercise 1.4. Assume that all the information in that exercise still holds, except assume now that Aba has an exponential utility function with a risk tolerance of $100,000. Determine Aba's preferred course of action.

2.2   This is a continuation of Exercise 1.5. Assume that all the information in that exercise still holds, except assume now that Kezo has an exponential utility function with a risk tolerance of $750,000. Determine Kezo's preferred ordering alternative using this utility function.

2.3   This is a continuation of the preceding exercise. (That is, assume that Kezo has an exponential utility function with a risk tolerance of $750,000.) In an effort to attract Kezo's order, KEC Electronics has revised its offer as follows: At no increase in price, KEC will now provide Kezo with the right to cancel its entire order for a 10% fee after the outcome of the antidumping suit is known. However, KEC will not be able to accept any additional orders from Kezo once the outcome of the suit is known. Thus, for example, if Kezo has agreed to purchase 250,000 PAL chips from KEC at $2.00 per chip, Kezo can cancel the order by paying $50,000. This ability to cancel the order is potentially of interest to Kezo because it knows that AM Chips would be able to supply PAL chips after the outcome of the antidumping suit is known in time for Kezo to fill the Tarja order. However, Kezo knows that AM will increase the price of its chips if an antidumping tax is imposed. In particular, if a 50% tax is imposed, then AM will increase its chip price by 15%. If a 100% tax is imposed, then AM will increase its chip price by 20%. Finally, if a 200% tax is imposed, then AM will increase its chip price by 25%. Assuming that all other information given in the preceding exercise is still valid, determine Kezo's preferred alternative for the initial order of PAL chips as well as what Kezo should do if the antidumping tax is imposed.