CHAPTER 9

Initial Conditions

Each stock in a model must be given an initial value. Once this is done, the simulation program will calculate the time history of each model variable for the specified time period. Sometimes initial values for the stocks can be quickly determined, but in other situations this task can be complicated and tedious. This chapter discusses two specific difficulties that often arise in specifying initial conditions for the stocks in a model.

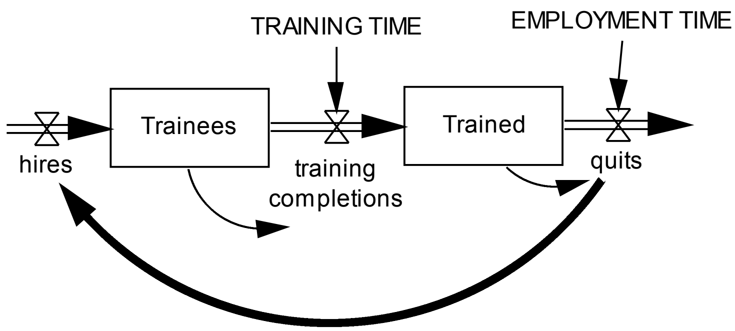
9.1 Initializing a Model to Equilibrium

Many models are specified for a process that is in *equilibrium*. That is, the values of the variables in the process are not changing. Often a model is being constructed for the process in order to estimate the impacts of making changes to the structure or operating policies of the process, and in such situations the first step in the modeling effort is to set the model up so that it reproduces the behavior of the existing process. This requires that the model be in equilibrium.

Figure 9.1 illustrates a simple model of this type for a personnel process which includes trainees and trained personnel. Trainees require an average of 3 months to train, and they stay in employment for an average of 3 years (36 months) once they are trained. Hiring is done to replace trained employees who quit, and the number of people to hire is determined using an exponential average of the quits over the last 6 months (26 weeks). It is known that there are currently 1000 trained employees, and it is estimated that there are 250 trainees.

When the model is run, the curves shown in Figure 9.1c result. Since it is desired to have the model in equilibrium, clearly something is wrong since the number of trainees and trained personnel both change over time. If the model were in equilibrium, these would remain constant.

The difficulty has resulted from the process used to set the initial conditions for the two stocks Trainees and Trained. For the process to be in equilibrium, none of the model variables can change, and some thought shows that this means that *the inflows and outflows for each stock must be equal.* For the model in Figure 9.1, this means that hires must be equal to training completions (for



a. Stock and flow diagram

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(01) EMPLOYMENT PERIOD = 36
(02) FINAL TIME  = 50
(03) hires = SMOOTH(quits, 26)
(04) INITIAL TIME  = 0
(05) quits = Trained / EMPLOYMENT PERIOD
(06) SAVEPER  = TIME STEP
(07) TIME STEP  = 0.125
(08) Trained = INTEG (+training completions-quits, 1000)
(09) Trainees = INTEG (hires-training completions, 250)
(10) training completions = Trainees / TRAINING PERIOD
(11) TRAINING PERIOD = 3
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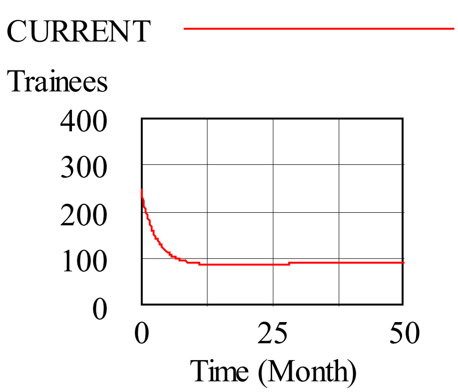
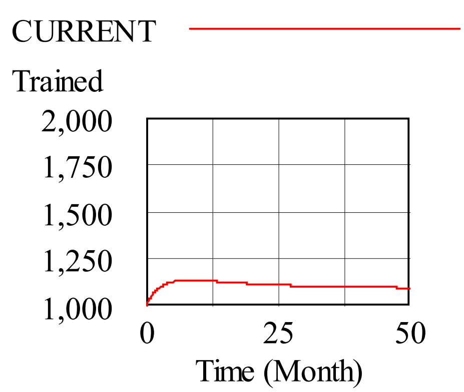
b. Vensim equations



a. Trainees



b. Trained

Figure 9.1 *Personnel training model*

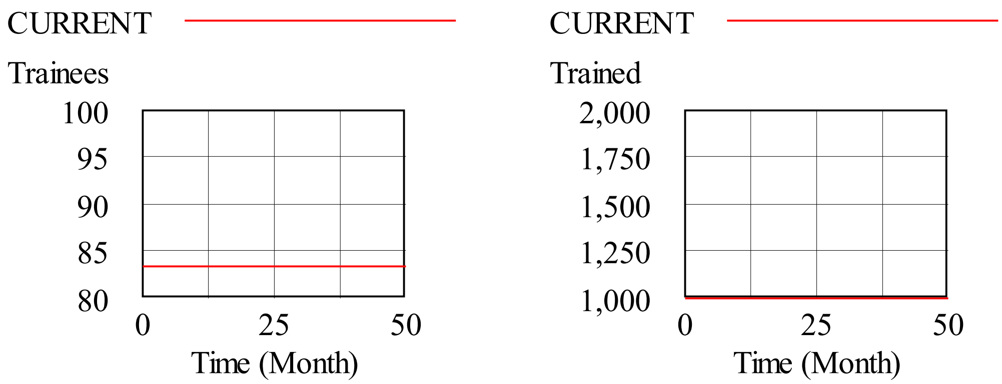


Figure 9.2 *Personnel training model (modified initial conditions)*

the inflows and outflows to Trainees to be equal), and training completions must be equal to quits (for the inflows and outflows to Trained to be equal.

However, as equations (03) and (10) in Figure 9.1b show, the flow training completions is equal to Trainees/TRAINING TIME, and the flow quits is equal to Trained/EMPLOYMENT TIME. Therefore, it is not possible to set both of the stocks Trainees and Trained independently. Specifically, for the rates training completion and quits to be equal it must be true that

$$\frac{\text{Trainees}}{\text{TRAINING TIME}} = \frac{\text{Trained}}{\text{EMPLOYMENT TIME}}$$

Thus, for the values of Trained, TRAINING TIME, and EMPLOYMENT TIME specified above, it must be true that Trainees $= 1\ 000 \times (3\ \ 36) = 83\ 3$, which is considerable different from the value of 250 that was assumed in the Figure 9.1 model.

Rather than doing all this arithmetic by hand, it may make sense to enter the equation for the initial value of Trainees into the equation for this stock. This requires replacing equation (09) in Figure 9.1b with

```
(09) Trainees = INTEG (hires-training completions,
                 Trained * (TRAINING PERIOD / EMPLOYMENT PERIOD))
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The results of making this change to the Figure 9.1 model are shown in Figure 9.2. Now the model is in equilibrium.

We have not discussed how to set the initial value for the flow hires. This is an exponentially smoothed value of quits. The SMOOTH function in Vensim is specified to have an initial output value equal to its input value. The initial value of the input to this SMOOTH is the initial value of quits, and so the initial value of hires (the output of the SMOOTH function) will be equal to the initial value of quits, which is (by the analysis above) equal to the initial value of training completions. Thus, the initial values of hires and training completions are equal, and hence the inflow and outflow for Trainees are equal, as they must be if the process is to be in equilibrium.

9.2 Simultaneous Initial Conditions

A second type of difficulty that can arise in specifying initial conditions involves the presence of simultaneous equations. Vensim and other similar simulation programs cannot solve simultaneous equations, and therefore it is not possible to set up initial conditions which require solving simultaneous equations. Such simultaneous equations can occur when a causal loop structure in a model includes only auxiliary variables. Figure 9.3 illustrates a model with this difficulty.

This is a model where the quality of a product is determined by the testing that is done on the product, and the order rate is impacted by the quality perceived by the customer. When the quality perceived by the customer is equal to one, there is an order rate of 10,000 units per month. In order to provide a product with a quality of one, each unit shipped must receive a testing effort equal to one hour of testing. Thus, at a shipping rate of 10,000 units per month, a TESTING CAPACITY of 10,000 hours per month is required. The quality perceived by the customer is an exponential smooth of the actual quality of the product shipped with a smoothing period of 6 months. That is, the customer perception of product quality changes more slowly than the rate at which actual product quality changes.

Everything appears to be correct in this model for it to be initialized to equilibrium. The TESTING CAPACITY is equal to 10,000 hours per month, which is the capacity required to maintain a product quality equal to one, which in turn is the quality required to have an order rate of 10,000 units per month. Hence it appears that the model in equilibrium. However, when you attempt to run the model, Vensim provides the following message: Model has errors and cannot be simulated. Do you want to correct the errors? If you click the Yes button, you see a further message that says there are Simultaneous initial value equations.

The difficulty is that there are simultaneous initial conditions involving the variables quality perceived by customers, order rate, and quality of product shipped. When Vensim attempts to solve for any of these variables, it ends up circling around back to the same variable. We know that the conditions specified in the Figure 9.3b equations are consistent, and they imply an order rate of 10,000 units per month with a quality of one, but Vensim is not able to determine this.

The SMOOTHI function is provided by Vensim to handle such situations. This has the same functionality as the SMOOTH function except that it allows you to specify an initial value for the output of the function. (Recall that with the SMOOTH function the initial output value of the function is equal to the initial input value.) The difficulty with simultaneous initial values is resolved by replacing equation (05) in Figure 9.3b with the following:

```
(5) quality perceived by customers =
        SMOOTHI(quality of product shipped, 6, 1)
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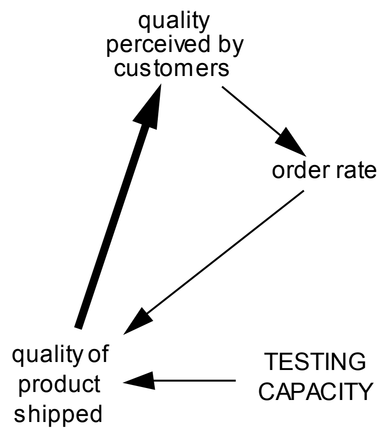
This specifies that the initial output for the SMOOTH function will be equal to one. Vensim can then use this initial output to set the value for quality perceived by customers to one, and then can use this to determine the initial values for the other variables.



a. Stock and flow diagram

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(1) FINAL TIME  = 50
(2) INITIAL TIME  = 0
(3) order rate = 10000 * quality perceived by customers
(4) quality of product shipped = TESTING CAPACITY /order rate
(5) quality perceived by customers
      = SMOOTH(quality of product shipped, 6)
(6) SAVEPER  = TIME STEP
(7) TESTING CAPACITY = 10000
(8) TIME STEP  = 0.125
```

b. Vensim equations

Figure 9.3 *Quality impacts ordering*