Distribution Constraints on Resource Allocation of PEV Load in the Power Grid

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***Abstract*— Social and economic pressures are inducing the proliferation of plug-in electric vehicles (PEVs) in the market. As the power grid has a limited supply of electricity, this problem can be formulated as a resource allocation problem, minimizing costs of charging. In this paper we model the PEV charging problem to minimize the total cost of charging, taking into account distribution constraints. These constraints model the maximum power that distribution-lines can carry as a function of time. Moreover, we develop an algorithm to schedule the PEVs aiming at minimizing the total charging cost, subject to these instantaneous distribution constraints. Although the proposed algorithm is a suboptimal one, it is of a polynomial complexity in the number of PEVs in the system.**

Index Terms—PEV, resource allocation, electricity markets

I. Introduction

Social and economic pressures are inducing the proliferation of plug-in electric vehicles (PEVs). PEV sales have been increasing by 80% every year since 2000, causing a shift from crude oil demand to electricity demand from the power grid [1], [2]. Some sources project that up to 18% of the power grid load in 2030 will be PEV load, inherently much more variable than traditional loads in the residential and commercial sector [3], [4]. Without any changes to the current power grid, such high levels of PEV penetration can cause price spikes within the electricity market, or even threaten the stability of the power system. For example, PEV owners may choose to immediately charge their vehicles after arriving home in the time window of 4-6PM, causing a spike in the load profile. As this is already a daily peak demand window, this would force more expensive reserves to be used, raising overall prices for generation and consumption of electricity, and in the worst case, causing frequency and voltage fluctuations resulting in a drastically increased probability of blackouts [5]. With data from the National Household Travel Survey (NHTS), it can be shown that uncontrolled PEV charging or even a demand response induced delayed charging method would drastically increase the peak consumption in a given region [6]. But as, with simple coordination, the PEV load profile could be shaped. With sophisticated control algorithms, the problem may be eliminated.

Research into the viability of communicating with and controlling a large number of PEVs in a distribution system show that mass PEV charging is possible with a low-bandwidth broadcast (as infrequent as once every 3 minutes), such that only minor modifications to the existing power communication infrastructure is needed [7]. With intelligent, bi-directional controls that will be implemented using the smart grid's communication infrastructure, algorithms can be devised to mitigate the spike and use the PEV batteries to the power system operator's advantage.

These problems are generally applied in the context of wide scale slow-charging of PEVs, assuming that the infrastructure and available charging options do not change, as well as ignoring the distribution constraints imposed by the power grid. Residential distribution systems, including transformers, have stringent current limits that are set by physical or economic constraints, so a large increase in residential load from PEVs will have to be accounted for in problem formulations. Solutions such as fast-charging stations and battery-switch stations have also been proposed in accommodating heavy penetration of PEVs to encourage consumer-confidence in PEV penetration by offering a quick and efficient way to charge the vehicle in everyday use [8, 9]. These methods, however, only add larger power spikes to PEV charging, underscoring the need to form efficient algorithms to charge PEVs on a massive scale at the distribution level.

Energy in the power grid can be seen as a limited resource, distributed to different clients at different times according to pricing, capacity, and other power system constraints. Given a certain penetration level of PEVs, the available grid power must be efficiently allocated among penetrating PEVs. Thus, the problem can be formulated as a resource allocation problem, with a focus on minimizing PEV charging cost, minimizing risk of load mismatch, or both.

The rest of this paper is organized as follows: Section II discusses the related work in the literature that has been carried out before. We formulate the problem in Section III, discuss our proposed algorithm in section IV, and mention the simulation results in Section V. Finally, the conclusion of the paper is in Section VI.

II. Related Work

One common approach that optimizes of PEV charging cost for a power system includes game theoretic models with a deregulated energy market, sometimes with a vehicle-to-aggregator or vehicle-to-grid component that adds bi-directional charge capabilities between the PEV and the power grid [10-12]. More related to this paper is minimizing cost through aggregator scheduling under a variety of different scenarios or assumptions. For example, the authors in [13] take into account a vehicle-to-grid system where an aggregator can use the capacity of electric vehicles to bid into the real-time energy market, elaborating further on the role of ancillary services in scheduling and cost optimization in [14]. Other work explores unidirectional

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scheduling, using time constraints on PEV loads to determine optimal load management [15].

Optimization of power system cost with an aggregator may consider day-ahead prices with an assumption of perfect day-ahead load forecasting or a stochastic model treating the real-time load as a random variable. In [6], a framework for aggregators of PEVs is formed where load scheduling is performed based on day-ahead prices and forecasted load, then the dynamic dispatch algorithm is applied. The proposed algorithm tries to minimize the total cost of PEV charging, with constraints ensuring that each PEV is fully charged during its plug-in period and that the charging rate values do not exceed battery constraints. The solution taken in this optimization problem was to "rank" (i.e. sort) the available slots (those slots between the arrival and departure of user i) according to a ranking function. This ranking function ranks the slots in an ascending order of their price rates.

Balancing both the PEV charging cost minimization and the load mismatch has been explored in [16]. This charging problem was set up as a two-stage optimization problem, separated into risk-aware day-ahead scheduling and risk-aware real-time dispatch. The authors use the term 'risk-aware' to denote that the risk of load mismatch is represented in both their scheduling and their real-time dispatch, so day-ahead forecasting is not assumed perfect. The charging period of each PEV is divided into slots of equal length. The supply model assumes a specific amount of power scheduled in the day-ahead market for each slot according to the day-ahead price and the forecast for the PEV demands. As the forecasting is not assumed to be perfect, the actual demand will differ from the forecasted demand, resulting in a real-time price for bought or canceled excess power, respectively. The risk of load mismatch is manifested as the difference between P_t^{da}, the day-ahead scheduled power, and P_t. The PEV model assumes a different energy demand for each PEV, with a maximum charging rate and a set plug-in time and charging deadline.

One assumption in the problem setup is the necessity for the aggregator to charge all PEVs to a full charge state before disconnecting them from the grid, not allowing for optimization using partial charges. Thus, the authors assume that the energy demand can be met in the specified time slots. Another assumption is that the PEVs will stay connected for the entire duration of the 10 PM to 7 AM timeframe, so the problem does not have PEV arrival/connection time as a stochastic variable. First, the risk-aware day ahead scheduling problem is shown as an optimization problem, minimizing cost. Due to the non-convexity of the objective function, the problem was recast as the optimization of a stochastic, two-stage linear program.

Distribution limits of charging must be considered. Other papers have examined charging cost optimization of PEVs assuming an upper limit to the charging of the physical battery, but not the distribution system limits that would be imposed by a few factors, such as transformer power limits combined with consumer demand patterns. The study most related to this problem appears in a conference paper submitted by both Manitoba Hydro and the Electric Power Research Institute (EPRI), where the impact of increased PEV penetration was examined within the Canadian province of Manitoba with collected data and PEV adoption statistics [17]. In this study, the authors used the NHTS travel survey data to establish the assumptions that characterize the demand of consumer PEVs over a day. PEV penetration levels from 0%-20% were studied at two different voltage charging levels, 120V and 240V. Also, a few main charging scenarios were considered, including coincident PEV charging and diversified charging. Coincident PEV charging, where the PEVs immediately charge at their 'arrival' time, was split into a peak hour and off-peak hour situation, and diversified charging assumed that 20% of the load was applied during each of the 5 hours as a type of demand response tactic. Emergency ratings of distribution service transformers were exceeded for many of the conditions, especially for 240V charging situations, and voltage also dipped drastically. This study underlines the need for the incorporation of distribution constraints into any PEV charging problem, as it is shown even at 10% PEV penetration to be a limiting factor in charging.

III. Distribution Constraints

The general optimization problem, without any distribution constraints, can be formulated as:

$$\begin{array}{lll} \text{minimize} & \Delta \sum_{i=1}^{N} \sum_{k=1}^{K} \tau_k p_{i,k} & \\ \text{subject to} & \sum_{k=1}^{K} p_{i,k} = p_i l_i & (1) \\ & p_{i,k} \in \{0, p_i\} \quad\quad \forall i,k & (2) \\ & p_{i,k} = 0 \quad\quad \forall\, i,k, \quad a_i < k \le e_i & (3) \\ \text{variables} & \{p_{i,k}\} & \end{array}$$

where $p_{i,k}$ is the amount of power supplied to PEV i in slot k, Δ is the length of the slot, τ_k is the electricity charging price in slot k over a total of K slots, l_i is the number of slots allocated to PEV i, and N is the total number of PEVs to be charged in a section of the distribution grid.

The total amount of power allocated to PEV i through all K timeslots must match the number of charging times, as shown in (1). The charging rate $p_{i,k}$ is either set at a maximum rate p_i or zero from constraints in the charging hardware in (2). The constraint (3) shows that charging must be zero in periods when PEV i is disconnected from the power grid, or outside of the arrival and departure deadlines.

For this optimization problem, we impose a constraint to capture the effect of distribution capabilities. The constraint added guarantees that at each time-slot k we have a maximum amount of power that the users can totally drive from the power grid. This maximum power P_k^{max} is dictated by the power grid according to the time of the day. So, at peak time-slots, the distribution lines will have smaller P_k^{max} than those at off-peak time-slots. The optimization problem with distribution constraints becomes,

$$\begin{array}{ll} \text{minimize} & \Delta \sum_{i=1}^{N} \sum_{k=1}^{K} \tau_k p_{i,k} \\ \text{subject to} & \sum_{k=1}^{K} p_{i,k} = p_i l_i \\ & p_{i,k} \in \{0, p_i\} \quad\quad \forall i,k \\ & p_{i,k} = 0 \quad\quad \forall\, i,k, \quad a_i < k \le e_i \\ & \sum_{i=1}^{N} p_{i,k} \le P_k^{max} \quad\quad \forall k \\ \text{variables} & \{p_{i,k}\} \end{array}$$

where as before, $p_{i,k}$ is the amount of power supplied to PEV i in slot k, Δ is the length of the slot, τ_k is the electricity charging price in slot k over a total of K slots, l_i is the number of slots allocated to PEV i, and N is the total number of PEVs to be charged in a section of the distribution grid. The final constraint listed is the added distribution constraint.

IV. Algorithm with Distribution Constraints

Prior work in PEV dispatch algorithms do not completely account for distribution constraints since they allocate the timeslots for the arriving users regardless of the transformer or distribution line loading. Thus, we devise a new dispatch algorithm that minimizes the total power allocated to all users in all timeslots incorporating distribution constraints. The proposed dispatch algorithm is detailed in Algorithm 1. Algorithm 2 is a feasibility check function called by Algorithm 1 that will be discussed later in this section.

Algorithm 1 Dispatch

Input:
- a. T: Set of already-existing users in the system that have not completed charging yet.
- b. s: Current time-slot.
- c. τ_k: Charging prices.
- d. P_k^{max}: Maximum power limit due to distribution system requirements for slot k.
- e. For each user i:
  - i. Remaining charging time-slots l_i.
  - ii. Charging-rate p_i.
  - iii. Departure time slot e_i.

1. **Loop** (if any new users arrived) **Do**:
   - a. $T = T \cup \{New\ Arrivals\}$.
   - b. $H_i = \{\}$ (Set carrying time slots allocated to user i).
   - c. **While** (T is not empty) **Do**:
     - i. Find the user having the maximum charging-rate p_i among all users $\in T$
       $$i_{max} = argmax_{i \in T} p_i.$$
     - ii. Find the set of time-slots $S_{i_{max}}$ that are feasible for user i_{max} (i.e. slots that can accommodate i_{max})
       $$S_{i_{max}} = \{k: P_k^{max} \geq p_{i_{max}}\}$$
     - iii. Find the users having deadline before $e_{i_{max}}$.
       $$B = \{i: e_i < e_{i_{\max}}\}$$
     - iv. **While** ($l_{i_{max}} > 0$) **Do**:
       1. **For** all the time-slots in $S_{i_{max}}$ **do:**
          - a. $k^* = argmin_{k \in S_{i_{max}} - H_{i_{max}}} \tau_k$
          - b. **If Check_Feasibility** $\left(B, \{l_i\}_{i \in B}, \{p_i\}_{i \in B}, \{P_k^{max}\}_{k < e_{i_{max}}}, p_{i_{max}}\right)$=1
            Then $H_{i_{max}} = H_{i_{max}} \cup \{k^*\}$,
            $l_{i_{max}} = l_{i_{max}} - 1$.
            $P_{k^*}^{max} = P_{k^*}^{max} - p_{i_{max}}$
            Exit For-Loop
          - c. **Else:** $S_{i_{max}} = S_{i_{max}} - k^*$
          - d. **End If**
       2. **End For**
     - v. **End While**
   - d. $T_a = T_a - \{i_{max}\}$
   - e. **End While**
2. **End Loop**

Algorithm 2 Check_Feasibility$\left(B, l_i, p_i, P_k^{max}, p_{i_{max}}\right)$

Input: $\left(B, l_i, p_i, P_k^{max}, p_{i_{max}}\right)$ (Note: variables here are local variables only)
1. Feasible = 1
2. **While** (B is not empty) **Do**:
   - a. B_{early} is the set of users having the earliest departure time, i.e.
     $$B_{early} = \underset{i \in B}{\operatorname{argmin}}\, e_i$$
   - b. **Loop:** (to check feasibility of each user in B_{early} starting with the user having the minimum charging rate p_i)
     - i. $i_{min} = \underset{i \in B_{early}}{\operatorname{argmin}}\, p_i$
     - ii. $K_{i_{min}} = \{k: P_k^{max} \geq p_{i_{min}}\}$
     - iii. **If** $\left(|K_{i_{min}}| < l_{i_{min}}\right)$
       Then: Feasible=0, **End Loop, End While**
     - iv. **Else:**
       1. Rank the slots in $K_{i_{min}}$ in ascending order of P_k^{max}. Call it $R_{K_{i_{min}}}\left(\{P_k^{max}\}_{k \in K_{i_{min}}}\right)$.
       2. $P_k^{max} = P_k^{max} - p_{i_{min}} \quad \forall k \in \left\{k: R_{K_{i_{min}}}(P_k^{max}) \leq l_{i_{min}}\right\}$
       3. $B_{early} = B_{early} - \{i_{min}\}$
       4. $B = B - \{i_{min}\}$
     - v. **End If**
   - c. **End Loop**
3. **End While**
   Return (Feasible)

When new PEVs arrive to the system at slot s, the algorithm allocates timeslots to these new users. The main idea of the algorithm stems from the fact that we want to allocate time slots to users having higher charging rate p_i first, giving less priority to users having lower charging rates. To understand this, consider two users with $p_1 = 100$ units while $p_2 = 10$ unit. Assume that both users have arrived, and will leave, in the same timeslots. If two timeslots are to be allocated to these two users (and the cheapest of them can only provide 100 units of power), then we would favor user 1 to be assigned to the slot with the smaller charging price τ_k. This will yield minimum total charging price regardless of the amount of timeslots required by each user (i.e. the amount of energy to fully charge their batteries).

However, users who arrive at a given timeslot might not depart simultaneously. If user 1 will depart after user 2, the system must compromise between meeting user 2's deadline and finding an allocation with the minimum cost. If we do not take this compromise into account (i.e. if we assign timeslots based on their prices only), then the cheapest timeslots may not be able to accommodate both users simultaneously. When those slots are assigned to user 1, user 2 may not be allocated any slots, since the latter has an earlier departure time. To tackle this problem in the algorithm, we choose the user i_{max} who has the highest charging rate p_k, and allocate the user i the cheapest timeslots in an iterative fashion. During each iteration we **Check** (using **Check_Feasibility** algorithm) if this assignment will be feasible or not. That is, we check if the users who will depart the system before user i_{max} (i.e. users in the set B) can still be accommodated in the remaining time-slots after this cheapest timeslot is assigned to user i_{max} or not. If not then the second cheapest timeslot is checked, continuing down time-slots until a suitable timeslot is found. Once this user is fulfilled (i.e. he is allocated all the timeslots he needs), then we move on to the user with the second highest charging rate, until assigning time-slots to all users.

With this in mind, in order to implement the algorithm online, we need to not only allocate time slots to the newly arriving users but to the old users in the system as well. That is, we must take into consideration the users that have arrived in previous time slots and have not completed their charging yet. Those users have already been allocated time slots before slot s (that they have already used), as well as slots after s that they haven't used yet. Those latter may not be the optimum allocation given the newly arriving EVs. Thus, to take these old users into consideration, we consider them as newly arriving users, but the amount of power they require is the amount of power remaining to fully charge their batteries at this time slot s (l_i). Although this algorithm satisfies all the constraints of the aforementioned optimization problem, it is a sub-optimal algorithm with respect to minimizing the objective function.

V. Simulation Results

We have simulated the system assuming a slotted structure, where each time-slot is 0.5 hours, for one day (i.e. $K = 48$ time-slots in the system), while the number of users is set at $N = 20$ users. The maximum distribution power constraint P_k^{max} is chosen with a uniformly random distribution over the interval $[1,1000]$ units ($\forall k$). The charging power rate p_i, the departure time-slot e_i, and the number of time-slots l_i to fully charge user i are all assigned with a uniformly random distribution between $[1, 200]$ units, [1,K], and $[1, e_i]$ respectively. In Fig. 1, we can find that the maximum load of the EV charging on the distribution system, after time-slot allocation via the proposed algorithm, has not exceeded the power distribution constraint that is set by the distribution system. On the same figure, we plot the load curve after allocating the time-slots in the same system but with ignoring the distribution constraints. In other words, allocating the time-slots to minimize the total charging cost of the aggregate users. This latter indeed is optimum with respect to minimizing the total cost, but we can see that there are some particular time-slots of the day (e.g. 4-5 am in Fig. 1) where the load exceeded the maximum value that the distribution lines can hold.

The total power due to the proposed algorithm is found to be about 30% more than that when ignoring the distribution constraints. This comes with the advantage of protecting the distribution system from getting over-loaded, including the safety of the transformers, lines, and protection hardware spread about the system.

To calculate the complexity order of the proposed algorithm, we can find that we loop over N users, assigning the user up to K time-slots. For each assigned time-slot, we call the **Check_Feasibility** algorithm up to K times. **Check_Feasibility** has complexity of $O(NK)$. Thus, the complexity of the proposed algorithm is of $O(N^2K^3)$, which is a polynomial-time complexity. Although the complexity seems fifth order complexity, we expect that the future EV charging system will increase dramatically in the number of users N, while the number of time-slots K will be limited. This is because a single user is not expected to be charging, plugged-in for more than a few hours, even with a large number of PEVs in the system. Thus the complexity of the system is expected to be of $O(N^2)$ (i.e. quadratic complexity).

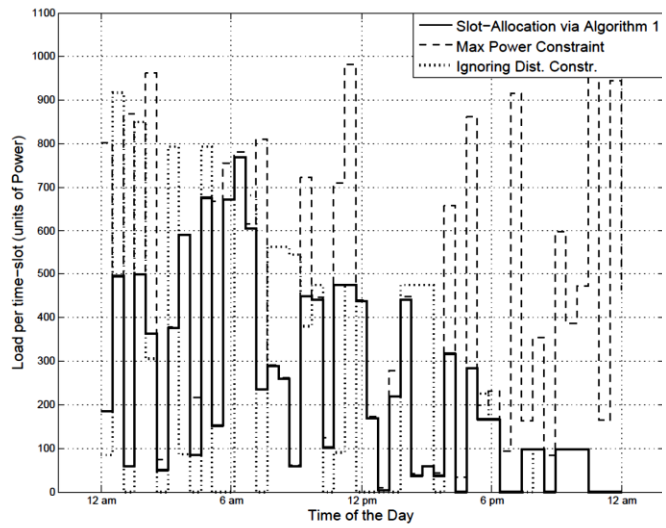

Fig. 1. Allocated loads from the algorithm from simulation

VI. Conclusions

The resource allocation problem for PEV charging is inherently a non-convex, stochastic optimization problem. However, the problem can be restructured and reformulated to arrive at near-optimal solutions. Research shows that with enough electric vehicle penetration, charging PEVs will create a spike in demand, which is especially harmful to the stability and economic efficiency of the power system during peak load hours. One solution to this problem is to allow an aggregator to coordinate the charging of the PEVs, aimed at minimizing the charging cost. This will automatically impose the PEVs to be charged in the valley of the current power load profile, or at least will not result in an overload to the power grid. Prior work finds the scheduling and dispatch algorithm that minimizes the total PEV charging cost based on the day-ahead expected prices and load, introducing the risk of load-imbalance in the optimization formulation. Presented is an algorithm that finds allocates the time slots of users in system where the distribution-line constraints must be taken into consideration. With polynomial complexity, this algorithm will be able to serve as a basis for realistic charging algorithms that will tackle the problems inherent in power systems with heavy amounts of PEV charging.

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