Throughput Optimization in Multichannel Cognitive Radios With Hard-Deadline Constraints

Ahmed E. Ewaisha and Cihan Tepedelenlio˘glu, Member, IEEE

Abstract—In a cognitive radio scenario, we consider a single secondary user (SU) accessing a multichannel system. The SU senses the channels sequentially to detect if a primary user (PU) is occupying the channels and stops its search to access a channel if it offers a significantly high throughput. The optimal stopping rule and power control problem is considered. The problem is formulated as an SU’s throughput-maximization problem under power, interference, and packet delay constraints. We first show the effect of the optimal stopping rule on packet delay and then solve this optimization problem for both the overlay system, where the SU transmits only at the spectrum holes, and the underlay system, where tolerable interference (or tolerable collision probability) is allowed. We provide closed-form expressions for the optimal stopping rule and show that the optimal power control strategy for this multichannel problem is a modified waterfilling approach. We extend the work to a multi-SU scenario and show that when the number of SUs is large, the complexity of the solution becomes smaller than that of the single-SU case. We discuss the application of this problem in typical networks where packets simultaneously arrive and have the same departure deadline. We further propose an online adaptation policy to the optimal stopping rule that meets the packets’ hard-deadline constraint and, at the same time, gives higher throughput than the offline policy.

Index Terms—Delay constraint, optimal channel selection, optimal stopping rule, stochastic optimization, water filling.

I. INTRODUCTION

C OGNITIVE radio (CR) systems are emerging wireless communication systems that allow efficient spectrum utilization [2]. This is due to the use of transceivers that are capable of detecting the presence of licensed (primary) users. The secondary users (SUs) use the frequency bands originally dedicated for the primary users (PUs) for their own transmission. Once the PU’s activity is detected on some frequency channel, the SU refrains from any further transmission on this channel. This may result in service disconnection for the SUs, thus degrading the quality of service (QoS). On the other hand, if the SUs have access to other channels, the QoS can be improved if these channels are efficiently utilized.

The problem of multiple channels in CR systems has gained attention in recent works due to the challenges associated with the sensing and access mechanisms in a multichannel CR system. Practical hardware constraints on the SUs’ transceivers may prevent them from sensing multiple channels simultaneously to detect the state of these channels (free/busy). This leads the SU to sensing the channels sequentially and then deciding which channel should be used for transmission [3], [4]. In a time-slotted system, if sequential channel sensing is employed, the SU senses the channels one at a time and stops sensing when a channel is found free. However, due to independent fading among channels, the SU is allowed to skip a free channel if its quality, which is measured by its power gain, is low and sense another channel seeking the possibility of a higher future gain. Otherwise, if the gain is high, the SU stops at this free channel to begin transmission. The question of when to stop sensing can be formulated as an optimal stopping rule problem [4]–[7]. In [5], Sabharwal et al. presented the optimal stopping rule for this problem in a non-CR system. The work in [4] develops an algorithm to find the optimal order by which channels are to be sequentially sensed in a CR scenario, whereas that in [6] studies the case where the SUs are allowed to transmit on multiple contiguous channels simultaneously. The authors presented the optimal stopping rule for this problem in a nonfading wireless channel. Transmissions on multiple channels simultaneously may be a strong assumption for low-cost transceivers, particularly when they cannot sense multiple channels simultaneously.

In general, if a perfect sensing mechanism is adopted, the SU will not cause interference to the PU since the former transmits only on spectrum holes (referred to as an overlay system). Nevertheless, if the sensing mechanism is imperfect, or if the SU’s system is an underlay system (where the SU uses the channels as long as the interference to the PU is tolerable), the transmitted power needs to be controlled to prevent harmful interference to the PU. In [8] and [9], power control is considered, and it is shown that the optimal power control strategy is a waterfilling approach under some interference constraint imposed on the SU transmitter. However, all of the aforementioned works study single-channel systems that cannot be extended to multiple channels in a straightforward manner. A multiuser CR system was considered in [10] in a time-slotted system. To allocate the frequency channel to one of the SUs, Hu et al. proposed a contention mechanism that does not depend on the SUs’ channel gains, thus neglecting the advantage of multiuser diversity. A major challenge in a multichannel system is the sequential nature of the sensing where the SU needs to take a decision to stop and begin transmission or continue sensing.
based on the information it has so far. This decision needs to trade off between waiting for a potentially higher throughput and taking advantage of the current free channel. Moreover, if transmission takes place on a given channel, the SU needs to decide on the amount of power transmitted to maximize its throughput, given some average interference and average power constraints.

In this paper, we model the overlay and underlay scenarios of a multichannel CR system, which are sensed sequentially. The problem is solved first for a single SU, and then, we discuss extensions to a multi-SU scenario. For the single-SU case, the problem is formulated as a joint optimal-stopping-rule and power-control problem with the goal of maximizing the SU’s throughput subject to average power and average interference constraints. This formulation results in increasing the expected service time of the SU’s packets. The expected service time is the average number of time slots that pass while the SU attempts to find a free channel, before successfully transmitting a packet. The increase in service time is due to skipping free channels, due to their poor gain, hoping to find a future channel of sufficiently high gain. If no channels having a satisfactory gain were found, the SU will not be able to transmit its packet and will have to wait for longer time to find a satisfactory channel. This increase in service time increases the queuing delay. Thus, we solve the problem subject to a bound on the expected service time that controls the delay (we note that, in this paper, we use the word delay to refer to the service time). In the multi-SU case, we show that the solution to the single-SU problem can be directly applied to the multi-SU system with a minor modification. We also show that the complexity of the solution decreases when the system has a large number of SUs.

To the best of our knowledge, this is the first work to study the joint power-control and optimal-stopping-rule problem in a multichannel CR system. Our contribution in this work is the formulation of a joint power-control and optimal-stopping-rule problem that also incorporates a delay constraint and presents a low-complexity solution in the presence of an interference/collision constraint from the SU to the PU due to the imperfect-sensing mechanism. The preliminary results in [1] consider an overlay framework for the single-user case while neglecting sensing errors. However, in this work, we also study the problem in the underlay scenario, where interference is allowed from the secondary transmitter (ST) to the primary receiver (PR) and extends to the multi-SU case. We also generalize the solution to the multi-SU case when the number of SUs is large. We discuss the applicability of our formulation in typical delay-constrained scenarios where packets arrive simultaneously and have the same deadline. We show that the proposed algorithm can be used to solve this problem offline to maximize the throughput and meet the deadline constraint at the same time. Moreover, we propose an online algorithm that gives higher throughput compared with the offline approach while meeting the deadline constraint.

The rest of this paper is organized as follows: The overlay system model and the underlying assumptions are presented in Section II. In Section III, the problem is mathematically formulated, the main objective is stated, and the solution to the overlay problem is proposed. Then, in Section IV, the underlay system model is discussed, and the optimal solution is presented. In Section V, the extension to multiple SUs is discussed. In Section VI, the delay constraint is generalized to the case where multiple packets arrive at the same time and have the same deadline. An online adaptation solution is also proposed, which maximizes the throughput subject to a delay constraint. Finally, numerical results are shown in Section VII, whereas Section VIII concludes this paper.

Throughout the sequel, we use bold fonts for vectors and an asterisk to denote that $x^*$ is the optimal value of $x$; all logarithms are natural, whereas the expected value operator is denoted $E[\cdot]$ and is taken with respect to all the random variables in its argument. Finally, we use $(x)^+ \triangleq \max(x,0)$ and $\mathbb{R}$ to denote the set of real numbers.

II. OVERLAY SYSTEM MODEL

Consider a PU network that has licensed access to $M$ orthogonal frequency channels. Time is slotted with a time-slot duration of $T_s$ seconds. The SU’s network consists of a single ST (SU and ST will be used interchangeably) attempting to send real-time data to its intended secondary receiver (SR) through one of the channels licensed to the PU. Before a time slot begins, the SU is assumed to have ordered the channels according to some sequence (we note that the method of ordering the channels is outside the scope of this work; see [4] for further details about channel ordering), which is labeled $1, \ldots, M$. The set of channels is denoted by $\mathcal{M} = \{1, \ldots, M\}$. Before the SU attempts to transmit its packet over channel $i$, it senses this channel to determine its availability “state,” which is described by a Bernoulli random variable $b_i$ with parameter $\theta_i$ ($\theta_i$ is called the availability probability of channel $i$). If $b_i = 0$ (which happens with probability $\theta_i$), then channel $i$ is free, and the SU may transmit over it until the ongoing time slot ends. If $b_i = 1$, channel $i$ is busy, and the SU proceeds to sense channel $i + 1$. Channel availabilities are statistically independent across frequency channels and across time slots.

We assume that the SU has limited capabilities in the sense that no two channels can be sensed simultaneously. This may be the case when considering radios having a single sensing module with a fixed bandwidth, so that it can be tuned to only one frequency channel at a time. The reader is referred to [11]–[13] for detailed information on advanced spectrum sensing techniques. Therefore, at the beginning of a given time slot, the SU selects a channel, e.g., channel 1, senses it for $\tau$ seconds ($\tau \ll T_s/M$), and detects if it is free. Otherwise, the SU skips this channel and senses channel 2, and so on, until it finds a free channel. If all channels are busy (i.e., the PU has transmission activities on all $M$ channels), then this time slot will be considered “blocked.” In this case, the SU waits for the following time slot and begins sensing following the same channel sensing sequence. As the sensing duration increases, the transmission phase duration decreases, which then decreases the throughput. However, we cannot arbitrarily decrease the value of $\tau$ since this decreases the reliability of the sensing outcome. This tradeoff has been extensively studied in the literature, e.g., [14] and [15]. In this paper, we study the
When the instantaneous channel gain is high, this allows channel selection and channel probing.

The impact of sequential channel sensing on the throughput rather than that of the sensing duration on the throughput. Hence, we assume that \( \tau \) is a fixed parameter and is not optimized over. For details on the tradeoff between throughput and sensing duration in this sequential sensing problem, the reader is referred to [16].

Fig. 1 shows a potential scenario where the SU senses \( k^* \) channels, skips the first \( k^* - 1 \), and uses the \( k^* \)th channel for transmission until the end of this ongoing time slot. In this scenario, the SU “stops” at the \( k^* \)th channel, for some \( k^* \in \mathcal{M} \). Stopping at channel \( i \) depends on two factors: 1) the availability of channel \( b_i \) and 2) the instantaneous power gain \( \gamma_i \). Clearly, \( b_i \) and \( \gamma_i \) are random variables that change from one time slot to another. Hence, \( i^* \), which depends on these two factors, is a random variable. More specifically, it depends on the states \( b_1,\ldots,b_M \) along with the gains of each channel \( \gamma_1,\ldots,\gamma_M \).

To understand why, consider that the SU senses channel \( i \), finds it free, and probes its gain \( \gamma_i \). If \( \gamma_i \) is found to be low, then the SU skips channel \( i \) (although free) and senses channel \( i + 1 \). This is to take advantage of the possibility that \( \gamma_j \gg \gamma_i \) for \( j > i \). On the other hand, if \( \gamma_i \) is sufficiently large, the SU stops at channel \( i \) and begins transmission. In that latter case, \( k^* = i \). Defining the two random vectors \( \mathbf{b} = [b_1,\ldots,b_M]^T \) and \( \mathbf{\gamma} = [\gamma_1,\ldots,\gamma_M]^T \), \( k^* \) is a deterministic function of \( \mathbf{b} \) and \( \mathbf{\gamma} \).

We define the stopping rule by defining a threshold \( \gamma_{th}(i) \) to which each \( \gamma_i \) is compared when the \( i \)th channel is found free. If \( \gamma_i \geq \gamma_{th}(i) \), the SU “stops” and transmits at channel \( i \). Otherwise, channel \( i \) is skipped, and channel \( i + 1 \) is sensed. In the extreme case when \( \gamma_{th}(i) = 0 \), the SU will not skip channel \( i \) if it is found free. Increasing \( \gamma_{th}(i) \) allows the SU to skip channel \( i \) whenever \( \gamma_i < \gamma_{th}(i) \), to search for a better channel, thus potentially increasing the throughput. Setting \( \gamma_{th}(i) \) too large allows channel \( i \) to be skipped even if \( \gamma_i \) is high. This constitutes the tradeoff in choosing the thresholds \( \gamma_{th}(i) \). The optimal values of \( \gamma_{th}(i) \in \mathcal{M} \) determine the optimal stopping rule.

Let \( P_i(\gamma) \) denote the power transmitted at the \( i \)th channel when the instantaneous channel gain is \( \gamma_i \), if channel \( i \) was chosen for transmission. Since the SU can transmit on one channel at a time, the power transmitted at any time slot at channel \( i \) is \( P_i(\gamma_i)1(i=k^*) \), where \( 1(i=k^*) = 1 \) if \( i = k^* \) and 0 otherwise. Define \( c_i = 1 - (i\tau/T_s) \) as the fraction of the time slot remaining for the SU’s transmission if the SU transmits on the \( i \)th channel in the sensing sequence. The average power constraint is \( \mathbb{E}_{\gamma} [c_i F_{\gamma^*}(\gamma_{th}(i))] \leq P_{avg} \), where the expectation is with respect to the random vectors \( \gamma \) and \( b \).

We will henceforth drop the subscript from the expected value operator \( \mathbb{E} \). This expectation can be recursively calculated from

\[
S_i(\Gamma_{th}(i), \mathbf{P}_i) = \theta_i c_i \int_{\gamma_{th}(i)}^{\infty} P_i(\gamma) f_\gamma(\gamma) d\gamma + [1 - \theta_i F_{\gamma_i}(\gamma_{th}(i))] S_{i+1}(\Gamma_{th}(i+1), \mathbf{P}_{i+1})
\]

\[
i \in \mathcal{M}, \text{ where } \mathbf{P}_1 \triangleq [P_i(\gamma), \ldots, P_M(\gamma)]^T \text{ and } \Gamma_{th}(i) \triangleq \left[ \gamma_{th}(i), \ldots, \gamma_{th}(M) \right]^T \text{ are the vectors of the power functions and thresholds, respectively, with } S_{M+1}(\Gamma_{th}(M+1), \mathbf{P}_{M+1}) \triangleq 0, F_{\gamma_i}(\gamma) \text{ is the probability density function (pdf) of the gain } \gamma_i \text{ of channel } i, \text{ and } F_{\gamma_i}(\gamma) \triangleq \int_{-\infty}^{\gamma} f_\gamma(\xi) d\xi \text{ is the complementary cumulative distribution function.}
\]

The SU’s average throughput is defined as \( \mathbb{E}[c_k \log(1 + P_k(\gamma_{th}(k^*)))] \). Similar to the average power, we denote the expected throughput as \( U_1(\Gamma_{th}(1), \mathbf{P}_1) \), which can be derived using the following recursive formula:

\[
U_i(\Gamma_{th}(i), \mathbf{P}_i) = \theta_i c_i \int_{\gamma_{th}(i)}^{\infty} \log (1 + P_i(\gamma)) f_\gamma(\gamma) d\gamma + [1 - \theta_i F_{\gamma_i}(\gamma_{th}(i))] U_{i+1}(\Gamma_{th}(i+1), \mathbf{P}_{i+1})
\]

\[
i \in \mathcal{M}, \text{ where } U_{M+1}(\Gamma_{th}(1), \mathbf{P}_{M+1}) \triangleq 0. U_1(\Gamma_{th}(1), \mathbf{P}_1) \text{ represents the expected data rate of the SU as a function of the threshold vector } \Gamma_{th}(1) \text{ and the power function vector } \mathbf{P}_1.
\]

If the SU skips all channels, either due to being busy, due to their low gain, or due to a combination of both, then the current time slot is said to be blocked. The SU has to wait for the following time slot to begin searching for a free channel again. This results in a delay in serving (transmitting) the SU’s packet. Define delay \( D \) as the number of time slots the SU consumes before successfully transmitting a packet. That is, \( D = 1 \) is a random variable that represents the number of consecutively blocked time slots. In real-time applications, there may exist some average delay requirement \( D_{max} \) on the packets that must not be exceeded. Since the availability of each channel is independent across time slots, \( D \) follows a geometric distribution having \( \mathbb{E}[D] = (P_{success})^{-1} \), where \( P_{success} = 1 - P_{Blocking} \). In other words, \( P_{success} \) is the probability that the SU finds a free channel with gain...
that is high enough so that it does not skip all $M$ channels in a time slot. It is given by $P_{T}^T[\text{Success}] \triangleq p_t(\Gamma_{th}(1))$, which can be recursively calculated using the following equation:

$$p_i(\Gamma_{th}(i)) = \theta_i F_i(\gamma_{th}(i)) + [1 - \theta_i F_i(\gamma_{th}(i))] p_{t+1}(\Gamma_{th}(i+1)) \quad (3)$$

$i \in M$, where $p_{M+1} = 0$. Here, $p_t(\Gamma_{th}(i))$ is the probability of transmission on channel $i$, $i+1, \ldots$, or $M$.

### III. Problem Statement and Proposed Solution

From (2), we see that the SU’s expected throughput $U_1$ depends on the threshold vector $\Gamma_{th}(1)$ and the power vector $P_1$. The goal is to find the optimum values of $\Gamma_{th}(1) \in \mathbb{R}^M$ and functions $P_1$ that maximize $U_1$ subject to an average power constraint and an expected packet delay constraint. The delay constraint can be written as $E[D] \leq \bar{D}_{\max}$ or, equivalently, $p_t(\Gamma_{th}(1)) \geq 1/\bar{D}_{\max}$. Mathematically, the problem becomes

$$\text{maximize} \quad U_1(\Gamma_{th}(1), P_1)$$

$$\text{subject to} \quad S_1(\Gamma_{th}(1), P_1) \leq P_{\text{avg}}$$

$$p_t(\Gamma_{th}(1)) \geq \frac{1}{D_{\max}}$$

$$\text{variables} \quad \Gamma_{th}(1), P_1 \quad (4)$$

where the first constraint represents the average power constraint, whereas the second constraint is a bound on the average packet delay. We allow the power $P_t$ to be an arbitrary function of $\gamma_t$ and optimize over this function to maximize the throughput subject to average power and delay constraints. Although (4) is not proven to be convex, we provide closed-form expressions for the optimal threshold and power function vectors. To this end, we first calculate the Lagrangian associated with (4). Let $\lambda_P$ and $\lambda_D$ be the dual variables associated with the constraints in problem (4). The Lagrangian for (4) becomes

$$L(\Gamma_{th}(1), P_1, \lambda_P, \lambda_D)$$

$$= U_1(\Gamma_{th}(1), P_1) - \lambda_P (S_1(\Gamma_{th}(1), P_1) - P_{\text{avg}})$$

$$+ \lambda_D (p_t(\Gamma_{th}(1)) - \frac{1}{\bar{D}_{\max}})$$

$$= \frac{1}{\bar{D}_{\max}}$$

Differentiating (5) with respect to each of the primal variables $P_i(\gamma)$ and $\gamma_{th}(i)$ and equating the resulting derivatives to zero, we obtain the Karush–Kuhn–Tucker (KKT) equations, below which are necessary conditions for optimality [18], [19], i.e.,

$$P_i^*(\gamma) = \left( \frac{1}{\lambda_P} - \frac{1}{\gamma} \right)^+ \quad , \quad \gamma \geq \gamma_{th}(i) \quad (6)$$

$$\log \left(1 + \left( \frac{1}{\lambda_P} - \frac{1}{\gamma_{th}(i)} \right)^+ \right) = -\frac{1}{\lambda_P} \gamma_{th}(i)$$

$$= U_{i+1}^* - \lambda_P S_{i+1}^* - \lambda_D (1 - p_{i+1}^*) \quad (7)$$

$$S_i^* \leq P_{\text{avg}}, \quad p_i^* \geq \frac{1}{D_{\max}}, \quad \lambda_P \geq 0, \quad \lambda_D \geq 0$$

$$\lambda_P \cdot (S_i^* - P_{\text{avg}}) = 0 \quad (8)$$

$$\lambda_D \cdot (p_i^* - \frac{1}{D_{\max}}) = 0 \quad (9)$$

$$i \in M$$. We use $U_{i+1}^* = U_{i+1}(\Gamma_{th}(i+1), P_{i+1}^*)$, while $S_{i+1}^* = S_{i+1}(\Gamma_{th}(i+1), P_{i+1}^*)$ and $p_{i+1}^* = p_{i+1}(\Gamma_{th}(i+1))$ for brevity in the sequel. We note that $U_{M+1}(\cdot, \cdot) = S_{M+1}(\cdot, \cdot) = P_{M+1}(\cdot, \cdot) \triangleq 0$ by definition. We observe that these KKT equations involve the primal ($\Gamma_{th}(1)$ and $P_1^*$) and the dual ($\lambda_P^*$ and $\lambda_D^*$) variables. Our approach is to find a closed-form expression for the primal variables in terms of the dual variables and then propose a low-complexity algorithm to obtain the solution for the dual variables. The optimality of this approach is discussed in Section III-C, where we show that, loosely speaking, the KKT equations provide a unique solution to the primal–dual variables. Hence, based on this unique solution and on the fact that the KKT equations are necessary conditions for the optimal solution, this solution is not only necessary but also sufficient and, hence, optimal.

### A. Solving for Primal Variables

Equation (6) is a waterfilling strategy with a slight modification due to the condition $\gamma > \gamma_{th}(i)$. This condition comes from the sequential sensing of the channels, which is absent in the classic waterfilling strategy [17]. Equation (6) gives a closed-form solution for $P_1$. On the other hand, the entries of the vector $\Gamma_{th}(1)$ are found via the set of equations (7). Note that (7) indeed forms a set of $M$ equations, each solves for one of $\gamma_{th}(i), i \in M$. We refer to this set as the “threshold-finding” equations. For a given value of $i$, solving for $\gamma_{th}(i)$ requires knowledge of only $\gamma_{th}(i+1)$ through $\gamma_{th}(M)$ and does not require knowing $\gamma_{th}(1)$ through $\gamma_{th}(i-1)$. Thus, these $M$ equations can be solved using back-substitution starting from $\gamma_{th}(M)$. To solve for $\gamma_{th}(i)$, we use the fact that $\gamma_{th}(i) \geq \lambda_P^*$, which is proven in the following lemma.

**Lemma 1:** The optimal solution of problem (4) satisfies $\gamma_{th}(i) \geq \lambda_P^* \forall i \in M$.

**Proof:** See Appendix A for the proof. 

The intuition behind Lemma 1 is as follows. If, for some channel $i$, $\gamma_{th}(i) < \lambda_P^*$ was possible, and the instantaneous gain $\gamma_i$ happened to fall in the range $[\gamma_{th}(i), \lambda_P^*]$ at a given time slot, then the SU will not skip channel $i$ since $\gamma_i > \gamma_{th}(i)$. However, the power transmitted on channel $i$ is $P_i(\gamma_{th}(i)) = (1/\lambda_P^* - 1/\gamma_{th}(i))^+ = 0$ since $\gamma_i < \lambda_P^*$. This means that the SU will neither skip nor transmit on channel $i$, which does not make sense from the SU’s throughput perspective. To overcome this event, the SU needs to set $\gamma_{th}(i)$ at least as large as $\lambda_P^*$, so that whenever $\gamma_i < \lambda_P^*$, the SU skips channel $i$ rather than transmitting with zero power.

Lemma 1 allows us to remove the $(\cdot)^+$ sign in (7) when solving for $\gamma_{th}(i)$. Rewriting (7), we get

$$-\lambda_P^* \log \left(1 - \frac{U_{i+1}^* - \lambda_P^* S_{i+1}^* - \lambda_D^* \cdot (1 - p_{i+1}^*)}{c_i} \right) = 0$$

$$\log \left(1 - \frac{U_{i+1}^* - \lambda_P^* S_{i+1}^* - \lambda_D^* \cdot (1 - p_{i+1}^*)}{c_i} \right) = 0$$

Equation (11) is now of the form $W \log(W) = c$, whose solution is $W = W_0(c)$, where $W_0(x)$ is the principal branch
of the Lambert W function [20] and is given by

$$W_0(x) = \sum_{n=1}^{\infty} \left(-\frac{n}{n!}\right)^n x^n.$$  

The only solution to (11), which satisfies Lemma 1, is given for \(i \in M\) by

$$\gamma_{th}(i) = \frac{-\lambda_{P}^{\gamma}}{W_0\left(-\exp\left(-\frac{U_{c+1}^{-1} \gamma_{D} \bar{S}_{c+1}^{-1} \lambda_{D} (1-p_{c+1})^{n+1} - 1)}{c_i} \right)\right)}.$$  

Hence, \(\Gamma_{th}^{\gamma}(1)\) and \(P_{i}^{\gamma}\) are found via (12) and (6), respectively, which are one-to-one mappings from the dual variables \((\lambda_{P}^{\gamma}, \lambda_{D}^{\gamma})\). Moreover, if we had an instantaneous power constraint \(P_{i}(\gamma) \leq P_{\text{max}}\), we could write down the Lagrangian and solve for \(P_{i}(\gamma)\). The details are similar to the case without an instantaneous power constraint and are, thus, omitted for brevity. The reader is referred to [9] for a similar proof. The expression for \(P_{i}^{\gamma}(\gamma)\) is given by

$$P_{i}^{\gamma}(\gamma) = \begin{cases} \left(\frac{1}{\lambda_{P}^{\gamma}} - \frac{1}{\gamma}\right)^{+} & \text{if } \frac{1}{\lambda_{P}^{\gamma}} - \frac{1}{\gamma} < P_{\text{max}} \\ \text{otherwise.} & \end{cases}$$  

Since the optimal primal variables are explicit functions of the optimal dual variables, once the optimal dual variables are found, the optimal primal variables are found, and the optimization problem is solved. We now discuss how to solve for these dual variables.

### B. Solving for Dual Variables

The optimum dual variable \(\lambda_{P}^{\gamma}\) must satisfy (9). Thus, if \(\lambda_{P}^{\gamma} > 0\), then we need \(S_{1}^{\gamma} - P_{\text{avg}} = 0\). This equation can be solved using any suitable root-finding algorithm. Hence, we propose Algorithm 1 that uses bisection [21]. In each iteration \(n\), the algorithm calculates \(S_{1}^{\gamma}\), given that \(\lambda_{P}^{\gamma} = \lambda_{P}^{(n)}\), and, given some fixed \(\lambda_{D}^{\gamma}\), compares it to \(P_{\text{avg}}\) to update \(\lambda_{P}^{(n+1)}\) accordingly. The algorithm terminates when \(S_{1}^{\gamma} = P_{\text{avg}}\), i.e., \(\lambda_{P}^{(n)} = \lambda_{P}^{\gamma}\). The superiority of this algorithm over the exhaustive search is due to the use of the bisection algorithm that does not go over all the search space of \(\lambda_{P}^{\gamma}\). For the bisection to converge, there must exist a single solution for equation \(S_{1}^{\gamma} = P_{\text{avg}}\). This is proven in Theorem 1.

**Theorem 1:** \(S_{1}^{\gamma}\) is decreasing in \(\lambda_{P}^{\gamma}\) in \([0, \infty)\) given some fixed \(\lambda_{D}^{\gamma} \geq 0\). Moreover, the optimal value \(\lambda_{P}^{\gamma}\) satisfying \(S_{1}^{\gamma} = P_{\text{avg}}\) is upper bounded by \(\lambda_{P}^{\max} \triangleq \sum_{i=1}^{M} \theta_{i} c_i / P_{\text{avg}}\).

**Proof:** See Appendix B for the proof.

We note that Algorithm 1 can be systematically modified to call any other root-finding algorithm (e.g., the secant algorithm [21] that converges faster than the bisection algorithm).

**Algorithm 1 Finding \(\lambda_{P}^{\gamma}\) given some \(\lambda_{D}^{\gamma}\)**

1. Initialize \(n \leftarrow 1\), \(\lambda_{P}^{\min} \leftarrow 0\), \(\lambda_{P}^{\max} \leftarrow \sum_{i=1}^{M} \theta_{i} c_i / P_{\text{avg}}\), \(\lambda_{P}^{(1)} \leftarrow (\lambda_{P}^{\min} + \lambda_{P}^{\max}) / 2\).
2. while \(|S_{1}^{\gamma} - P_{\text{avg}}| > \epsilon\) do
3. Calculate \(S_{1}^{\gamma}\) given that \(\lambda_{P}^{\gamma} = \lambda_{P}^{(n)}\). Call it \(S^{(n)}\).
4. if \(S^{(n)} - P_{\text{avg}} > 0\) then
5. \(\lambda_{P}^{\min} \leftarrow \lambda_{P}^{(n)}\)
6. else
7. \(\lambda_{P}^{\max} = \lambda_{P}^{(n)}\)
8. end if
9. \(\lambda_{P}^{(n+1)} \leftarrow (\lambda_{P}^{\min} + \lambda_{P}^{\max}) / 2\)
10. \(n \leftarrow n + 1\)
11. end while
12. \(\lambda_{P}^{\gamma} \leftarrow \lambda_{P}^{(n)}\)

Now, to search for \(\lambda_{D}^{\gamma}\), we state the following lemma.

**Lemma 2:** The optimum value \(\lambda_{D}^{\gamma}\) that solves problem (4) satisfies \(0 \leq \lambda_{D}^{\gamma} < \lambda_{D}^{\max}\), where

$$\lambda_{D}^{\max} = c_1 [\log(t) - t + 1] + U_{2}^{\max} / 1 - p_{2}^{\max}$$  

with \(t \triangleq \min(\lambda_{P}^{\max}, \bar{F}_{\gamma}^{-1}(1/\theta_{1} \bar{D}_{\max})) / (\bar{F}_{\gamma}^{-1}(1/\theta_{1} \bar{D}_{\max}))\), and \(U_{2}^{\max}\) is an upper bound on \(U_{2}\) and is given by \((\infty) \log(\gamma / \lambda_{P}^{\max}) / (\sum_{i=1}^{M} \theta_{i} c_i)\), whereas \(p_{2}^{\max}\) is an upper bound on \(p_{2}\) and is given by \(\sum_{i=2}^{M} \prod_{j=2}^{M} (1 - \theta_{j}) \theta_{i}\).

**Proof:** See Appendix C.

Lemma 2 gives an upper bound on \(\lambda_{D}^{\gamma}\). This bound decreases the search space of \(\lambda_{D}^{\gamma}\) drastically instead of searching over \(\mathbb{R}\). Thus, the solution of problem (4) can be summarized in three steps: 1) Fix \(\lambda_{D}^{\gamma} \in [0, \lambda_{D}^{\max}]\) and find the corresponding optimum \(\lambda_{P}^{\gamma}\) using Algorithm 1. 2) Substitute the pair \((\lambda_{P}^{\gamma}, \lambda_{D}^{\gamma})\) in (6) and (12) to get the power and threshold functions and then evaluate \(U_{1}^{\gamma}\) from (2). 3) Repeat steps 1 and 2 for other values of \(\lambda_{D}^{\gamma}\) until reaching the optimum \(\lambda_{D}^{\gamma}\) that satisfies \(p_{1}^{\gamma} = 1 / \bar{D}_{\max}\). If there are multiple \(\lambda_{D}^{\gamma}\)’s satisfying \(p_{1}^{\gamma} = 1 / \bar{D}_{\max}\), then the optimum value is that which gives the highest \(U_{1}^{\gamma}\).

Although the order by which the channels are sensed is assumed fixed, the proposed algorithm can be modified to optimize over the sensing order by a relatively low-complexity sorting algorithm. In particular, the dynamic programming proposed in [4] can be called by Algorithm 1 to order the channels. The complexity of the sorting algorithm alone is \(O(2^{M})\) compared with \(O(M!)\) of the exhaustive search to sort the \(M\) channels. The modification to our proposed algorithm would be in step 3 of Algorithm 1, where \(S_{1}^{\gamma}\) would be optimized over the number of channels (as well as \(\Gamma_{th}^{\gamma}(1)\)).

**C. Optimality of the Proposed Solution**

Since the problem in (4) is not proven to be convex, the KKT conditions provide only necessary conditions for optimality and need not be sufficient [22]. This means that there might exist multiple solutions (i.e., multiple solutions for the primal and/or dual variables) satisfying the KKT conditions, at least one of which is optimal. However, since Theorem 1 proves that there exists one unique solution to \(\lambda_{P}^{\gamma}\) given \(\lambda_{D}^{\gamma}\), then \(\Gamma_{th}^{\gamma}(1)\) and \(P_{1}^{\gamma}\) are unique as well [from (6) and (12)] given some \(\lambda_{D}^{\gamma}\). Hence, by sweeping \(\lambda_{D}^{\gamma}\) over \([0, \lambda_{D}^{\max}]\), we have a unique solution satisfying the KKT conditions, which means that the KKT conditions are sufficient as well and that our approach is optimal for problem (4).
IV. UNDERLAY SYSTEM

In the overlay system, the SU tries to locate the free channels at each time slot to access these spectrum holes without interfering with the PUs. Recently, the Federal Communications Commission has allowed the SUs to interfere with the PU’s network, as long as this interference does not harm the PUs [23]. If the interference from the SU measured at the PU’s receiver is below the tolerable level, then the interference is deemed acceptable.

To model the interference at the PR, we assume that the SU uses a channel sensing technique that produces the sufficient statistic $\gamma_i$ at channel $i$ [24], [25]. The SU is assumed to know the distribution of $z_i$ given that channel $i$ is free and busy, namely, $f_{z|b}(z_i|b_i=0)$ and $f_{z|b}(z_i|b_i=1)$, respectively. For brevity, we omit subscript $i$ from $b_i$ whenever it is clear from the context. The value of $z_i$ indicates how confident the SU is in the presence of the PU at channel $i$. Thus, the SU stops at channel $i$ according to how likely it is and how much data rate it will gain from this channel (i.e., according to $z_i$ and $\gamma_i$, respectively). Hence, when the SU senses channel $i$ to acquire $z_i$, the channel gain $\gamma_i$ is probed and compared to some function $\gamma_{th}(i,z_i)$, if $\gamma_i \geq \gamma_{th}(i,z_i)$ transmission occurs on channel $i$; otherwise, channel $i$ is skipped, and $i+1$ is sensed. Potentially, $\gamma_{th}(i,z_i)$ is a function in the statistic $z_i$. This means that, at channel $i$, for each possible value that $z_i$ might take, there is a corresponding threshold $\gamma_{th}(i,z_i)$. Before formulating the problem, we note that this model can capture the overlay with the distribution of $z_i$ to the PU’s receiver, aggregated over all $i$. Moreover, let the average interference from the SU’s transmitter to the PU’s receiver, be

$$I_i(\Gamma_{th}(i,z),P_1)$$

where $I_{M+1}(\Gamma_{th}(M+1,z),P_{M+1}) \triangleq 0$. This interference model is based on the assumption that the channel gain from the SU’s transmitter to the PU’s receiver is known at the SU’s transmitter. This is the case for reciprocal channels when the PR acts as a transmitter and transmits training data to its intended primary transmitter (when it is acting as a receiver) [26]. The ST overhears these training data and estimates the channel from itself to the PR. Moreover, the gain at each channel from the ST to the PR is assumed unity for presentation simplicity. This could be easily extended to the case of nonunity gain by multiplying the first term in (16) by the gain from the ST to the PR at channel $i$. Finally, $p_1(\Gamma_{th}(1,z))$ is the probability of a successful transmission in the current time slot and can be calculated using

$$p_1(\Gamma_{th}(1,z)) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_z(\gamma) d\gamma f_x(z) dz$$

where $p_i^{\text{skip}}(\Gamma_{th}(i+1,z),P_{i+1}) \triangleq 0$. Given this background, the problem is

$$\max_{\Gamma_{th}(1,z),\bar{P}_1} \quad U_1(\Gamma_{th}(1,z),\bar{P}_1)$$

subject to

$$I_i(\Gamma_{th}(i+1,z),P_{i+1}) \leq I_{th}(i+1)$$

variables $\Gamma_{th}(1,z),\bar{P}_1$. (18)

Let $\lambda_I$ and $\lambda_D$ be the Lagrange multipliers associated with the interference and delay constraints of problem (18), respectively. Problem (18) is more challenging compared with the overlay case. This is because, unlike those in (4), the thresholds in (18) are functions rather than constants. The KKT conditions for (18) are given by

$$P_i^+(\gamma) = \frac{1}{\lambda_I^+ \Pr[b_i = 1|z]} - 1, \quad i \in \mathcal{M}.$$  \hspace{1cm} (19)

$$\gamma_{th}(i,z) = \frac{-\lambda_D^+ \Pr[b_i = 1|z]}{W_0(\exp(\frac{U_{th}(1,\lambda_I^+)+\lambda_D^+(1-\Pr[b_i = 1|z])}{\lambda_I^+}-1)), \quad i \in \mathcal{M}}$$  \hspace{1cm} (20)
where \( U_{i+1}^* \triangleq U_i(\Gamma_{th}(1, z), P_i^*(\gamma)) \), \( I_{i+1}^* \triangleq I_i(\Gamma_{th}(1, z), P_i^*(\gamma)) \), and \( p_{i+1}^* \triangleq p_i(\Gamma_{th}(1, z)) \), whereas \( \Pr \{ b_i = 1 \mid z \} \) is the conditional probability that channel \( i \) is busy given \( z \) and is given by

\[
\Pr \{ b_i = 1 \mid z \} = \frac{(1 - \theta_i) f_{z/b}(z) b_i = 1}{f_{z}(z)}.
\]  \( \text{(21)} \)

Note that \( P_i^*(\gamma) \) is increasing in \( \gamma \) and is upper bounded by the term \( 1/(\lambda_i^* \Pr \{ b_i = 1 \mid z \}) \). Hence, as \( \Pr \{ b_i = 1 \mid z \} \) increases, the SU’s maximum power becomes more limited, i.e., the maximum power decreases. This is because the PU is more likely to be occupying channel \( i \). Thus, the power transmitted from the SU should decrease to protect the PU.

Algorithm 1 can also be used to find \( \lambda_i^* \). Only a single modification is required in the algorithm, which is that \( S_1 \) would be replaced by \( I_1^* \). Thus, the solution of problem \( \text{(18)} \) can be summarized in three steps:

1) Fix \( \lambda_D^* \in \mathbb{R}^+ \) and find the corresponding optimum \( \lambda_i^* \) using the modified version of Algorithm 1.
2) Substitute the pair \( \lambda_i^* \), \( \lambda_D^* \) in \( \text{(19)} \) and \( \text{(20)} \) to get the power and threshold functions and then evaluate \( \lambda_i^* \) from \( \text{(15)} \).
3) Repeat steps 1 and 2 for other values of \( \lambda_D^* \) until reaching the optimum \( \lambda_i^* \) that satisfies \( p_i^* = 1/D_{\text{max}} \), and if there are multiple \( \lambda_i^* \)’s satisfying \( p_i^* = 1/D_{\text{max}} \), then the optimum value is that which gives the highest \( U_i^* \). This approach yields the optimal solution. Next, Theorem 2 asserts the monotonicity of \( I_i^* \) in \( \lambda_i^* \), which allows using the bisection to find \( \lambda_i^* \) given some fixed \( \lambda_D^* \).

\textbf{Theorem 2:} \( I_i^* \) is decreasing in \( \lambda_i^* \in [0, \infty) \) given some fixed \( \lambda_D^* \geq 0 \).

\textbf{Proof:} We differentiate \( I_i^* \) with respect to \( \lambda_i^* \) given that \( P_i^*(\gamma) \) and \( \gamma_{th}(i, z) \) are given by \( \text{(19)} \) and \( \text{(20)} \), respectively, and then show that this derivative is negative. The proof is omitted since it follows the same lines in Theorem 1.

Although the interference power constraint is sufficient for the problem to prevent the power functions from going to infinity, in some applications, one may have an additional power constraint on the SUs. Hence, problem \( \text{(18)} \) can be modified to introduce an average power constraint that is given by \( S_1(\Gamma_{th}(1, z), P_1) \leq P_{\text{avg}} \), where

\[
S_1(\Gamma_{th}(i, z), P_i) = c_i \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P_i(\gamma) f_{\gamma}(\gamma) d\gamma f_{z}(z) dz + \sum_{l=1}^{\text{skip}} S_{i+1}(\Gamma_{th}(i + 1, z), P_{i+1}).
\]  \( \text{(22)} \)

It can be easily shown that the solution to the modified problem is similar to that presented in \( \text{(19)} \) and \( \text{(20)} \), which is

\[
P_i^*(\gamma) = \frac{1}{(\lambda_p^* + \lambda_D^* \Pr \{ b_i = 1 \mid z \}) - \frac{1}{\gamma} + (\lambda_p^* + \lambda_D^* \Pr \{ b_i = 1 \mid z \})}
\]  \( \text{(23)} \)

\[
\gamma_{th}(i, z) = \frac{W_0}{\exp \left( \left( \frac{U_{i+1}^* - \gamma_{th}(i, z) - \lambda_D^* S_{i+1}^* - \lambda_P^* (1 - p_{i+1}^*)}{c_i} \right) \right) + 1)}.
\]  \( \text{(24)} \)

\( \forall i \in \mathcal{M} \), where \( S_i^* \triangleq S_i(\Gamma_{th}(i, z), P_i^*(\gamma)) \). This solution is more general since it takes into account both the average interference and the average power constraint apart from the delay constraint. Moreover, it allows for the case where the power constraint is inactive, which happens if the PU has a strict average interference constraint. In this case, the optimum solution would result in \( \lambda_p^* = 0 \), making \( \text{(23)} \) and \( \text{(24)} \) identical to \( \text{(19)} \) and \( \text{(20)} \), respectively.

\section{Multiple Secondary Users}

Here, we show how our single-SU framework can be extended to multiple SUs in a multiuser diversity framework without increase in the complexity of the algorithm. We will show that when the number of SUs is high, with slight modifications to the definitions of the throughput, power, and probability of success, the single-SU optimization problem in \( \text{(4)} \) [or \( \text{(18)} \)] can capture the multi-SU scenario. Moreover, the proposed solution for the overlay model still works for the multi-SU scenario. Finally, at the end of this section, we will show that the proposed algorithm provides a throughput-optimal and delay-optimal solution with even lower complexity for finding the thresholds compared with the single-SU case if the number of SUs is large.

Consider a CR network with \( L \) SUs associated with a centralized secondary base station (BS) in a downlink overlay scenario. Before describing the system model, we would like to note that when we say that channel \( i \) will be sensed, this means that each user will independently sense channel \( i \) and feed the sensing outcome back to the BS to make a global decision. Although we neglect sensing errors in this section, the analysis will work similarly in the presence of sensing errors by using the underlay model. At the beginning of each time slot, the \( L \) SUs sense channel 1. If it is free, each SU observes it free with no errors and probes the instantaneous channel gain and feeds it back to the BS. The BS compares the maximum received channel gain among the \( L \) received channel gains to \( \gamma_{th}(1) \). Channel 1 is assigned to the user having the maximum channel gain if this maximum gain is higher than \( \gamma_{th}(1) \), whereas the remaining \( L - 1 \) users continue to sense channel 2. On the other hand, if the maximum channel gain is less than \( \gamma_{th}(1) \), channel 1 is skipped, and channel 2 is sensed by all \( L \) users.

Unlike the case in the single-SU scenario where only a single channel is claimed per time slot, in this multi-SU system, the BS can allocate more than one channel in one time slot such that each SU is not allocated more than one channel and each channel is not allocated to more than one SU. Based on this scheme, the expected per-SU throughput \( U_i^L \) is calculated from

\[
U_i^L = \frac{\theta_i c_i}{\gamma_{th}(i)} \int_{\gamma_{th}(i)}^{\infty} \log \left( 1 + P_i(\gamma) \right) f_i(\gamma) d\gamma
\]

\[+ \theta_i \bar{F}_i(\gamma_{th}(i)) \left( 1 - \frac{1}{L} \right) U_{i+1}^{L-1} + \left( 1 - \theta_i \bar{F}_i(\gamma_{th}(i)) \right) U_{i+1}^L \]

\( i \in \mathcal{M} \) and \( l \in \{L - i + 1, \ldots, L\} \) with initialization \( U_{M+1}^L = 0 \). Here, \( f_i(\gamma) \) represents the density of the maximum gain.
The solution of the unconstrained problem is given by setting \( \lambda \) as the complementary cumulative distribution function. We study the case where \( L \gg M \); thus, when a channel is allocated to a user, we can assume that the remaining number of users is still \( L \). Thus, we approximate \( L \) with \( L \forall \in \{ L - i, \ldots, L \} \) and \( \forall i \in M \). Similar to the throughput derived in (25), we could write the exact expressions for the per-SU average power and per-SU probability of transmission. Furthermore, since \( L \gg M \), we can approximate \( S_i^L \) with \( S_i^L \) and \( p_i^L \) with \( p_i^L \), \( \forall i \in \{ L - i + 1, \ldots, L \} \) and \( \forall i \in M \). The per-SU expected throughput \( U_i^L \), the average power \( S_i^L \), and the probability of transmission \( P_i^L \) can be derived from

\[
U_i^L(\Theta_{th}(i), P_i) = \frac{\frac{\lambda_i}{L}}{\gamma_{th}(i)} \int_0^\infty \log(1 + P_i(\gamma) f_L(\gamma)) d\gamma
+ \left[ 1 - \frac{\frac{\lambda_i}{L} F_L(\gamma_{th}(i))}{L} \right] U_{i+1}^L(\Theta_{th}(i+1), P_{i+1})
\]

\[
S_i^L(\Theta_{th}(i), P_i) = \frac{\frac{\lambda_i}{L}}{\gamma_{th}(i)} \int_0^\infty P_i(\gamma) f_L(\gamma) d\gamma
+ \left[ 1 - \frac{\frac{\lambda_i}{L} F_L(\gamma_{th}(i))}{L} \right] S_{i+1}^L(\Theta_{th}(i+1), P_{i+1})
\]

\[
p_i^L(\Theta_{th}(i)) = \frac{\frac{\lambda_i}{L} F_L(\gamma_{th}(i))}{\gamma_{th}(i)} \left[ 1 - \frac{\frac{\lambda_i}{L} F_L(\gamma_{th}(i))}{L} \right] p_{i+1}^L(\Theta_{th}(i+1))
\]

\( i \in M \), respectively, with \( U_{M+1}^L = S_{M+1}^L = p_{M+1}^L = 0 \). To formulate the multi-SU optimization problem, we replace \( U_1 \), \( S_1 \), and \( p_1 \) in (4) with \( U_i^L \), \( S_i^L \), and \( p_i^L \) derived in (26)–(28), respectively. Taking the Lagrangian and following the same procedure in Section III, we arrive at the solution for \( P_i^* \) and \( \gamma_{th}^*(i) \), as given by (6) and (12), respectively. Hence, (6) and (12) represent the optimal solution for the multi-SU scenario. The details are omitted since they follow those of the single-SU case discussed in Section III.

Next, we show that this solution is optimal with respect to the delay as well as the throughput when \( L \) is large. We show this by studying the system after ignoring the delay constraint and show that the resulting solution of this system (which is what we refer to as the unconstrained problem) is also delay optimal. The solution of the unconstrained problem is given by setting \( \lambda_D = 0 \) in (12), arriving at

\[
\gamma_{th}^*(i) | \lambda_D = 0 = \frac{-\lambda_P}{W_0 - \exp \left( \frac{(U_i^L - \lambda_P S_i^L)^2}{e_i} - 1 \right)}
\]

\( \forall i \in M \). As the number of SUs increases, the per-user expected throughput \( U_i^L \) decreases, since these users share the total throughput. Moreover, \( U_i^L \) decreases as well \( \forall i \in M \) decreasing the value of \( \gamma_{th}^*(i) \) [from (29)] until reaching its minimum (i.e., \( \gamma_{th}^*(i) = \lambda_P \); the right-hand side of (29) is minimum when its denominator is as much negative as possible, that is, when \( W_0(x) = -1 \) since \( W_0(x) \geq -1, \forall x \in \mathbb{R} \) as \( L \rightarrow \infty \). From (28), it can be easily shown that \( p_i^L(\Theta_{th}(1)) \) is monotonically decreasing in \( \gamma_{th}(i) \) \( \forall i \in M \). Thus, the minimum possible average delay (corresponding to the maximum \( p_i^L(\Theta_{th}(1)) \)) occurs when \( \gamma_{th}(i) \) is at its minimum possible value for all \( i \in M \). Consequently, having \( \gamma_{th}^*(i) = \lambda_P^* \) means that the system is at the optimum delay point. That is, the unconstrained problem cannot achieve any smaller delay with an additional delay constraint. Hence, the multi-SU problem, which is formulated by adding a delay constraint to the unconstrained problem, achieves the optimum delay performance when \( L \) is asymptotically large.

Recall that the overall complexity of the solution for the single-SU case is due to three factors: 1) evaluating the Lambert W function in Algorithm 1; 2) the bisection algorithm in Algorithm 1; and 3) the search over \( \lambda_D \). On the other hand, the complexity of the solution for the multi-SU case asymptotically decreases (as \( L \rightarrow \infty \)). This is because of two reasons: 1) When \( L \gg M \), \( \gamma_{th}^*(i) = \lambda_P^* \forall i \in M \). This means that we will not have to evaluate the Lambert W function in (12), but instead, we set \( \gamma_{th}^*(i) = \lambda_P^* \) since \( L \gg M \). 2) When \( \gamma_{th}^*(i) = \lambda_P^* \), there will be no need to find \( \lambda_D^* \) since the delay is minimum (we recall that in the single-SU case, we need to calculate \( \lambda_D^* \) to substitute it in (12) to evaluate \( \gamma_{th}^*(i) \), but in the multi-SU case, \( \gamma_{th}^*(i) = \lambda_P^* \).

VI. GENERALIZATION OF DEADLINE CONSTRAINTS

In the overlay and underlay schemes discussed thus far, we were assuming that each packet has a hard deadline of one time slot. If a packet is not delivered as soon as it arrives at the ST, then it is dropped from the system. However, in real-time applications, data arrive at the ST’s buffer on a frame-by-frame structure. This means that multiple packets (constituting the same frame) arrive simultaneously rather than one at a time. A frame consists of a fixed number of packets, and each packet fits into exactly one time slot of duration \( T_s \). Each frame has its own deadline, and thus, packets belonging to the same frame have the same deadline [27]. This deadline represents the maximum number of time slots by which the packets belonging to the same frame need to be transmitted, on average.

Here, we solve this problem for the overlay scenario. The solution presented in Section III can be thought of as a special case of the problem presented in this section, where the deadline was equal to one time slot, and each frame consists of one packet. We show that the solution presented in Section III can be used to solve this generalized problem in an offline fashion (i.e., before attempting to transmit any packet of the frame). Moreover, we propose an online update algorithm that updates the thresholds and power functions in each time slot and show that this outperforms the offline solution.
A. Offline Solution

Assume that each frame consists of $K$ packets and that the entire frame has a deadline of $t_f$ time slots ($t_f > K$). If the SU does not succeed in transmitting the $K$ packets before the $t_f$ time slots, then the whole frame is considered wasted. Since instantaneous channel gains and PU’s activities are independent across time slots, the probability that the SU succeeds in transmitting the frame in $t_f$ time slots or less is given by

$$P_{\text{frame}}(K, t_f) = \sum_{n=K}^{t_f} \binom{t_f}{n} p^n (1-p)^{t_f-n}$$

where $p$ is the probability of transmitting a packet on some channel in a single time slot and is given by (3) or (17) if the SU’s system was overlay or underlay, respectively. $P_{\text{frame}}(K, t_f)$ represents the probability of finding $K$ or more free time slots out of a total of $t_f$ time slots.

To guarantee some QoS for the real-time data, the SU needs to keep the probability of successful frame transmission above a minimum value denoted as $r_{\text{min}}$, that is $P_{\text{frame}} \geq r_{\text{min}}$. Hence, the problem becomes a throughput-maximization problem subject to some average power and QoS constraints, as follows:

$$\begin{align*}
\text{maximize} & \quad U_1(\Gamma_{\text{th}}(1), P_1) \\
\text{subject to} & \quad S_1(\Gamma_{\text{th}}(1), P_1) \leq P_{\text{avg}} \\
& \quad P_{\text{frame}}(K, t_f) \geq r_{\text{min}} \\
\text{variables} & \quad \Gamma_{\text{th}}(1), P_1.
\end{align*}$$

This is the optimization problem assuming an overlay system since we used (2) and (1) for throughput and power, respectively. It can also be systematically modified to the case of an underlay system. Since there exists a one-to-one mapping between $P_{\text{frame}}(K, t_f)$ and $p$, then there exists a value for $D_{\text{max}}$ such that the inequality $p \geq 1/D_{\text{max}}$ is equivalent to the QoS inequality $P_{\text{frame}}(K, t_f) \geq r_{\text{min}}$. That is, we can replace inequality $P_{\text{frame}}(K, t_f) \geq r_{\text{min}}$ by $p \geq 1/D_{\text{max}}$ for some $D_{\text{max}}$ that depends on $r_{\text{min}}, K,$ and $t_f$ that are known a priori. Consequently, problem (31) is reduced to the simpler, yet equivalent, single-time-slot problem (4), and the SU can solve for $P_1$ and $\Gamma_{\text{th}}(1)$ vectors following the approach proposed in Section III. The SU solves this problem offline (i.e., before the beginning of the frame transmission) and uses this solution each time slot of the $t_f$ time slots. With this offline scheme, the SU will be able to meet the QoS and the average power constraint requirements, as well as maximize its throughput.

B. Online Power-and-Threshold Adaptation

In problem (4), we have seen that as $1/D_{\text{max}}$ decreases, the system becomes less stringent in terms of the delay constraint. This results in an increase in the average throughput $U_1^\ast$. With this in mind, let us assume, in the generalized delay model, that, at time slot 1, the SU succeeds in transmitting a packet. Thus, at time slot 2, the SU has $K - 1$ remaining packets to be transmitted in $t_f - 1$ time slots. Moreover, from the properties of (30), $P_{\text{frame}}(K - 1, t_f - 1) > P_{\text{frame}}(K, t_f)$. This means that the system becomes less stringent in terms of the QoS constraint after a successful packet transmission. This advantage appears in the form of higher throughput. To see how we can make use of this advantage, define $P_{\text{frame}}(K(t), t_f - t + 1)$ as

$$P_{\text{frame}}(K(t), t_f - t + 1) = \sum_{n=K(t)}^{t_f - t + 1} \binom{t_f - t + 1}{n} p(t)^n (1-p(t))^{t_f - t + 1 - n}$$

where $K(t)$ is the remaining number of packets before time slot $t \in \{1, \ldots, t_f\}$, and $p(t)$ is the probability of successful transmission at time slot $t$. At each time slot $t \in \{1, \ldots, t_f\}$, the SU modifies the QoS constraint to be $P_{\text{frame}}(K(t), t_f - t + 1) \geq r_{\text{min}}$ instead of $P_{\text{frame}}(K, t_f) \geq r_{\text{min}}$, which was used in the offline adaptation, and solves the following problem:

$$\begin{align*}
\text{maximize} & \quad U_1(\Gamma_{\text{th}}(1), P_1) \\
\text{subject to} & \quad S_1(\Gamma_{\text{th}}(1), P_1) \leq P_{\text{avg}} \\
& \quad P_{\text{frame}}(K(t), t_f - t + 1) \geq r_{\text{min}} \\
\text{variables} & \quad \Gamma_{\text{th}}(1), P_1.
\end{align*}$$

This results in an increase in the average throughput $U_1^\ast$, which is exponentially distributed as $f_{U_1^\ast}(\tau) = \exp(-\tau/\bar{\tau})/\bar{\tau}$, where $\bar{\tau}$ is the average channel gain and is set to be 1, unless otherwise specified.

Fig. 2 plots the expected throughput $U_1^\ast$ for the overlay scenario after solving problem (4). $U_1^\ast$ is plotted using (2), which represents the average number of bits transmitted divided by the average time required to transmit those bits, taking into account the time wasted due to the blocked time slots. We plot $U_1^\ast$ with $D_{\text{max}} = 1.02T_s$ and with $D_{\text{max}} = \infty$ (i.e., neglecting the delay constraint). We refer to the former problem as a constrained problem and to the latter as an unconstrained problem. We also compare the performance to the nonoptimump-stopping-rule case (No-OSR), where the SU transmits at the first available channel. We expect the No-OSR case to have the best delay and the worst throughput performances. We can see...
that the unconstrained problem has the best throughput among all constrained problems.

Examining the constrained problem for different sensing orders of the channels, we observe that when the channels are sorted in an ascending order of $\theta_i$, the throughput is higher. This is because a channel $i$ has a higher chance of being skipped if put at the beginning of the order compared with the case if put at the end of the order. This is a property of the problem no matter how the channels are ordered, i.e., this property holds even if all channels have equal values of $\theta_i$. Hence, it is more favorable to put the high-quality channels at the end of the sensing order so that they are not put in a position of being frequently skipped. However, this is not necessarily optimum order, which is out of the scope of this work and is left as a future work for this delay-constrained optimization problem.

We also plot the expected throughput of a simple stopping rule that we call the $K$-out-of-$M$ scheme, where we choose the highest $K$ channels in availability probability and ignore the remaining channels as if they do not exist in the system. The SU senses those $K$ channels sequentially; probes the gain of each free channel, if any; and transmits on the channel with the highest gain. This scheme has a constant fraction $K\tau/T_s$ of time wasted each slot. However, it has the advantage of choosing the best channel among multiple available channels. In Fig. 2, we can see that the degradation of the throughput when $K = 5$ compared with the optimal stopping rule scheme. The reason is twofold: 1) due to the constant wasted fraction of time and 2) ignoring the remaining channels that could potentially be free with a high gain if they were considered, as opposed to the constrained problem.

The delay is shown in Fig. 3 for both the constrained and unconstrained problems. We see that the unconstrained problem suffers around 6% increase in the delay, at $P_{avg} = 10$, compared with the constrained problem.

Studying the system performance under low average channel gain is essential. A low average channel gain represents an SU’s channel being in a permanent deep fade or if there is a relatively high interference level at the SR. Fig. 4 shows $\gamma_{th}^*(i)$ versus $\bar{\gamma}$. At low $\bar{\gamma}$, the throughput is expected to be small; hence, $\gamma_{th}^*(i)$ is close to its minimum value $\lambda_P$, so that even if $\gamma_i$ is relatively small, $i$ should not be skipped. In other words, at low average channel gain, the expected throughput is small; thus, a relatively low instantaneous gain will be satisfactory for stopping at channel $i$. While when the average channel gain increases, $\gamma_{th}^*(i)$ should increase so that only high instantaneous gains should lead to stopping at channel $i$. In both cases, i.e., high and low $\bar{\gamma}$, there is still a tradeoff between choosing a high versus a low value of $\gamma_{th}^*(i)$.

The sensing channel (i.e., the channel between the PT and the ST over which the ST overhears the PT activity) is modeled
as additive white Gaussian noise with unit variance. The distributions of the energy detector output $z$ (average energy of $N$ samples sampled from this sensing channel) under the free and busy hypotheses are the chi-square and the noncentral chi-square, which are given by

\[ f_{z|b_i = 0} = \left( \frac{N}{\sigma^2} \right)^N \frac{z^{N-1}}{(N-1)!} \exp \left( -\frac{NZ}{\sigma^2} \right) \]

\[ f_{z|b_i = 1} = \left( \frac{N}{\sigma^2} \right)^{\frac{z}{E}} \frac{z^{N-1}}{(E)^{N-1}} \exp \left( -\frac{NZ + E\gamma}{\sigma^2} \right) \]

\[ \times \left( \frac{2N\sqrt{Ez}}{\sigma^2} \right) \]

where $\sigma^2$, which is set to 1, is the variance of the Gaussian noise of the energy detector, $E$ is the amount of energy received by the ST due to the activity of the PT and is taken as $E = 2\sigma^2$ throughout the simulations, whereas $I_{\text{Bessel}}^{\text{Bes}}(x)$ is the modified Bessel function of the first kind and $i$th order, and $N = 10$.

The main problem we are addressing in this paper is the optimal stopping rule that dictates the SU when to stop sensing and when to start transmitting. As we have seen, this is identified by the threshold vector $\Gamma_{\text{th}}(1, z)$. If the SU does not consider the optimal stopping rule problem and rather transmits as soon as it detects a free channel, then it will be wasting future opportunities of possibly higher throughput. Hence, we expect degradation in the throughput. We plot the two scenarios in Fig. 5 for the underlay system with no delay constraint.

For the multi-SU scenario, numerical analysis was run for the case of $L = 30$ SUs and $M = 10$ channels. We assumed that the fading channels are i.i.d. among users and among frequency channels. Each channel is exponentially distributed with unity average channel gain. Moreover, since $L$ is large, the distribution of the maximum gain among $L$ random gains converges in distribution to the Gumbel distribution [28] having a cumulative distribution function of $\exp(-\exp(-\gamma/\gamma))$. The per-user throughput $U_L^{L_t}$ is plotted in Fig. 6, where the throughput values of the delay-constrained and the unconstrained optimization problems coincide. This is because when $L \gg M$, the solution of the unconstrained problem is delay optimal as well. Hence, adding a delay constraint does not sacrifice the throughput when $L$ is large. Moreover, the delay performance shown in Fig. 7 shows that the delay does not change with and without considering the average delay constraint since the system is already delay (and throughput) optimal.

We have simulated the system for the online algorithm in Section VI for $K(1) = 2$ packets and $t_f = 4$ time slots. We simulated the system at $r_{\text{min}} = 0.95$, which means that the QoS of the SU requires that at least 95% of the frames be successfully transmitted. Fig. 8 shows the improvement in the throughput of the online over the offline adaptation. This is because the SU adapts the power and thresholds at each time slot to allocate the remaining resources (i.e., remaining time slots) according to the remaining number of packets and the desired QoS. This comes at the expense of resolving the problem at each time slot (i.e., $t_f$ times more).
In this paper, we have formulated the problem of a CR system having a single SU that sequentially tests $M$ potentially available channels, originally licensed to the PU’s network, to transmit delay-constrained data. The unique challenge with delay-constrained data is by which each packet has a deadline that needs to be transmitted, on average. Thus, there is a tradeoff to transmit delay-constrained data. The unique challenge with an algorithm that takes sequential multiple channels into account.

In the overlay scheme, the SU was allowed to transmit on free channels only. We have seen that the optimal power control underlay was found to be throughput optimal and delay optimal at the same time. Our algorithm can reach this solution with smaller than the number of channels. Moreover, the optimum solution is a variable in the problem. Hence, the use of the bisection method in Algorithm 1 is not guaranteed to be optimal.

The solution to the delay-unconstrained problem. Then, we generalized the problem to consider packets arriving simultaneously and having the same deadline to model typical data. A low-complexity online power-and-threshold adaptation solution was proposed, and simulation results showed its performance superiority over the offline solution.

While the problem of finding the optimal sensing order of the channels is outside the scope of this work, one could still rely on previous work that addressed this problem. The work in [4] proposes a dynamic programming algorithm for this problem but without any delay constraints and, moreover, while fixing $P_i(\gamma) = 1 \forall i \in \mathcal{M}$. Based on the closed-form expressions of the proposed approach for the threshold and power functions, one could still find the optimum sensing sequence using this dynamic programming algorithm, given some fixed $\lambda_P$ and $\lambda_D$ (as mentioned in Section III-B). However, finding the optimum $(\lambda_P^*, \lambda_D^*)$ is still an open question. This is because the monotonicity of $S_1^*$ in $\lambda_P^*$ is not proven when the sensing order is a variable in the problem. Hence, the use of the bisection method in Algorithm 1 is not guaranteed to be optimal.

APPENDIX A
PROOF OF LEMMA 1

We carry out the proof by contradiction. Assume, for some $i$, that $\gamma^*_{th}(i) < \lambda_P^*$. Thus, the reward starting from channel $i$, $U_i([\gamma^*_{th}(i), \gamma^*_th(i + 1), \ldots, \gamma^*_th(M)]^T, P_i^*)$ becomes

\[ \theta_i c_i \int_{\gamma_{th}(i)}^{\gamma^*_{th}(i)} \log(1 + P^*_i \gamma) f_\gamma(\gamma) d\gamma + \theta_i U_{i+1}^* \int_{0}^{\lambda_P^*} f_\gamma(\gamma) d\gamma + (1 - \theta_i) U_{i+1}^* \]

(36)

\[ \leq \theta_i c_i \int_{\lambda_P^*}^{\infty} \log(1 + P^*_i \gamma) f_\gamma(\gamma) d\gamma + \theta_i U_{i+1}^* \int_{0}^{\lambda_P^*} f_\gamma(\gamma) d\gamma + (1 - \theta_i) U_{i+1}^* \]

(37)

\[ = U_i ([\lambda_P^*, \gamma^*_{th}(i + 1), \ldots, \gamma^*_{th}(M)]^T, P_i^*) \]

(38)

Inequality (37) follows by adding the term $\theta_i \int_{\gamma_{th}(i)}^{\lambda_P^*} f_\gamma(\gamma) d\gamma) U_{i+1}^*$ to (36), whereas (38) follows by the definition of the right-hand side of (37). Using (2), we can calculate the reward $U_{i-1}$ for both the left- and right-hand sides of the previous inequality. Thus, the following inequality holds:

\[ U_{i-1} \left( [\gamma^*_{th}(i - 1), \gamma^*_{th}(i), \ldots, \gamma^*_{th}(M)]^T, P_{i-1}^* \right) \leq U_i \left( [\gamma^*_{th}(i - 1), \lambda_P^*, \ldots, \gamma^*_{th}(M)]^T, P_i^* \right) \]

(39)

Carrying out the last step recursively by $i - 2$ more times, we find the following relation:

\[ U_1 \left( [\gamma^*_{th}(1), \gamma^*_{th}(i - 1), \gamma^*_{th}(i), \ldots, \gamma^*_{th}(M)]^T, P_1^* \right) \leq U_1 \left( [\gamma^*_{th}(1), \lambda_P^*, \ldots, \gamma^*_{th}(M)]^T, P_1^* \right) \]

(40)

which contradicts with the fact that $\gamma^*_{th}(i)$ is optimal. □
APPENDIX B

PROOF OF THEOREM 1

We first get $S^*_i$, $U^*_i$, and $p^*_i$ by substituting equations $\gamma^*_\text{th}(i)$ and $P^*_i(\gamma)$ in (1)–(3), respectively. Then, we differentiate with respect to $\lambda^*_p$, treating $\lambda^*_D$ as a constant, yielding

$$\frac{\partial S^*_i}{\partial \lambda^*_p} = -\theta_i f_\gamma (\gamma^*_\text{th}(i)) \frac{\partial \gamma^*_\text{th}(i)}{\partial \lambda^*_p} (c_i P^*_i(\gamma^*_\text{th}(i))) - \frac{S^*_{i+1}}{(\lambda^*_p)^2}$$

$$+ \left(1 - \theta_i F_\gamma (\gamma^*_\text{th}(i)) \frac{\partial S^*_{i+1}}{\partial \lambda^*_p} \right) \left[ \lambda^*_p (c_i P^*_i(\gamma^*_\text{th}(i))) - S^*_i + \lambda^*_D (1 - p^*_i) \right]$$

and

$$\frac{\partial U^*_i}{\partial \lambda^*_p} = -\theta_i f_\gamma (\gamma^*_\text{th}(i)) \frac{\partial \gamma^*_\text{th}(i)}{\partial \lambda^*_p} \left(1 - p^*_i \right)$$

$$+ \left(1 - \theta_i F_\gamma (\gamma^*_\text{th}(i)) \frac{\partial p^*_i}{\partial \lambda^*_p} \right) \left[ \lambda^*_p (c_i P^*_i(\gamma^*_\text{th}(i))) - S^*_i + \lambda^*_D (1 - p^*_i) \right]$$

respectively. Multiplying (41) by $-\lambda^*_p$ and (43) by $\lambda^*_D$ and then adding them to (42), we can easily show that, for all $i \in M$

$$\frac{\partial U^*_i}{\partial \lambda^*_p} = \lambda^*_P \frac{\partial S^*_i}{\partial \lambda^*_p} + \lambda^*_D \frac{\partial p^*_i}{\partial \lambda^*_p} = 0.$$ 

(45)

We now find the derivative of $\gamma^*_\text{th}(i)$ with respect to $\lambda^*_p$, by differentiating both sides of (7) with respect to $\lambda^*_p$, while treating $\lambda^*_D$ as a constant, then using (45), and then rearranging, we get

$$\frac{\partial \gamma^*_\text{th}(i)}{\partial \lambda^*_p} = \frac{c_i P^*_i(\gamma^*_\text{th}(i)) - S^*_{i+1}}{c_i \lambda^*_p (\gamma^*_\text{th}(i)) P^*_i(\gamma^*_\text{th}(i))}$$

(46)

Substituting (46) in (41), we get

$$\frac{\partial S^*_i}{\partial \lambda^*_p} = -\alpha_i \left[c_i P^*_i(\gamma^*_\text{th}(i)) - S^*_{i+1}\right]^2 - \theta_i c_i \frac{F_\gamma (\gamma^*_\text{th}(i))}{(\lambda^*_p)^2}$$

$$+ \left(1 - \theta_i F_\gamma (\gamma^*_\text{th}(i)) \frac{\partial S^*_{i+1}}{\partial \lambda^*_p} \right) \left(\lambda^*_p \gamma^*_\text{th}(i)\right)$$

(47)

where $\alpha_i$ is given by

$$\alpha_i = \frac{\theta_i f_\gamma (\gamma^*_\text{th}(i))}{c_i \lambda^*_p (\gamma^*_\text{th}(i)) P^*_i(\gamma^*_\text{th}(i))} \geq 0.$$ 

(48)

Now, evaluating (47) at $i = M$ and $i = M - 1$, we get

$$\frac{\partial S^*_M}{\partial \lambda^*_p} = -\alpha_M \left[c_M P^*_M (\gamma^*_\text{th}(M))\right]^2 - \theta_M c_M \frac{F_\gamma (\gamma^*_\text{th}(M))}{(\lambda^*_p)^2}$$

(49)

$$\frac{\partial S^*_M}{\partial \lambda^*_p} = -\alpha_{M-1} \left[c_{M-1} P^*_n (\gamma^*_\text{th}(M-1)) - S^*_M\right]^2 - \theta_{M-1} c_M \frac{F_\gamma (\gamma^*_\text{th}(M-1))}{(\lambda^*_p)^2}$$

$$+ \left(1 - \theta_{M-1} F_\gamma (\gamma^*_\text{th}(M-1)) \frac{\partial S^*_M}{\partial \lambda^*_p} \right) \left[\lambda^*_p \gamma^*_\text{th}(M)\right]$$

(50)

respectively. We can see that $(\partial S^*_M/\partial \lambda^*_p) < 0$; hence, $(\partial S^*_M/\partial \lambda^*_p) < 0$. By induction, let us assume that $(\partial S^*_M/\partial \lambda^*_p) < 0$. From (47), we get that

$$\frac{\partial S^*_i}{\partial \lambda^*_p} = -\alpha_i \left[c_i P^*_i(\gamma^*_\text{th}(i)) - S^*_{i+1}\right]^2 - \theta_i c_i \frac{F_\gamma (\gamma^*_\text{th}(i))}{(\lambda^*_p)^2}$$

$$+ \left(1 - \theta_i F_\gamma (\gamma^*_\text{th}(i)) \frac{\partial S^*_{i+1}}{\partial \lambda^*_p} \right) < 0$$

(51)

since all its terms are negative. Finally, we find that $(\partial S^*_i/\partial \lambda^*_p) < 0$, indicating that $S^*_i$ is monotonically decreasing in $\lambda^*_p$ given any fixed $\lambda^*_D > 0$.

Now, to get an upper bound on $\lambda^*_p$, we know that

$$S^*_i = \theta_i c_i \int_{\gamma^*_\text{th}(i)}^{\infty} \frac{1}{\lambda^*_p} f_\gamma (\gamma) d\gamma + \left[1 - \theta_i F_\gamma (\gamma^*_\text{th}(i)) \right] S^*_{i+1}.$$ 

(52)

We can upper bound the first term in (52) by $\theta_i c_i / \lambda^*_p$, while $\left[1 - \theta_i F_\gamma (\gamma^*_\text{th}(i)) \right] < 1$. Using these two bounds, we can write $S^*_i < \sum_{i=1}^{M} \theta_i c_i / \lambda^*_p$. However, since $S^*_1 = \lambda^*_D$, the upper bound on $\lambda^*_p$, which was mentioned in Theorem 1, follows. □

APPENDIX C

PROOF OF LEMMA 2

We provide a proof sketch for this bound. We know that, at the optimal point, $p^*_1 = (1/\bar{D}_{\text{max}})$ and that $p^*_1 = \theta_1 F_\gamma (\gamma^*_\text{th}(1)) + (1 - \theta_1 F_\gamma (\gamma^*_\text{th}(1))) p^*_2$. However, since the second term in the latter equation is always positive, then

$$\theta_1 F_\gamma (\gamma^*_\text{th}(1)) < \frac{1}{\bar{D}_{\text{max}}}.$$ 

(53)

Substituting (12) in (53) and rearranging, we can upper bound $\lambda^*_p$ by

$$c_1 \left[\log \left(\frac{\lambda^*_p}{\bar{F}_{\gamma^{-1}}(\bar{D}_{\text{max}})}\right) - \frac{\lambda^*_p}{\bar{F}_{\gamma^{-1}}(\bar{D}_{\text{max}})} + 1\right] + U^* - \lambda^*_p S^*_2.$$ 

$$1 - p^*_2$$

We can easily upper bound log $(\lambda^*_p/\bar{F}_{\gamma^{-1}}(1/(\theta_1 D_{\text{max}})))$ by substituting $\lambda^*_\text{max}$ for $\lambda^*_p$, when $\lambda^*_p < F_{\gamma^{-1}}(1/(\theta_1 D_{\text{max}}))$ and by 1 otherwise. Moreover, it can also be shown that $U^*_2 < U^*_\text{max}$, $p^*_2 < p^*_\text{max}$ and that $\lambda^*_p S^*_2 > 0$, and from Theorem 1, we have $\lambda^*_p < \lambda^*_\text{max}$; the proof then follows. □
References


Ahmed E. Ewaisha was born in Cairo, Egypt, in 1987. He received the B.S. degree in electrical engineering (with honors, ranked top 5% in his class) from Alexandria University, Alexandria, Egypt, in 2009 and the M.S. degree in electrical engineering from Nile University, 6th of October City, Egypt, which is considered the first research-based university in Egypt, in 2011. He is currently working toward the Ph.D. degree in delay analysis in cognitive radio networks with the Ira A. Fulton School of Engineering, Arizona State University, Tempe, AZ, USA.

His research interests span a wide area of wireless and wired communication networks, including stochastic optimization, power allocation, cognitive radio networks, resource allocation, and quality-of-service guarantees in data networks.

Cihan Tepedelenliglu (S’97–M’01) was born in Ankara, Turkey, in 1973. He received the B.S. degree in electrical engineering (with highest honors) from the Florida Institute of Technology, Melbourne, FL, USA, in 1995; the M.S. degree in electrical engineering from The University of Virginia, Charlottesville, VA, USA, in 1998; and the Ph.D. degree in electrical and computer engineering from the University of Minnesota, Minneapolis, MN, USA.

From January 1999 to May 2001, he was a Research Assistant with the University of Minnesota.

He is currently an Associate Professor of electrical engineering with Arizona State University, Tempe, AZ, USA.

Dr. Tepedelenliglu received the National Science Foundation Early Career Grant in 2001. He has served as an Associate Editor for several IEEE TRANSACTIONS, including the IEEE TRANSACTIONS ON COMMUNICATIONS, the IEEE SIGNAL PROCESSING LETTERS, and the IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY. His research interests include statistical signal processing, system identification, wireless communications, estimation and equalization algorithms for wireless systems, multiantenna communications, orthogonal frequency-division multiplexing, ultrawideband systems, distributed detection and estimation, and data mining for photovoltaic systems.