Throughput Optimization in Multichannel Cognitive Radios With Hard-Deadline Constraints

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***Abstract*—In a cognitive radio scenario, we consider a single secondary user (SU) accessing a multichannel system. The SU senses the channels sequentially to detect if a primary user (PU) is occupying the channels and stops its search to access a channel if it offers a significantly high throughput. The optimal stopping rule and power control problem is considered. The problem is formulated as an SU's throughput-maximization problem under power, interference, and packet delay constraints. We first show the effect of the optimal stopping rule on packet delay and then solve this optimization problem for both the overlay system, where the SU transmits only at the spectrum holes, and the underlay system, where tolerable interference (or tolerable collision probability) is allowed. We provide closed-form expressions for the optimal stopping rule and show that the optimal power control strategy for this multichannel problem is a modified waterfilling approach. We extend the work to a multi-SU scenario and show that when the number of SUs is large, the complexity of the solution becomes smaller than that of the single-SU case. We discuss the application of this problem in typical networks where packets simultaneously arrive and have the same departure deadline. We further propose an online adaptation policy to the optimal stopping rule that meets the packets' hard-deadline constraint and, at the same time, gives higher throughput than the offline policy.**

***Index Terms*—Delay constraint, optimal channel selection, optimal stopping rule, stochastic optimization, water filling.**

I. Introduction

COGNITIVE radio (CR) systems are emerging wireless communication systems that allow efficient spectrum utilization [2]. This is due to the use of transceivers that are capable of detecting the presence of licensed (primary) users. The secondary users (SUs) use the frequency bands originally dedicated for the primary users (PUs) for their own transmission. Once the PU's activity is detected on some frequency channel, the SU refrains from any further transmission on this channel. This may result in service disconnection for the SUs, thus degrading the quality of service (QoS). On the other hand, if the SUs have access to other channels, the QoS can be improved if these channels are efficiently utilized.

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The problem of multiple channels in CR systems has gained attention in recent works due to the challenges associated with the sensing and access mechanisms in a multichannel CR system. Practical hardware constraints on the SUs' transceivers may prevent them from sensing multiple channels simultaneously to detect the state of these channels (free/busy). This leads the SU to sensing the channels sequentially and then deciding which channel should be used for transmission [3], [4]. In a time-slotted system, if sequential channel sensing is employed, the SU senses the channels one at a time and stops sensing when a channel is found free. However, due to independent fading among channels, the SU is allowed to skip a free channel if its quality, which is measured by its power gain, is low and sense another channel seeking the possibility of a higher future gain. Otherwise, if the gain is high, the SU stops at this free channel to begin transmission. The question of when to stop sensing can be formulated as an optimal stopping rule problem [4]–[7]. In [5], Sabharwal *et al.* presented the optimal stopping rule for this problem in a non-CR system. The work in [4] develops an algorithm to find the optimal order by which channels are to be sequentially sensed in a CR scenario, whereas that in [6] studies the case where the SUs are allowed to transmit on multiple contiguous channels simultaneously. The authors presented the optimal stopping rule for this problem in a nonfading wireless channel. Transmissions on multiple channels simultaneously may be a strong assumption for low-cost transceivers, particularly when they cannot sense multiple channels simultaneously.

In general, if a perfect sensing mechanism is adopted, the SU will not cause interference to the PU since the former transmits only on spectrum holes (referred to as an overlay system). Nevertheless, if the sensing mechanism is imperfect, or if the SU's system is an underlay system (where the SU uses the channels as long as the interference to the PU is tolerable), the transmitted power needs to be controlled to prevent harmful interference to the PU. In [8] and [9], power control is considered, and it is shown and that the optimal power control strategy is a waterfilling approach under some interference constraint imposed on the SU transmitter. However, all of the aforementioned works study single-channel systems that cannot be extended to multiple channels in a straightforward manner. A multiuser CR system was considered in [10] in a time-slotted system. To allocate the frequency channel to one of the SUs, Hu *et al.* proposed a contention mechanism that does not depend on the SUs' channel gains, thus neglecting the advantage of multiuser diversity. A major challenge in a multichannel system is the sequential nature of the sensing where the SU needs to take a decision to stop and begin transmission or continue sensing based on the information it has so far. This decision needs to trade off between waiting for a potentially higher throughput and taking advantage of the current free channel. Moreover, if transmission takes place on a given channel, the SU needs to decide on the amount of power transmitted to maximize its throughput, given some average interference and average power constraints.

In this paper, we model the overlay and underlay scenarios of a multichannel CR system, which are sensed sequentially. The problem is solved first for a single SU, and then, we discuss extensions to a multi-SU scenario. For the single-SU case, the problem is formulated as a joint optimal-stopping-rule and power-control problem with the goal of maximizing the SU's throughput subject to average power and average interference constraints. This formulation results in increasing the expected service time of the SU's packets. The expected service time is the average number of time slots that pass while the SU attempts to find a free channel, before successfully transmitting a packet. The increase in service time is due to skipping free channels, due to their poor gain, hoping to find a future channel of sufficiently high gain. If no channels having a satisfactory gain were found, the SU will not be able to transmit its packet and will have to wait for longer time to find a satisfactory channel. This increase in service time increases the queuing delay. Thus, we solve the problem subject to a bound on the expected service time that controls the delay (we note that, in this paper, we use the word delay to refer to the service time). In the multi-SU case, we show that the solution to the single-SU problem can be directly applied to the multi-SU system with a minor modification. We also show that the complexity of the solution decreases when the system has a large number of SUs.

To the best of our knowledge, this is the first work to study the joint power-control and optimal-stopping-rule problem in a multichannel CR system. Our contribution in this work is the formulation of a joint power-control and optimal-stopping-rule problem that also incorporates a delay constraint and presents a low-complexity solution in the presence of an interference/collision constraint from the SU to the PU due to the imperfect-sensing mechanism. The preliminary results in [1] consider an overlay framework for the single-user case while neglecting sensing errors. However, in this work, we also study the problem in the underlay scenario, where interference is allowed from the secondary transmitter (ST) to the primary receiver (PR) and extends to the multi-SU case. We also generalize the solution to the multi-SU case when the number of SUs is large. We discuss the applicability of our formulation in typical delay-constrained scenarios where packets arrive simultaneously and have the same deadline. We show that the proposed algorithm can be used to solve this problem offline to maximize the throughput and meet the deadline constraint at the same time. Moreover, we propose an online algorithm that gives higher throughput compared with the offline approach while meeting the deadline constraint.

The rest of this paper is organized as follows: The overlay system model and the underlying assumptions are presented in Section II. In Section III, the problem is mathematically formulated, the main objective is stated, and the solution to the overlay problem is proposed. Then, in Section IV, the underlay system model is discussed, and the optimal solution is presented. In Section V, the extension to multiple SUs is discussed. In Section VI, the delay constraint is generalized to the case where multiple packets arrive at the same time and have the same deadline. An online adaptation solution is also proposed, which maximizes the throughput subject to a delay constraint. Finally, numerical results are shown in Section VII, whereas Section VIII concludes this paper.

Throughout the sequel, we use bold fonts for vectors and an asterisk to denote that x^* is the optimal value of x; all logarithms are natural, whereas the expected value operator is denoted $\mathbb{E}[\cdot]$ and is taken with respect to all the random variables in its argument. Finally, we use $(x)^+ \triangleq \max(x, 0)$ and $\mathbb{R}$ to denote the set of real numbers.

II. Overlay System Model

Consider a PU network that has licensed access to M orthogonal frequency channels. Time is slotted with a time-slot duration of T_s seconds. The SU's network consists of a single ST (SU and ST will be used interchangeably) attempting to send real-time data to its intended secondary receiver (SR) through one of the channels licensed to the PU. Before a time slot begins, the SU is assumed to have ordered the channels according to some sequence (we note that the method of ordering the channels is outside the scope of this work; see [4] for further details about channel ordering), which is labeled $1, \ldots, M$. The set of channels is denoted by $\mathcal{M} = \{1, \ldots, M\}$. Before the SU attempts to transmit its packet over channel i, it senses this channel to determine its availability "state," which is described by a Bernoulli random variable b_i with parameter θ_i (θ_i is called the availability probability of channel i). If $b_i = 0$ (which happens with probability θ_i), then channel i is free, and the SU may transmit over it until the ongoing time slot ends. If $b_i = 1$, channel i is busy, and the SU proceeds to sense channel $i + 1$. Channel availabilities are statistically independent across frequency channels and across time slots.

We assume that the SU has limited capabilities in the sense that no two channels can be sensed simultaneously. This may be the case when considering radios having a single sensing module with a fixed bandwidth, so that it can be tuned to only one frequency channel at a time. The reader is referred to [11]–[13] for detailed information on advanced spectrum sensing techniques. Therefore, at the beginning of a given time slot, the SU selects a channel, e.g., channel 1, senses it for τ seconds ($\tau \ll T_\mathrm{s}/M$), and detects if it is free. Otherwise, the SU skips this channel and senses channel 2, and so on, until it finds a free channel. If all channels are busy (i.e., the PU has transmission activities on all M channels), then this time slot will be considered "blocked." In this case, the SU waits for the following time slot and begins sensing following the same channel sensing sequence. As the sensing duration increases, the transmission phase duration decreases, which then decreases the throughput. However, we cannot arbitrarily decrease the value of τ since this decreases the reliability of the sensing outcome. This tradeoff has been extensively studied in the literature, e.g., [14] and [15]. In this paper, we study the

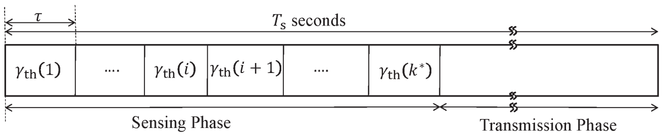

Fig. 1. Sensing and transmission phases in one time slot. The SU senses each channel for τ seconds, determines its state, and then probes the gain if the channel is found free. The sensing phase ends if the probed gain $\gamma_i > \gamma_{\mathrm{th}}(i)$, in which case $k^* = i$. Hence, k^* is a random variable that depends on the channel states and gains.

impact of sequential channel sensing on the throughput rather than that of the sensing duration on the throughput. Hence, we assume that τ is a fixed parameter and is not optimized over. For details on the tradeoff between throughput and sensing duration in this sequential sensing problem, the reader is referred to [16].

The fading channel between ST and SR is assumed to be flat fading with independent and identically distributed (i.i.d.) channel gains across the M channels. To achieve higher data rates, the SU adapts its data rate according to the instantaneous power gain of the channel before beginning transmission on this channel. To do this, once the SU finds a free channel, e.g., channel i, the gain γ_i is probed. The data rate will be proportional to $\log(1 + P_i(\gamma_i)\gamma_i)$, where $P_i(\gamma_i)$ is the power transmitted by the SU at channel i as a function of the instantaneous gain [17]. Fig. 1 shows a potential scenario where the SU senses k^* channels, skips the first $k^* - 1$, and uses the k^*th channel for transmission until the end of this ongoing time slot. In this scenario, the SU "stops" at the k^*th channel, for some $k^* \in \mathcal{M}$. Stopping at channel i depends on two factors: 1) the availability of channel b_i and 2) the instantaneous power gain γ_i. Clearly, b_i and γ_i are random variables that change from one time slot to another. Hence, k^*, which depends on these two factors, is a random variable. More specifically, it depends on the states $[b_1, \ldots, b_M]$ along with the gains of each channel $[\gamma_1, \ldots, \gamma_M]$. To understand why, consider that the SU senses channel i, finds it free, and probes its gain γ_i. If γ_i is found to be low, then the SU skips channel i (although free) and senses channel $i+1$. This is to take advantage of the possibility that $\gamma_j \gg \gamma_i$ for $j > i$. On the other hand, if γ_i is sufficiently large, the SU stops at channel i and begins transmission. In that latter case, $k^* = i$. Defining the two random vectors $\underline{b} = [b_1, \ldots, b_M]^T$ and $\underline{\gamma} = [\gamma_1, \ldots, \gamma_M]^T$, k^* is a deterministic function of $\underline{b}$ and $\underline{\gamma}$.

We define the stopping rule by defining a threshold $\gamma_{\mathrm{th}}(i)$ to which each γ_i is compared when the ith channel is found free. If $\gamma_i \geq \gamma_{\mathrm{th}}(i)$, the SU "stops" and transmits at channel i. Otherwise, channel i is skipped, and channel $i+1$ is sensed. In the extreme case when $\gamma_{\mathrm{th}}(i) = 0$, the SU will not skip channel i if it is found free. Increasing $\gamma_{\mathrm{th}}(i)$ allows the SU to skip channel i whenever $\gamma_i < \gamma_{\mathrm{th}}(i)$, to search for a better channel, thus potentially increasing the throughput. Setting $\gamma_{\mathrm{th}}(i)$ too large allows channel i to be skipped even if γ_i is high. This constitutes the tradeoff in choosing the thresholds $\gamma_{\mathrm{th}}(i)$. The optimal values of $\gamma_{\mathrm{th}}(i)$ $i \in \mathcal{M}$ determine the optimal stopping rule.

Let $P_i(\gamma)$ denote the power transmitted at the ith channel when the instantaneous channel gain is γ, if channel i was chosen for transmission. Since the SU can transmit on one channel at a time, the power transmitted at any time slot at channel i is $P_i(\gamma_i)\mathbb{1}(i = k^*)$, where $\mathbb{1}(i = k^*) = 1$ if $i = k^*$ and 0 otherwise. Define $c_i \triangleq 1 - (i\tau/T_s)$ as the fraction of the time slot remaining for the SU's transmission if the SU transmits on the ith channel in the sensing sequence. The average power constraint is $\mathbb{E}_{\underline{\gamma},\underline{b}}[c_{k^*} P_{k^*}(\gamma_{k^*})] \leq P_{\mathrm{avg}}$, where the expectation is with respect to the random vectors $\underline{\gamma}$ and $\underline{b}$. We will henceforth drop the subscript from the expected value operator $\mathbb{E}$. This expectation can be recursively calculated from

$$S_i(\mathbf{\Gamma}_{\mathrm{th}}(i), \mathbf{P}_i) = \theta_i c_i \int_{\gamma_{\mathrm{th}}(i)}^{\infty} P_i(\gamma) f_{\gamma_i}(\gamma)\, d\gamma + \left[1 - \theta_i \bar{F}_{\gamma_i}(\gamma_{\mathrm{th}}(i))\right] S_{i+1}(\mathbf{\Gamma}_{\mathrm{th}}(i+1), \mathbf{P}_{i+1}) \quad (1)$$

$i \in \mathcal{M}$, where $\mathbf{P}_i \triangleq [P_i(\gamma), \ldots, P_M(\gamma)]^T$ and $\mathbf{\Gamma}_{\mathrm{th}}(i) \triangleq [\gamma_{\mathrm{th}}(i), \ldots, \gamma_{\mathrm{th}}(M)]^T$ are the vectors of the power functions and thresholds, respectively, with $S_{M+1}(\mathbf{\Gamma}_{\mathrm{th}}(M+1), \mathbf{P}_{M+1}) \triangleq 0$, $f_{\gamma_i}(\gamma)$ is the probability density function (pdf) of the gain γ_i of channel i, and $\bar{F}_{\gamma_i}(x) \triangleq \int_x^\infty f_{\gamma_i}(\gamma)\, d\gamma$ is the complementary cumulative distribution function. The first term in (1) is the average power transmitted at channel i given that channel is chosen for transmission (i.e., given that $k^* = i$). The second term represents the case where channel i is skipped and channel $i+1$ is sensed. It can be shown that $S_1(\mathbf{\Gamma}_{\mathrm{th}}(1), \mathbf{P}_1) = \mathbb{E}[c_{k^*} P_{k^*}(\gamma)]$. Moreover, we will also drop the index i from the subscript of $f_{\gamma_i}(\gamma)$ and $\bar{F}_{\gamma_i}(\gamma)$ since channels suffer i.i.d. fading. Although we have only included an average power constraint in our problem, we will modify, after solving the problem, the solution to include an instantaneous power constraint as well.

The SU's average throughput is defined as $\mathbb{E}[c_{k^*} \log(1 + P_{k^*}(\gamma_{k^*})\gamma_{k^*})]$. Similar to the average power, we denote the expected throughput as $U_1(\mathbf{\Gamma}_{\mathrm{th}}(1), \mathbf{P}_1)$, which can be derived using the following recursive formula:

$$U_i(\mathbf{\Gamma}_{\mathrm{th}}(i), \mathbf{P}_i) = \theta_i c_i \int_{\gamma_{\mathrm{th}}(i)}^{\infty} \log\left(1 + P_i(\gamma)\gamma\right) f_\gamma(\gamma)\, d\gamma + \left[1 - \theta_i \bar{F}_\gamma(\gamma_{\mathrm{th}}(i))\right] U_{i+1}\left(\mathbf{\Gamma}_{\mathrm{th}}(i+1), \mathbf{P}_{i+1}\right) \quad (2)$$

$i \in \mathcal{M}$, with $U_{M+1}(\cdot,\cdot) \triangleq 0$. $U_1(\mathbf{\Gamma}_{\mathrm{th}}(1), \mathbf{P}_1)$ represents the expected data rate of the SU as a function of the threshold vector $\mathbf{\Gamma}_{\mathrm{th}}(1)$ and the power function vector $\mathbf{P}_1$.

If the SU skips all channels, either due to being busy, due to their low gain, or due to a combination of both, then the current time slot is said to be blocked. The SU has to wait for the following time slot to begin searching for a free channel again. This results in a delay in serving (transmitting) the SU's packet. Define delay D as the number of time slots the SU consumes before successfully transmitting a packet. That is, $D - 1$ is a random variable that represents the number of consecutively blocked time slots. In real-time applications, there may exist some average delay requirement $\bar{D}_{\max}$ on the packets that must not be exceeded. Since the availability of each channel is independent across time slots, D follows a geometric distribution having $\mathbb{E}[D] = (\Pr[\text{Success}])^{-1}$, where $\Pr[\text{Success}] = 1 - \Pr[\text{Blocking}]$. In other words, $\Pr[\text{Success}]$ is the probability that the SU finds a free channel with gain that is high enough so that it does not skip all M channels in a time slot. It is given by $\Pr[\text{Success}] \triangleq p_1(\mathbf{\Gamma}_{\text{th}}(1))$, which can be recursively calculated using the following equation:

$$p_i(\mathbf{\Gamma}_{\text{th}}(i)) = \theta_i \bar{F}_\gamma(\gamma_{\text{th}}(i)) + \left[1 - \theta_i \bar{F}_\gamma(\gamma_{\text{th}}(i))\right] p_{i+1}(\mathbf{\Gamma}_{\text{th}}(i+1)) \quad (3)$$

$i \in \mathcal{M}$, where $p_{M+1} \triangleq 0$. Here, $p_i(\mathbf{\Gamma}_{\text{th}}(i))$ is the probability of transmission on channel i, $i+1, \ldots$, or M.

III. Problem Statement and Proposed Solution

From (2), we see that the SU's expected throughput U_1 depends on the threshold vector $\mathbf{\Gamma}_{\text{th}}(1)$ and the power vector $\mathbf{P}_1$. The goal is to find the optimum values of $\mathbf{\Gamma}_{\text{th}}(1) \in \mathbb{R}^M$ and functions $\mathbf{P}_1$ that maximize U_1 subject to an average power constraint and an expected packet delay constraint. The delay constraint can be written as $\mathbb{E}[D] \leq \bar{D}_{\max}$ or, equivalently, $p_1(\mathbf{\Gamma}_{\text{th}}(1)) \geq 1/\bar{D}_{\max}$. Mathematically, the problem becomes

$$\begin{aligned} &\text{maximize} && U_1(\mathbf{\Gamma}_{\text{th}}(1), \mathbf{P}_1) \\ &\text{subject to} && S_1(\mathbf{\Gamma}_{\text{th}}(1), \mathbf{P}_1) \leq P_{\text{avg}} \\ &&& p_1(\mathbf{\Gamma}_{\text{th}}(1)) \geq \frac{1}{\bar{D}_{\max}} \\ &\text{variables} && \mathbf{\Gamma}_{\text{th}}(1), \mathbf{P}_1 \end{aligned} \quad (4)$$

where the first constraint represents the average power constraint, whereas the second constraint is a bound on the average packet delay. We allow the power P_i to be an arbitrary function of γ_i and optimize over this function to maximize the throughput subject to average power and delay constraints. Although (4) is not proven to be convex, we provide closed-form expressions for the optimal threshold and power function vectors. To this end, we first calculate the Lagrangian associated with (4). Let λ_P and λ_D be the dual variables associated with the constraints in problem (4). The Lagrangian for (4) becomes

$$\begin{aligned} L\left(\mathbf{\Gamma}_{\text{th}}(1), \mathbf{P}_1, \lambda_P, \lambda_D\right) &= U_1\left(\mathbf{\Gamma}_{\text{th}}(1), \mathbf{P}_1\right) - \lambda_P\left(S_1(\mathbf{\Gamma}_{\text{th}}(1), \mathbf{P}_1) - P_{\text{avg}}\right) \\ &\quad + \lambda_D\left(p_1(\mathbf{\Gamma}_{\text{th}}(1)) - \frac{1}{\bar{D}_{\max}}\right). \end{aligned} \quad (5)$$

Differentiating (5) with respect to each of the primal variables $P_i(\gamma)$ and $\gamma_{\text{th}}(i)$ and equating the resulting derivatives to zero, we obtain the Karush–Kuhn–Tucker (KKT) equations, below which are necessary conditions for optimality [18], [19], i.e.,

$$P_i^*(\gamma) = \left(\frac{1}{\lambda_P^*} - \frac{1}{\gamma}\right)^+, \ \gamma > \gamma_{\text{th}}^*(i) \quad (6)$$

$$\log\left(1 + \left(\frac{1}{\lambda_P^*} - \frac{1}{\gamma_{\text{th}}^*(i)}\right)^+ \gamma_{\text{th}}^*(i)\right) - \lambda_P^* \left(\frac{1}{\lambda_P^*} - \frac{1}{\gamma_{\text{th}}^*(i)}\right)^+ = \frac{U_{i+1}^* - \lambda_P^* S_{i+1}^* - \lambda_D^* \cdot \left(1 - p_{i+1}^*\right)}{c_i} \quad (7)$$

$$S_1^* \leq P_{\text{avg}}, \ p_1^* \geq \frac{1}{\bar{D}_{\max}}, \ \lambda_P^* \geq 0, \ \lambda_D^* \geq 0 \quad (8)$$

$$\lambda_P^* \cdot (S_1^* - P_{\text{avg}}) = 0 \quad (9)$$

$$\lambda_D^* \cdot \left(p_1^* - \frac{1}{\bar{D}_{\max}}\right) = 0 \quad (10)$$

$i \in \mathcal{M}$. We use $U_{i+1}^* \triangleq U_{i+1}(\mathbf{\Gamma}_{\text{th}}^*(i+1), \mathbf{P}_{i+1}^*)$, while $S_{i+1}^* \triangleq S_{i+1}(\mathbf{\Gamma}_{\text{th}}^*(i+1), \mathbf{P}_{i+1}^*)$ and $p_{i+1}^* \triangleq p_{i+1}(\mathbf{\Gamma}_{\text{th}}^*(i+1))$ for brevity in the sequel. We note that $U_{M+1}(\cdot,\cdot) = S_{M+1}(\cdot,\cdot) = p_{M+1}(\cdot) \triangleq 0$ by definition. We observe that these KKT equations involve the primal ($\mathbf{\Gamma}_{\text{th}}^*(1)$ and $\mathbf{P}_1^*$) and the dual (λ_P^* and λ_D^*) variables. Our approach is to find a closed-form expression for the primal variables in terms of the dual variables and then propose a low-complexity algorithm to obtain the solution for the dual variables. The optimality of this approach is discussed in Section III-C, where we show that, loosely speaking, the KKT equations provide a unique solution to the primal–dual variables. Hence, based on this unique solution and on the fact that the KKT equations are necessary conditions for the optimal solution, this solution is not only necessary but also sufficient and, hence, optimal.

A. Solving for Primal Variables

Equation (6) is a waterfilling strategy with a slight modification due to having the condition $\gamma > \gamma_{\text{th}}(i)$. This condition comes from the sequential sensing of the channels, which is absent in the classic waterfilling strategy [17]. Equation (6) gives a closed-form solution for $\mathbf{P}_1$. On the other hand, the entries of the vector $\mathbf{\Gamma}_{\text{th}}^*(1)$ are found via the set of equations (7). Note that (7) indeed forms a set of M equations, each solves for one of $\gamma_{\text{th}}^*(i)$, $i \in \mathcal{M}$. We refer to this set as the "threshold-finding" equations. For a given value of i, solving for $\gamma_{\text{th}}^*(i)$ requires knowledge of only $\gamma_{\text{th}}^*(i+1)$ through $\gamma_{\text{th}}^*(M)$ and does not require knowing $\gamma_{\text{th}}^*(1)$ through $\gamma_{\text{th}}^*(i-1)$. Thus, these M equations can be solved using back-substitution starting from $\gamma_{\text{th}}^*(M)$. To solve for $\gamma_{\text{th}}^*(i)$, we use the fact that $\gamma_{\text{th}}^*(i) \geq \lambda_P^*$, which is proven in the following lemma.

Lemma 1: The optimal solution of problem (4) satisfies $\gamma_{\text{th}}^*(i) \geq \lambda_P^* \ \forall i \in \mathcal{M}$.

Proof: See Appendix A for the proof. □

The intuition behind Lemma 1 is as follows. If, for some channel i, $\gamma_{\text{th}}^*(i) < \lambda_P^*$ was possible, and the instantaneous gain γ_i happened to fall in the range $[\gamma_{\text{th}}^*(i), \lambda_P^*)$ at a given time slot, then the SU will not skip channel i since $\gamma_i > \gamma_{\text{th}}^*(i)$. However, the power transmitted on channel i is $P_i(\gamma_i) = (1/\lambda_P^* - 1/\gamma_i)^+ = 0$ since $\gamma_i < \lambda_P^*$. This means that the SU will neither skip nor transmit on channel i, which does not make sense from the SU's throughput perspective. To overcome this event, the SU needs to set $\gamma_{\text{th}}^*(i)$ at least as large as λ_P^* so that whenever $\gamma_i < \lambda_P^*$, the SU skips channel i rather than transmitting with zero power.

Lemma 1 allows us to remove the $(\cdot)^+$ sign in (7) when solving for $\gamma_{\text{th}}^*(i)$. Rewriting (7), we get

$$\frac{-\lambda_P^*}{\gamma_{\text{th}}^*(i)} \exp\left(\frac{-\lambda_P^*}{\gamma_{\text{th}}^*(i)}\right) = -\exp\left(-\frac{U_{i+1}^* - \lambda_P^* S_{i+1}^* - \lambda_D^* \cdot \left(1 - p_{i+1}^*\right)}{c_i} - 1\right), i \in \mathcal{M}. \quad (11)$$

Equation (11) is now of the form $W\exp(W) = c$, whose solution is $W = W_0(c)$, where $W_0(x)$ is the principal branch of the Lambert W function [20] and is given by $W_0(x) = \sum_{n=1}^{\infty}((-n)^{n-1}/n!)x^n$. The only solution to (11), which satisfies Lemma 1, is given for $i \in \mathcal{M}$ by

$$\gamma_{\text{th}}^*(i) = \frac{-\lambda_P^*}{W_0\left(-\exp\left(-\frac{\left(U_{i+1}^* - \lambda_P^* S_{i+1}^* - \lambda_D^*\left(1-p_{i+1}^*\right)\right)^+}{c_i} - 1\right)\right)}. \tag{12}$$

Hence, $\boldsymbol{\Gamma}_{\text{th}}^*(1)$ and $\mathbf{P}_1^*$ are found via (12) and (6), respectively, which are one-to-one mappings from the dual variables $(\lambda_P^*, \lambda_D^*)$. Moreover, if we had an instantaneous power constraint $P_i(\gamma) \leq P_{\max}$, we could write down the Lagrangian and solve for $P_i(\gamma)$. The details are similar to the case without an instantaneous power constraint and are, thus, omitted for brevity. The reader is referred to [9] for a similar proof. The expression for $P_i^*(\gamma)$ is given by

$$P_i^*(\gamma) = \begin{cases} \left(\frac{1}{\lambda_P^*} - \frac{1}{\gamma}\right)^+, & \text{if } \frac{1}{\lambda_P^*} - \frac{1}{\gamma} < P_{\max} \\ P_{\max}, & \text{otherwise.} \end{cases} \tag{13}$$

Since the optimal primal variables are explicit functions of the optimal dual variables, once the optimal dual variables are found, the optimal primal variables are found, and the optimization problem is solved. We now discuss how to solve for these dual variables.

B. Solving for Dual Variables

The optimum dual variable λ_P^* must satisfy (9). Thus, if $\lambda_P^* > 0$, then we need $S_1^* - P_{\text{avg}} = 0$. This equation can be solved using any suitable root-finding algorithm. Hence, we propose Algorithm 1 that uses bisection [21]. In each iteration n, the algorithm calculates S_1^*, given that $\lambda_P = \lambda_P^{(n)}$, and, given some fixed λ_D, compares it to P_{avg} to update $\lambda_P^{(n+1)}$ accordingly. The algorithm terminates when $S_1^* = P_{\text{avg}}$, i.e., $\lambda_P^{(n)} = \lambda_P^*$. The superiority of this algorithm over the exhaustive search is due to the use of the bisection algorithm that does not go over all the search space of λ_P. For the bisection to converge, there must exist a single solution for equation $S_1^* = P_{\text{avg}}$. This is proven in Theorem 1.

Theorem 1: S_1^* is decreasing in $\lambda_P^* \in [0, \infty)$ given some fixed $\lambda_D^* \geq 0$. Moreover, the optimal value λ_P^* satisfying $S_1^* = P_{\text{avg}}$ is upper bounded by $\lambda_P^{\max} \triangleq \sum_{i=1}^{M} \theta_i c_i / P_{\text{avg}}$.

Proof: See Appendix B for the proof. □

We note that Algorithm 1 can be systematically modified to call any other root-finding algorithm (e.g., the secant algorithm [21] that converges faster than the bisection algorithm).

Algorithm 1 Finding λ_P^* given some λ_D

1: Initialize $n \leftarrow 1$, $\lambda_P^{\min} \leftarrow 0$, $\lambda_P^{\max} \leftarrow \sum_{i=1}^{M} \theta_i c_i / P_{\text{avg}}$, $\lambda_P^{(1)} \leftarrow (\lambda_P^{\min} + \lambda_P^{\max})/2$
2: **while** $|S_1^* - P_{\text{avg}}| > \epsilon$ **do**
3: Calculate S_1^* given that $\lambda_P^* = \lambda_P^{(n)}$. Call it $S^{(n)}$.
4: **if** $S^{(n)} - P_{\text{avg}} > 0$ **then**
5: $\lambda_P^{\min} = \lambda_P^{(n)}$
6: **else**
7: $\lambda_P^{\max} = \lambda_P^{(n)}$
8: **end if**
9: $\lambda_P^{(n+1)} \leftarrow (\lambda_P^{\min} + \lambda_P^{\max})/2$
10: $n \leftarrow n + 1$
11: **end while**
12: $\lambda_P^* \leftarrow \lambda_P^{(n)}$

Now, to search for λ_D^*, we state the following lemma.

Lemma 2: The optimum value λ_D^* that solves problem (4) satisfies $0 \leq \lambda_D^* < \lambda_D^{\max}$, where

$$\lambda_D^{\max} \triangleq \frac{c_1 \left[\log(t) - t + 1\right] + U_2^{\max}}{1 - p_2^{\max}} \tag{14}$$

with $t \triangleq (\min(\lambda_P^{\max}, \bar{F}_\gamma^{-1}(1/\theta_1 \bar{D}_{\max})))/(\bar{F}_\gamma^{-1}(1/\theta_1 \bar{D}_{\max}))$, and $U_2^{\max}$ is an upper bound on U_2^* and is given by $(\int_{\lambda_P^{\max}}^{\infty} \log(\gamma/\lambda_P^{\max}) f_\gamma(\gamma)\, d\gamma)(\sum_{i=2}^{M} \theta_i c_i)$, whereas $p_2^{\max}$ is an upper bound on p_2^* and is given by $\sum_{i=2}^{M} \prod_{j=2}^{i-1}(1-\theta_j)\theta_i$.

Proof: See Appendix C. □

Lemma 2 gives an upper bound on λ_D^*. This bound decreases the search space of λ_D^* drastically instead of searching over $\mathbb{R}$. Thus, the solution of problem (4) can be summarized in three steps: 1) Fix $\lambda_D^* \in [0, \lambda_D^{\max})$ and find the corresponding optimum λ_P^* using Algorithm 1. 2) Substitute the pair $(\lambda_P^*, \lambda_D^*)$ in (6) and (12) to get the power and threshold functions and then evaluate U_1^* from (2). 3) Repeat steps 1 and 2 for other values of λ_D^* until reaching the optimum λ_D^* that satisfies $p_1^* = 1/\bar{D}_{\max}$. If there are multiple λ_D^*'s satisfying $p_1^* = 1/\bar{D}_{\max}$, then the optimum value is that which gives the highest U_1^*.

Although the order by which the channels are sensed is assumed fixed, the proposed algorithm can be modified to optimize over the sensing order by a relatively low-complexity sorting algorithm. In particular, the dynamic programming proposed in [4] can be called by Algorithm 1 to order the channels. The complexity of the sorting algorithm alone is $O(2^M)$ compared with $O(M!)$ of the exhaustive search to sort the M channels. The modification to our proposed algorithm would be in step 3 of Algorithm 1, where S_1^* would be optimized over the number of channels (as well as $\boldsymbol{\Gamma}_{\text{th}}^*(1)$).

C. Optimality of the Proposed Solution

Since the problem in (4) is not proven to be convex, the KKT conditions provide only necessary conditions for optimality and need not be sufficient [22]. This means that there might exist multiple solutions (i.e., multiple solutions for the primal and/or dual variables) satisfying the KKT conditions, at least one of which is optimal. However, since Theorem 1 proves that there exists one unique solution to λ_P^* given λ_D^*, then $\boldsymbol{\Gamma}_{\text{th}}^*(1)$ and $\mathbf{P}_1^*$ are unique as well [from (6) and (12)] given some λ_D^*. Hence, by sweeping λ_D^* over $[0, \lambda_D^{\max})$, we have a unique solution satisfying the KKT conditions, which means that the KKT conditions are sufficient as well and that our approach is optimal for problem (4).

IV. Underlay System

In the overlay system, the SU tries to locate the free channels at each time slot to access these spectrum holes without interfering with the PUs. Recently, the Federal Communications Commission has allowed the SUs to interfere with the PU's network, as long as this interference does not harm the PUs [23]. If the interference from the SU measured at the PU's receiver is below the tolerable level, then the interference is deemed acceptable.

To model the interference at the PR, we assume that the SU uses a channel sensing technique that produces the sufficient statistic z_i at channel i [24], [25]. The SU is assumed to know the distribution of z_i given that channel i is free and busy, namely, $f_{z|b}(z_i|b_i = 0)$ and $f_{z|b}(z_i|b_i = 1)$, respectively. For brevity, we omit subscript i from b_i whenever it is clear from the context. The value of z_i indicates how confident the SU is in the presence of the PU at channel i. Thus, the SU stops at channel i according to how likely busy it is and how much data rate it will gain from this channel (i.e., according to z_i and γ_i, respectively). Hence, when the SU senses channel i to acquire z_i, the channel gain γ_i is probed and compared to some function $\gamma_{\text{th}}(i, z_i)$, if $\gamma_i \geq \gamma_{\text{th}}(i, z_i)$ transmission occurs on channel i; otherwise, channel i is skipped, and $i+1$ is sensed. Potentially, $\gamma_{\text{th}}(i, z_i)$ is a function in the statistic z_i. This means that, at channel i, for each possible value that z_i might take, there is a corresponding threshold $\gamma_{\text{th}}(i, z)$. Before formulating the problem, we note that this model can capture the overlay with a sensing error model as a special case where $f_{z|b}(z|b_i = 1) = (1 - P_{\text{MD}})\delta(z - z_{\text{b}}) + P_{\text{MD}}\delta(z - z_{\text{f}})$ while $f_{z|b}(z|b_i = 0) = P_{\text{FA}}\delta(z - z_{\text{b}}) + (1 - P_{\text{FA}})\delta(z - z_{\text{f}})$, where P_{MD} and P_{FA} are the probabilities of missed detection and false alarm, respectively, whereas $\delta(z)$ is the Dirac delta function, and z_b and z_f represent the values that z takes when the channel is busy and free, respectively. Hence, the interference constraint, which will soon be described, can be modified to a detection probability constraint and/or a false-alarm probability constraint.

The SU's expected throughput is given by $U_1(\mathbf{\Gamma}_{\text{th}}(1, z), \mathbf{P}_1)$, which can be recursively calculated from

$$\begin{aligned} &U_i(\mathbf{\Gamma}_{\text{th}}(i, z), \mathbf{P}_i) \\ &\quad = c_i \int_{-\infty}^{\infty} \int_{\gamma_{\text{th}}(i,z)}^{\infty} \log\left(1 + P_i(\gamma)\gamma\right) f_\gamma(\gamma)\, d\gamma f_z(z)\, dz \\ &\qquad + p_i^{\text{skip}} U_{i+1}(\mathbf{\Gamma}_{\text{th}}(i+1, z), \mathbf{P}_{i+1}), \quad i \in \mathcal{M} \end{aligned} \tag{15}$$

where $U_{M+1}(\mathbf{\Gamma}_{\text{th}}(M+1, z), \mathbf{P}_{M+1}) \triangleq 0$, $\mathbf{\Gamma}_{\text{th}}(i, z) \triangleq [\gamma_{\text{th}}(i, z), \ldots, \gamma_{\text{th}}(M, z)]^T$, $f_z(z) \triangleq \theta_i f_{z|b}(z|b_i = 0) + (1 - \theta_i) f_{z|b}(z|b_i = 1)$ is the pdf of the random variable z_i, and $p_i^{\text{skip}} \triangleq \int_{-\infty}^{\infty} \int_0^{\gamma_{\text{th}}(i,z)} f_\gamma(\gamma)\, d\gamma f_z(z)\, dz$. The first term in (15) is the SU's throughput at channel i averaged over all realizations of z_i and that of $\gamma_i \geq \gamma_{\text{th}}(i, z)$. The second term is the average throughput when the SU skips channel i due to its low gain. Moreover, let the average interference from the SU's transmitter to the PU's receiver, aggregated over all M channels, be $I_1(\mathbf{\Gamma}_{\text{th}}(1, z), \mathbf{P}_1)$. This represents the total interference affecting the PU's network due to the existence of the SU. The SU is responsible for guaranteeing that this interference does not exceed a threshold I_{avg} dictated by the PU's network. $I_1(\mathbf{\Gamma}_{\text{th}}(1, z), \mathbf{P}_1)$ can be derived using the following recursive formula:

$$\begin{aligned} &I_i\left(\mathbf{\Gamma}_{\text{th}}(i, z), \mathbf{P}_i\right) \\ &\quad = (1 - \theta_i)\, c_i \int_{-\infty}^{\infty} \int_{\gamma_{\text{th}}(i,z)}^{\infty} P_i(\gamma) f_\gamma(\gamma)\, d\gamma f_{z|b}(z|b_i = 1)\, dz \\ &\qquad + p_i^{\text{skip}} I_{i+1}(\mathbf{\Gamma}_{\text{th}}(i+1, z), \mathbf{P}_{i+1}), \quad i \in \mathcal{M} \end{aligned} \tag{16}$$

where $I_{M+1}(\mathbf{\Gamma}_{\text{th}}(M+1, z), \mathbf{P}_{M+1}) \triangleq 0$. This interference model is based on the assumption that the channel gain from the SU's transmitter to the PU's receiver is known at the SU's transmitter. This is the case for reciprocal channels when the PR acts as a transmitter and transmits training data to its intended primary transmitter (when it is acting as a receiver) [26]. The ST overhears these training data and estimates the channel from itself to the PR. Moreover, the gain at each channel from the ST to the PR is assumed unity for presentation simplicity. This could be easily extended to the case of nonunity gain by multiplying the first term in (16) by the gain from the ST to the PR at channel i. Finally, $p_1(\mathbf{\Gamma}_{\text{th}}(1, z))$ is the probability of a successful transmission in the current time slot and can be calculated using

$$\begin{aligned} p_i\left(\mathbf{\Gamma}_{\text{th}}(i, z)\right) = \int_{-\infty}^{\infty} \int_{\gamma_{\text{th}}(i,z)}^{\infty} f_\gamma(\gamma)\, d\gamma f_z(z)\, dz \\ + p_i^{\text{skip}} p_{i+1}\left(\mathbf{\Gamma}_{\text{th}}(i+1, z)\right) \end{aligned} \tag{17}$$

$i \in \mathcal{M}$, $p_{M+1}(\mathbf{\Gamma}_{\text{th}}(M+1, z)) \triangleq 0$. Given this background, the problem is

$$\begin{aligned} \text{maximize} \quad & U_1(\mathbf{\Gamma}_{\text{th}}(1, z), \mathbf{P}_1) \\ \text{subject to} \quad & I_1(\mathbf{\Gamma}_{\text{th}}(1, z), \mathbf{P}_1) \leq I_{\text{avg}} \\ & p_1\left(\mathbf{\Gamma}_{\text{th}}(1, z)\right) \geq \frac{1}{\bar{D}_{\max}} \\ \text{variables} \quad & \mathbf{\Gamma}_{\text{th}}(1, z), \mathbf{P}_1. \end{aligned} \tag{18}$$

Let λ_I and λ_D be the Lagrange multipliers associated with the interference and delay constraints of problem (18), respectively. Problem (18) is more challenging compared with the overlay case. This is because, unlike those in (4), the thresholds in (18) are functions rather than constants. The KKT conditions for (18) are given by

$$\begin{aligned} &P_i^*(\gamma) \\ &\quad = \left(\frac{1}{\lambda_I^* \Pr[b_i = 1|z]} - \frac{1}{\gamma}\right)^+, \quad i \in \mathcal{M}. \end{aligned} \tag{19}$$

$$\begin{aligned} &\gamma_{\text{th}}^*(i, z) \\ &\quad = \frac{-\lambda_I^* \Pr\left[b_i = 1|z\right]}{W_0\!\left(-\exp\!\left(-\frac{\left(U_{i+1}^* - \lambda_I^* I_{i+1}^* - \lambda_D^*(1 - p_{i+1}^*)\right)^+}{c_i} - 1\right)\right)}, \quad i \in \mathcal{M} \end{aligned} \tag{20}$$

in addition to the primal feasibility, dual feasibility, and the complementary slackness equations given in (8)–(10), where $U_{i+1}^* \triangleq U_1(\boldsymbol{\Gamma}_{\text{th}}^*(1,z), P_1^*(\gamma))$, $I_{i+1}^* \triangleq I_1(\boldsymbol{\Gamma}_{\text{th}}^*(1,z), P_1^*(\gamma))$, and $p_{i+1}^* \triangleq p_1(\boldsymbol{\Gamma}_{\text{th}}^*(1,z))$, whereas $\Pr[b_i=1|z]$ is the conditional probability that channel i is busy given z_i and is given by

$$\Pr[b_i=1|z] = \frac{(1-\theta_i)\, f_{z|b}(z|b_i=1)}{f_z(z)}. \tag{21}$$

Note that $P_i^*(\gamma)$ is increasing in γ and is upper bounded by the term $1/(\lambda_I^* \Pr[b_i=1|z])$. Hence, as $\Pr[b_i=1|z]$ increases, the SU's maximum power becomes more limited, i.e., the maximum power decreases. This is because the PU is more likely to be occupying channel i. Thus, the power transmitted from the SU should decrease to protect the PU.

Algorithm 1 can also be used to find λ_I^*. Only a single modification is required in the algorithm, which is that S_1^* would be replaced by I_1^*. Thus, the solution of problem (18) can be summarized in three steps: 1) Fix $\lambda_D^* \in \mathbb{R}^+$ and find the corresponding optimum λ_I^* using the modified version of Algorithm 1. 2) Substitute the pair $(\lambda_I^*, \lambda_D^*)$ in (19) and (20) to get the power and threshold functions and then evaluate U_1^* from (15). 3) Repeat steps 1 and 2 for other values of λ_D^* until reaching the optimum λ_D^* that satisfies $p_1^* = 1/\bar{D}_{\max}$, and if there are multiple λ_D^*'s satisfying $p_1^* = 1/\bar{D}_{\max}$, then the optimum value is that which gives the highest U_1^*. This approach yields the optimal solution. Next, Theorem 2 asserts the monotonicity of I_1^* in λ_I^*, which allows using the bisection to find λ_I^* given some fixed λ_D^*.

Theorem 2: I_1^* is decreasing in $\lambda_I^* \in [0,\infty)$ given some fixed $\lambda_D^* \geq 0$.

Proof: We differentiate I_1^* with respect to λ_I^* given that $P_i^*(\gamma)$ and $\gamma_{\text{th}}^*(i,z)$ are given by (19) and (20), respectively, and then show that this derivative is negative. The proof is omitted since it follows the same lines in Theorem 1. $\square$

Although the interference power constraint is sufficient for the problem to prevent the power functions from going to infinity, in some applications, one may have an additional power constraint on the SUs. Hence, problem (18) can be modified to introduce an average power constraint that is given by $S_1(\boldsymbol{\Gamma}_{\text{th}}(1,z), \mathbf{P}_1) \leq P_{\text{avg}}$, where

$$S_i(\boldsymbol{\Gamma}_{\text{th}}(i,z), \mathbf{P}_i) = c_i \int_{-\infty}^{\infty} \int_{\gamma_{\text{th}}(i,z)}^{\infty} P_i(\gamma) f_\gamma(\gamma)\, d\gamma f_z(z)\, dz + p_i^{\text{skip}} S_{i+1}(\boldsymbol{\Gamma}_{\text{th}}(i+1,z), \mathbf{P}_{i+1}). \tag{22}$$

It can be easily shown that the solution to the modified problem is similar to that presented in (19) and (20), which is

$$P_i^*(\gamma) = \left(\frac{1}{\lambda_P^* + \lambda_I^* \Pr[b_i=1|z]} - \frac{1}{\gamma}\right)^+ \tag{23}$$

$$\gamma_{\text{th}}^*(i,z) = \frac{-(\lambda_P^* + \lambda_I^* \Pr[b_i=1|z])}{W_0\left(-\exp\left(-\frac{\left(U_{i+1}^* - \lambda_I^* I_{i+1}^* - \lambda_P^* S_{i+1}^* - \lambda_D^*(1-p_{i+1}^*)\right)^+}{c_i} - 1\right)\right)} \tag{24}$$

$\forall i \in \mathcal{M}$, where $S_i^* \triangleq S_i(\boldsymbol{\Gamma}_{\text{th}}^*(i,z), P_i^*(\gamma))$. This solution is more general since it takes into account both the average interference and the average power constraint apart from the delay constraint. Moreover, it allows for the case where the power constraint is inactive, which happens if the PU has a strict average interference constraint. In this case, the optimum solution would result in $\lambda_P^* = 0$, making (23) and (24) identical to (19) and (20), respectively.

V. Multiple Secondary Users

Here, we show how our single-SU framework can be extended to multiple SUs in a multiuser diversity framework without increase in the complexity of the algorithm. We will show that when the number of SUs is high, with slight modifications to the definitions of the throughput, power, and probability of success, the single-SU optimization problem in (4) [or (18)] can capture the multi-SU scenario. Moreover, the proposed solution for the overlay model still works for the multi-SU scenario. Finally, at the end of this section, we will show that the proposed algorithm provides a throughput-optimal and delay-optimal solution with even lower complexity for finding the thresholds compared with the single-SU case if the number of SUs is large.

Consider a CR network with L SUs associated with a centralized secondary base station (BS) in a downlink overlay scenario. Before describing the system model, we would like to note that when we say that channel i will be sensed, this means that each user will independently sense channel i and feed the sensing outcome back to the BS to make a global decision. Although we neglect sensing errors in this section, the analysis will work similarly in the presence of sensing errors by using the underlay model. At the beginning of each time slot, the L SUs sense channel 1. If it is free, each SU observes it free with no errors and probes the instantaneous channel gain and feeds it back to the BS. The BS compares the maximum received channel gain among the L received channel gains to $\gamma_{\text{th}}(1)$. Channel 1 is assigned to the user having the maximum channel gain if this maximum gain is higher than $\gamma_{\text{th}}(1)$, whereas the remaining $L-1$ users continue to sense channel 2. On the other hand, if the maximum channel gain is less than $\gamma_{\text{th}}(1)$, channel 1 is skipped, and channel 2 is sensed by all L users. Unlike the case in the single-SU scenario where only a single channel is claimed per time slot, in this multi-SU system, the BS can allocate more than one channel in one time slot such that each SU is not allocated more than one channel and each channel is not allocated to more than one SU. Based on this scheme, the expected per-SU throughput U_1^L is calculated from

$$U_i^l = \frac{\theta_i c_i}{l} \int_{\gamma_{\text{th}}(i)}^{\infty} \log(1 + P_i(\gamma)\gamma)\, f_l(\gamma)\, d\gamma + \theta_i \bar{F}_l(\gamma_{\text{th}}(i)) \left(1 - \frac{1}{l}\right) U_{i+1}^{l-1} + \left(1 - \theta_i \bar{F}_l(\gamma_{\text{th}}(i))\right) U_{i+1}^l \tag{25}$$

$i \in \mathcal{M}$ and $l \in \{L-i+1, \ldots, L\}$ with initialization $U_{M+1}^l = 0$. Here, $f_l(\gamma)$ represents the density of the maximum gain among l i.i.d. users' gains, whereas $\bar{F}_l(\gamma)$ is its complementary cumulative distribution function. We study the case where $L \gg M$; thus, when a channel is allocated to a user, we can assume that the remaining number of users is still L. Thus, we approximate l with L $\forall l \in \{L-i,\ldots,L\}$ and $\forall i \in \mathcal{M}$. Similar to the throughput derived in (25), we could write the exact expressions for the per-SU average power and per-SU probability of transmission. Furthermore, since $L \gg M$, we can approximate S_i^l with S_i^L and p_i^l with p_i^L, $\forall l \in \{L-i+1,\ldots,L\}$ and $\forall i \in \mathcal{M}$. The per-SU expected throughput U_1^L, the average power S_1^L, and the probability of transmission p_1^L can be derived from

$$U_i^L(\mathbf{\Gamma}_{\rm th}(i),\mathbf{P}_i) = \frac{\theta_i c_i}{L}\int_{\gamma_{\rm th}(i)}^{\infty}\log\left(1+P_i(\gamma)\gamma\right) f_L(\gamma)\,d\gamma + \left[1-\frac{\theta_i \bar{F}_L\left(\gamma_{\rm th}(i)\right)}{L}\right] U_{i+1}^L\left(\mathbf{\Gamma}_{\rm th}(i+1),\mathbf{P}_{i+1}\right) \quad (26)$$

$$S_i^L(\mathbf{\Gamma}_{\rm th}(i),\mathbf{P}_i) = \frac{\theta_i c_i}{L}\int_{\gamma_{\rm th}(i)}^{\infty} P_i(\gamma) f_L(\gamma)\,d\gamma + \left[1-\frac{\theta_i \bar{F}_L\left(\gamma_{\rm th}(i)\right)}{L}\right] S_{i+1}^L\left(\mathbf{\Gamma}_{\rm th}(i+1),\mathbf{P}_{i+1}\right) \quad (27)$$

$$p_i^L\left(\mathbf{\Gamma}_{\rm th}(i)\right) = \frac{\theta_i}{L}\bar{F}_L\left(\gamma_{\rm th}(i)\right) + \left[1-\frac{\theta_i \bar{F}_L(\gamma_{\rm th}(i))}{L}\right] p_{i+1}^L\left(\mathbf{\Gamma}_{\rm th}(i+1)\right) \quad (28)$$

$i \in \mathcal{M}$, respectively, with $U_{M+1}^L = S_{M+1}^L = p_{M+1}^L = 0$. To formulate the multi-SU optimization problem, we replace U_1, S_1, and p_1 in (4) with U_1^L, S_1^L, and p_1^L derived in (26)–(28), respectively. Taking the Lagrangian and following the same procedure in Section III, we arrive at the solution for P_i^* and $\gamma_{\rm th}^*(i)$, as given by (6) and (12), respectively. Hence, (6) and (12) represent the optimal solution for the multi-SU scenario. The details are omitted since they follow those of the single-SU case discussed in Section III.

Next, we show that this solution is optimal with respect to the delay as well as the throughput when L is large. We show this by studying the system after ignoring the delay constraint and show that the resulting solution of this system (which is what we refer to as the unconstrained problem) is also delay optimal. The solution of the unconstrained problem is given by setting $\lambda_D^* = 0$ in (12), arriving at

$$\gamma_{\rm th}^*(i)|_{\lambda_D^*=0} = \frac{-\lambda_P^*}{W_0\left(-\exp\left(-\frac{\left(U_{i+1}^{L*}-\lambda_P^* S_{i+1}^{L*}\right)^+}{c_i}-1\right)\right)} \quad (29)$$

$\forall i \in \mathcal{M}$. As the number of SUs increases, the per-user expected throughput U_1^L decreases, since these users share the total throughput. Moreover, U_i^L decreases as well $\forall i \in \mathcal{M}$ decreasing the value of $\gamma_{\rm th}^*(i)$ [from (29)] until reaching its minimum (i.e., $\gamma_{\rm th}^*(i) = \lambda_P^*$; the right-hand side of (29) is minimum when its denominator is as much negative as possible, that is, when $W_0(x) = -1$ since $W_0(x) \geq -1$, $\forall x \in \mathbb{R}$) as $L \to \infty$. From (28), it can be easily shown that $p_1^L(\mathbf{\Gamma}_{\rm th}(1))$ is monotonically decreasing in $\gamma_{\rm th}(i)$ $\forall i \in \mathcal{M}$. Thus, the minimum possible average delay (corresponding to the maximum $p_1^L(\mathbf{\Gamma}_{\rm th}(1))$) occurs when $\gamma_{\rm th}(i)$ is at its minimum possible value for all $i \in \mathcal{M}$. Consequently, having $\gamma_{\rm th}^*(i) = \lambda_P^*$ means that the system is at the optimum delay point. That is, the unconstrained problem cannot achieve any smaller delay with an additional delay constraint. Hence, the multi-SU problem, which is formulated by adding a delay constraint to the unconstrained problem, achieves the optimum delay performance when L is asymptotically large.

Recall that the overall complexity of the solution for the single-SU case is due to three factors: 1) evaluating the Lambert W function in Algorithm 1; 2) the bisection algorithm in Algorithm 1; and 3) the search over λ_D. On the other hand, the complexity of the solution for the multi-SU case asymptotically decreases (as $L \to \infty$). This is because of two reasons: 1) When $L \gg M$, $\gamma_{\rm th}^*(i) \to \lambda_P^* \forall i \in \mathcal{M}$. This means that we will not have to evaluate the Lambert W function in (12), but instead, we set $\gamma_{\rm th}^*(i) = \lambda_P^*$, since $L \gg M$. 2) When $\gamma_{\rm th}^*(i) = \lambda_P^*$, there will be no need to find λ_D^* since the delay is minimum (we recall that in the single-SU case, we need to calculate λ_D^* to substitute it in (12) to evaluate $\gamma_{\rm th}^*(i)$, but in the multi-SU case, $\gamma_{\rm th}^*(i) = \lambda_P^*$).

VI. Generalization of Deadline Constraints

In the overlay and underlay schemes discussed thus far, we were assuming that each packet has a hard deadline of one time slot. If a packet is not delivered as soon as it arrives at the ST, then it is dropped from the system. However, in real-time applications, data arrive at the ST's buffer on a frame-by-frame structure. This means that multiple packets (constituting the same frame) arrive simultaneously rather than one at a time. A frame consists of a fixed number of packets, and each packet fits into exactly one time slot of duration T_s. Each frame has its own deadline, and thus, packets belonging to the same frame have the same deadline [27]. This deadline represents the maximum number of time slots by which the packets belonging to the same frame need to be transmitted, on average.

Here, we solve this problem for the overlay scenario. The solution presented in Section III can be thought of as a special case of the problem presented in this section, where the deadline was equal to one time slot, and each frame consists of one packet. We show that the solution presented in Section III can be used to solve this generalized problem in an offline fashion (i.e., before attempting to transmit any packet of the frame). Moreover, we propose an online update algorithm that updates the thresholds and power functions in each time slot and show that this outperforms the offline solution.

A. Offline Solution

Assume that each frame consists of K packets and that the entire frame has a deadline of t_f time slots ($t_f > K$). If the SU does not succeed in transmitting the K packets before the t_f time slots, then the whole frame is considered wasted. Since instantaneous channel gains and PU's activities are independent across time slots, the probability that the SU succeeds in transmitting the frame in t_f time slots or less is given by

$$P_{\text{frame}}(K, t_f) = \sum_{n=K}^{t_f} \binom{t_f}{n} p^n (1-p)^{t_f - n} \tag{30}$$

where p is the probability of transmitting a packet on some channel in a single time slot and is given by (3) or (17) if the SU's system was overlay or underlay, respectively. $P_{\text{frame}}(K, t_f)$ represents the probability of finding K or more free time slots out of a total of t_f time slots.

To guarantee some QoS for the real-time data, the SU needs to keep the probability of successful frame transmission above a minimum value denoted $r_{\min}$, that is $P_{\text{frame}} \geq r_{\min}$. Hence, the problem becomes a throughput-maximization problem subject to some average power and QoS constraints, as follows:

$$\begin{aligned} &\text{maximize} && U_1(\boldsymbol{\Gamma}_{\text{th}}(1), \mathbf{P}_1) \\ &\text{subject to} && S_1(\boldsymbol{\Gamma}_{\text{th}}(1), \mathbf{P}_1) \leq P_{\text{avg}} \\ &&& P_{\text{frame}}(K, t_f) \geq r_{\min} \\ &\text{variables} && \boldsymbol{\Gamma}_{\text{th}}(1), \mathbf{P}_1. \end{aligned} \tag{31}$$

This is the optimization problem assuming an overlay system since we used (2) and (1) for throughput and power, respectively. It can also be systematically modified to the case of an underlay system. Since there exists a one-to-one mapping between $P_{\text{frame}}(K, t_f)$ and p, then there exists a value for $\bar{D}_{\max}$ such that the inequality $p \geq 1/\bar{D}_{\max}$ is equivalent to the QoS inequality $P_{\text{frame}}(K, t_f) \geq r_{\min}$. That is, we can replace inequality $P_{\text{frame}}(K, t_f) \geq r_{\min}$ by $p \geq 1/\bar{D}_{\max}$ for some $\bar{D}_{\max}$ that depends on $r_{\min}$, K, and t_f that are known *a priori*. Consequently, problem (31) is reduced to the simpler, yet equivalent, single-time-slot problem (4), and the SU can solve for $\mathbf{P}_1^*$ and $\boldsymbol{\Gamma}_{\text{th}}^*(1)$ vectors following the approach proposed in Section III. The SU solves this problem offline (i.e., before the beginning of the frame transmission) and uses this solution each time slot of the t_f time slots. With this offline scheme, the SU will be able to meet the QoS and the average power constraint requirements, as well as maximize its throughput.

B. Online Power-and-Threshold Adaptation

In problem (4), we have seen that as $1/\bar{D}_{\max}$ decreases, the system becomes less stringent in terms of the delay constraint. This results in an increase in the average throughput U_1^*. With this in mind, let us assume, in the generalized delay model, that, at time slot 1, the SU succeeds in transmitting a packet. Thus, at time slot 2, the SU has $K-1$ remaining packets to be transmitted in $t_f - 1$ time slots. Moreover, from the properties of (30), $P_{\text{frame}}(K-1, t_f-1) > P_{\text{frame}}(K, t_f)$. This means that the system becomes less stringent in terms of the QoS constraint after a successful packet transmission. This advantage appears in the form of higher throughput. To see how we can make use of this advantage, define $P_{\text{frame}}(K(t), t_f - t + 1)$ as

$$\begin{aligned} &P_{\text{frame}}\left(K(t), t_f - t + 1\right) \\ &\quad = \sum_{n=K(t)}^{t_f - t + 1} \binom{t_f - t + 1}{n} \left(p(t)\right)^n \left(1 - p(t)\right)^{t_f - t + 1 - n} \end{aligned} \tag{32}$$

where $K(t)$ is the remaining number of packets before time slot $t \in \{1, \ldots, t_f\}$, and $p(t)$ is the probability of successful transmission at time slot t. At each time slot $t \in \{1, \ldots t_f\}$, the SU modifies the QoS constraint to be $P_{\text{frame}}(K(t), t_f - t + 1) \geq r_{\min}$ instead of $P_{\text{frame}}(K, t_f) \geq r_{\min}$, which was used in the offline adaptation, and solves the following problem:

$$\begin{aligned} &\text{maximize} && U_1\left(\boldsymbol{\Gamma}_{\text{th}}(1), \mathbf{P}_1\right) \\ &\text{subject to} && S_1\left(\boldsymbol{\Gamma}_{\text{th}}(1), \mathbf{P}_1\right) \leq P_{\text{avg}} \\ &&& P_{\text{frame}}\left(K(t), t_f - t + 1\right) \geq r_{\min} \\ &\text{variables} && \boldsymbol{\Gamma}_{\text{th}}(1), \mathbf{P}_1 \end{aligned} \tag{33}$$

to obtain the power and threshold vectors. When the delay constraint in (33) is replaced by its equivalent constraint $p \geq 1/\bar{D}_{\max}$, the resulting problem can be solved using the overlay approach proposed in Section III without much increase in computational complexity since the power functions and thresholds are given in closed-form expressions. With this online adaptation, the average throughput U_1^* increases while still satisfying the QoS constraint.

VII. Numerical Results

We show the performance of the proposed solution for the overlay and underlay scenarios. The slot duration is taken to be unity (i.e., all time measurements are taken relative to the time-slot duration), whereas $\tau = 0.05T_{\text{s}}$. Here, we use $M = 10$ channels that suffer i.i.d. Rayleigh fading. The availability probability is taken as $\theta_i = 0.05i$ throughout the simulations. The power gain γ is exponentially distributed as $f_\gamma(\gamma) = \exp(\gamma/\bar{\gamma})/\bar{\gamma}$, where $\bar{\gamma}$ is the average channel gain and is set to be 1, unless otherwise specified.

Fig. 2 plots the expected throughput U_1^* for the overlay scenario after solving problem (4). U_1^* is plotted using (2), which represents the average number of bits transmitted divided by the average time required to transmit those bits, taking into account the time wasted due to the blocked time slots. We plot U_1^* with $\bar{D}_{\max} = 1.02T_{\text{s}}$ and with $\bar{D}_{\max} = \infty$ (i.e., neglecting the delay constraint). We refer to the former problem as a constrained problem and to the latter as an unconstrained problem. We also compare the performance to the nonoptimum-stopping-rule case (No-OSR), where the SU transmits at the first available channel. We expect the No-OSR case to have the best delay and the worst throughput performances. We can see

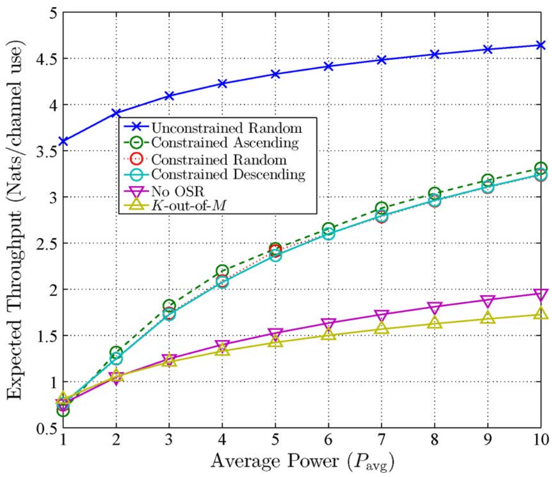

Fig. 2. Expected throughput for the overlay scenario for four cases: 1) **Proposed constrained problem**: with average delay constraint for three channel ordering possibilities (ascending ordering of channel availability probabilities, descending ordering, and random ordering); 2) **unconstrained problem** that ignores the delay constraint; 3) **nonoptimum stopping rule (No-OSR)**, where the SU transmits at the first free channel; and 4) **K-out-of-M scheme**, where the SU assumes that the system has only $K = 5$ channels and ignores the remaining $M - K$ channels.

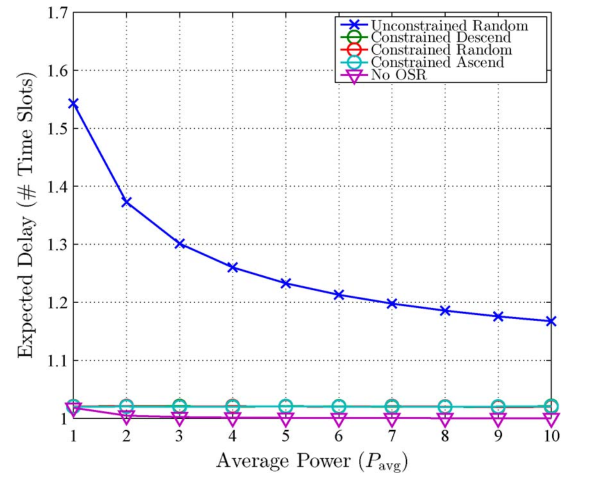

Fig. 3. Expected delay for the overlay scenario for problem (4). The unconstrained problem can suffer an arbitrary high delay. The constrained problem has a guaranteed average delay for all ordering strategies. The No-OSR scenario, on the other hand, has the best delay performance since the SU uses the first free channel.

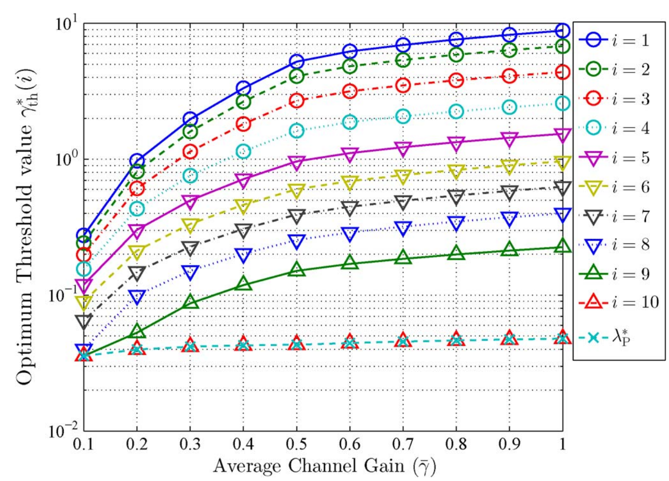

Fig. 4. Gap between the optimum threshold $\gamma^*_{\text{th}}(i)$ and its minimum value λ^*_P increases as the average gain increases. This is because as $\bar{\gamma}$ increases, U_{i+1} increases as well. Hence, $\gamma^*_{\text{th}}(i)$ increases so that only sufficiently high instantaneous gains should lead to stopping at channel i.

that the unconstrained problem has the best throughput among all constrained problems.

Examining the constrained problem for different sensing orders of the channels, we observe that when the channels are sorted in an ascending order of θ_i, the throughput is higher. This is because a channel i has a higher chance of being skipped if put at the beginning of the order compared with the case if put at the end of the order. This is a property of the problem no matter how the channels are ordered, i.e., this property holds even if all channels have equal values of θ_i. Hence, it is more favorable to put the high-quality channels at the end of the sensing order so that they are not put in a position of being frequently skipped. However, this is not necessarily optimum order, which is out of the scope of this work and is left as a future work for this delay-constrained optimization problem.

We also plot the expected throughput of a simple stopping rule that we call the K-out-of-M scheme, where we choose the highest K channels in availability probability and ignore the remaining channels as if they do not exist in the system. The SU senses those K channels sequentially; probes the gain of each free channel, if any; and transmits on the channel with the highest gain. This scheme has a constant fraction $K\tau/T_{\text{s}}$ of time wasted each slot. However, it has the advantage of choosing the best channel among multiple available channels. In Fig. 2, we can see that the degradation of the throughput when $K = 5$ compared with the optimal stopping rule scheme. The reason is twofold: 1) due to the constant wasted fraction of time and 2) ignoring the remaining channels that could potentially be free with a high gain if they were considered, as opposed to the constrained problem.

The delay is shown in Fig. 3 for both the constrained and unconstrained problems. We see that the unconstrained problem suffers around 6% increase in the delay, at $P_{\text{avg}} = 10$, compared with the constrained problem.

Studying the system performance under low average channel gain is essential. A low average channel gain represents an SU's channel being in a permanent deep fade or if there is a relatively high interference level at the SR. Fig. 4 shows $\gamma^*_{\text{th}}(i)$ versus $\bar{\gamma}$. At low $\bar{\gamma}$, the throughput is expected to be small; hence, $\gamma^*_{\text{th}}(i)$ is close to its minimum value λ^*_P so that even if γ_i is relatively small, i should not be skipped. In other words, at low average channel gain, the expected throughput is small; thus, a relatively low instantaneous gain will be satisfactory for stopping at channel i. While when the average channel gain increases, $\gamma^*_{\text{th}}(i)$ should increase so that only high instantaneous gains should lead to stopping at channel i. In both cases, i.e., high and low $\bar{\gamma}$, there is still a tradeoff between choosing a high versus a low value of $\gamma^*_{\text{th}}(i)$.

The sensing channel (i.e., the channel between the PT and the ST over which the ST overhears the PT activity) is modeled

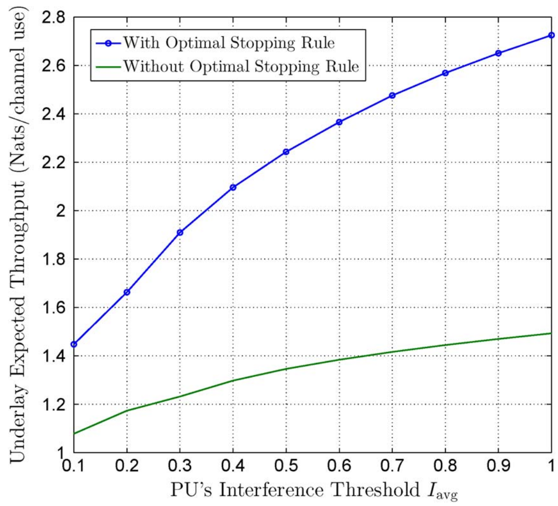

Fig. 5. Underlay expected throughput versus the average interference threshold I_{avg}. Two scenarios are shown: with and without the optimal stopping rule formulation. In the latter, the SU transmits as soon as a channel is found free.

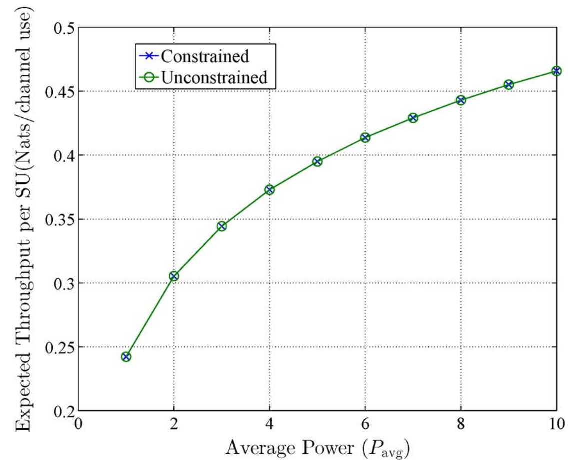

Fig. 6. Per-user throughput of the system at $L = 30$ SUs. The throughput values of the constrained and unconstrained problems coincide since the system is throughput (and delay) optimal.

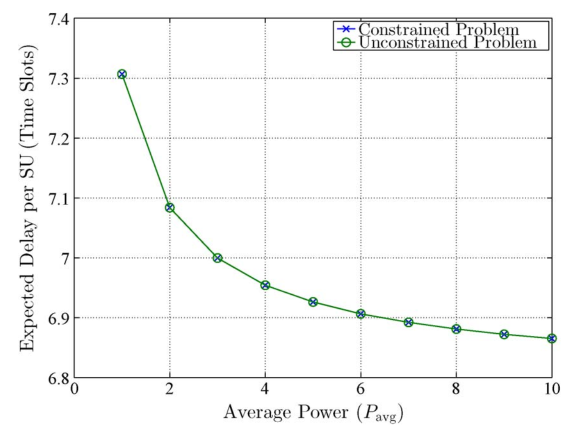

Fig. 7. Average delay seen by each user in the system at $L = 30$ SUs. The delay values of the constrained and unconstrained problems coincide since the system is delay (and throughput) optimal.

as additive white Gaussian noise with unit variance. The distributions of the energy detector output z (average energy of N samples sampled from this sensing channel) under the free and busy hypotheses are the chi-square and the noncentral chi-square, which are given by

$$f_{z|b}(z|b_i = 0) = \left(\frac{N}{\sigma^2}\right)^N \frac{z^{N-1}}{(N-1)!} \exp\left(\frac{-Nz}{\sigma^2}\right) \tag{34}$$

$$f_{z|b}(z|b_i = 1) = \left(\frac{N}{\sigma^2}\right)\left(\frac{z}{\mathcal{E}}\right)^{\frac{N-1}{2}} \exp\left(\frac{-N(z+\mathcal{E})}{\sigma^2}\right) I_{N-1}^{\text{Bes}} \times \left(\frac{2N\sqrt{\mathcal{E}z}}{\sigma^2}\right) \tag{35}$$

where σ^2, which is set to 1, is the variance of the Gaussian noise of the energy detector, $\mathcal{E}$ is the amount of energy received by the ST due to the activity of the PT and is taken as $\mathcal{E} = 2\sigma^2$ throughout the simulations, whereas $I_i^{\text{Bes}}(x)$ is the modified Bessel function of the first kind and ith order, and $N = 10$.

The main problem we are addressing in this paper is the optimal stopping rule that dictates the SU when to stop sensing and when to start transmitting. As we have seen, this is identified by the threshold vector $\mathbf{\Gamma}_{\text{th}}^*(1, z)$. If the SU does not consider the optimal stopping rule problem and rather transmits as soon as it detects a free channel, then it will be wasting future opportunities of possibly higher throughput. Hence, we expect degradation in the throughput. We plot the two scenarios in Fig. 5 for the underlay system with no delay constraint.

For the multi-SU scenario, numerical analysis was run for the case of $L = 30$ SUs and $M = 10$ channels. We assumed that the fading channels are i.i.d. among users and among frequency channels. Each channel is exponentially distributed with unity average channel gain. Moreover, since L is large, the distribution of the maximum gain among L random gains converges in distribution to the Gumbel distribution [28] having a cumulative distribution function of $\exp(-\exp(-\gamma/\bar{\gamma}))$. The per-user throughput U_1^{L*} is plotted in Fig. 6, where the throughput values of the delay-constrained and the unconstrained optimization problems coincide. This is because when $L \gg M$, the solution of the unconstrained problem is delay optimal as well. Hence, adding a delay constraint does not sacrifice the throughput when L is large. Moreover, the delay performance shown in Fig. 7 shows that the delay does not change with and without considering the average delay constraint since the system is already delay (and throughput) optimal.

We have simulated the system for the online algorithm in Section VI for $K(1) = 2$ packets and $t_f = 4$ time slots. We simulated the system at $r_{\min} = 0.95$, which means that the QoS of the SU requires that at least 95% of the frames be successfully transmitted. Fig. 8 shows the improvement in the throughput of the online over the offline adaptation. This is because the SU adapts the power and thresholds at each time slot to allocate the remaining resources (i.e., remaining time slots) according to the remaining number of packets and the desired QoS. This comes at the expense of resolving the problem at each time slot (i.e., t_f times more).

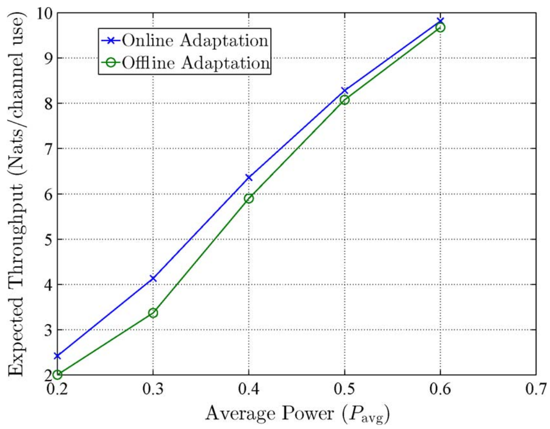

Fig. 8. Performance of the online adaptation algorithm for the general delay case.

VIII. Conclusion and Discussions

In this paper, we have formulated the problem of a CR system having a single SU that sequentially tests M potentially available channels, originally licensed to the PU's network, to transmit delay-constrained data. The unique challenge with delay-constrained data is by which each packet has a deadline that needs to be transmitted, on average. Thus, there is a tradeoff in either taking advantage of a free channel to transmit a packet but with low throughput at the current time or waiting for a future channel that has a considerably higher gain but might be busy. The sequential nature of the problem gave rise to an optimal stopping rule formulation for both overlay and underlay.

In the overlay scheme, the SU was allowed to transmit on free channels only. We have seen that the optimal power control strategy is solved by a modified version of the waterfilling algorithm that takes sequential multiple channels into account. Moreover, the solution of the optimal stopping rule was explicitly given via a set of equations obtained in closed-form expressions. These equations, although derived in a single-SU scenario, are shown to be valid in the multi-SU scenario as well.

In the underlay scheme, on the other hand, the SU is allowed to transmit on any channel even if it was busy as long as the average interference to the PU is tolerable. The solution to the underlay problem involved thresholds that were functions of the sensing instantaneous sufficient statistic. We have provided the optimal closed-form expressions for these thresholds and showed how they depend on the distribution of this sufficient statistic or, more precisely, on the probability of the channel being busy, given this sufficient statistic.

We also discussed the extension of our solution to multiple SUs. We showed that the proposed algorithm can apply to a multi-SU system when the number of SUs is sufficiently larger than the number of channels. Moreover, the optimum solution was found to be throughput optimal and delay optimal at the same time. Our algorithm can reach this solution with smaller complexity relative to the single-SU case.

Finally, our formulations for both the overlay and the underlay incorporated the average delay that the SU's packets experience before being transmitted. We showed that when the average delay is constrained in the optimization problem, we could achieve a relatively low packet delay compared with the delay-unconstrained problem. Then, we generalized the problem to consider packets arriving simultaneously and having the same deadline to model typical data. A low-complexity online power-and-threshold adaptation solution was proposed, and simulation results showed its performance superiority over the offline solution.

While the problem of finding the optimal sensing order of the channels is outside the scope of this work, one could still rely on previous work that addressed this problem. The work in [4] proposes a dynamic programming algorithm for this problem but without any delay constraints and, moreover, while fixing $P_i(\gamma) = 1 \; \forall i \in \mathcal{M}$. Based on the closed-form expressions of the proposed approach for the threshold and power functions, one could still find the optimum sensing sequence using this dynamic programming algorithm, given some fixed λ_P and λ_D (as mentioned in Section III-B). However, finding the optimum $(\lambda_P^*, \lambda_D^*)$ is still an open question. This is because the monotonicity of S_1^* in λ_P^* is not proven when the sensing order is a variable in the problem. Hence, the use of the bisection method in Algorithm 1 is not guaranteed to be optimal.

Appendix A
Proof of Lemma 1

We carry out the proof by contradiction. Assume, for some i, that $\gamma_{\text{th}}^*(i) < \lambda_P^*$. Thus, the reward starting from channel i, $U_i([\gamma_{\text{th}}^*(i), \gamma_{\text{th}}^*(i+1), \ldots, \gamma_{\text{th}}^*(M)]^T, \mathbf{P}_i^*)$ becomes

$$\theta_i c_i \int_{\gamma_{\text{th}}^*(i)}^{\infty} \log(1 + P_i^* \gamma) f_\gamma(\gamma)\, d\gamma + \theta_i U_{i+1}^* \int_0^{\gamma_{\text{th}}^*(i)} f_\gamma(\gamma)\, d\gamma + (1-\theta_i) U_{i+1}^* \tag{36}$$

$$\leq \theta_i c_i \int_{\lambda_P^*}^{\infty} \log\left(1 + P_i^* \gamma\right) f_\gamma(\gamma)\, d\gamma + \theta_i U_{i+1}^* \int_0^{\lambda_P^*} f_\gamma(\gamma)\, d\gamma + (1-\theta_i) U_{i+1}^* \tag{37}$$

$$= U_i\left([\lambda_P^*, \gamma_{\text{th}}^*(i+1), \ldots, \gamma_{\text{th}}^*(M)]^T, \mathbf{P}_i^*\right). \tag{38}$$

Inequality (37) follows by adding the term $\theta_i(\int_{\gamma_{\text{th}}^*(i)}^{\lambda_P^*} f_\gamma(\gamma)\, d\gamma) U_{i+1}^*$ to (36), whereas (38) follows by the definition of the right-hand side of (37). Using (2), we can calculate the reward U_{i-1} for both the left- and right-hand sides of the previous inequality. Thus, the following inequality holds:

$$U_{i-1}\left([\gamma_{\text{th}}^*(i-1), \gamma_{\text{th}}^*(i), \ldots, \gamma_{\text{th}}^*(M)]^T, \mathbf{P}_{i-1}^*\right) \leq U_{i-1}\left([\gamma_{\text{th}}^*(i-1), \lambda_P^*, \ldots, \gamma_{\text{th}}^*(M)]^T, \mathbf{P}_{i-1}^*\right). \tag{39}$$

Carrying out the last step recursively by $i-2$ more times, we find the following relation:

$$U_1\left([\gamma_{\text{th}}^*(1), \ldots, \gamma_{\text{th}}^*(i-1), \gamma_{\text{th}}^*(i), \ldots, \gamma_{\text{th}}^*(M)]^T, \mathbf{P}_1^*\right) \leq U_1\left([\gamma_{\text{th}}^*(1), \ldots, \gamma_{\text{th}}^*(i-1), \lambda_P^*, \ldots, \gamma_{\text{th}}^*(M)]^T, \mathbf{P}_1^*\right) \tag{40}$$

which contradicts with the fact that $\gamma_{\text{th}}^*(i)$ is optimal. □

APPENDIX B
PROOF OF THEOREM 1

We first get S_i^*, U_i^*, and p_i^* by substituting equations $\gamma_{\text{th}}^*(i)$ and $P_i^*(\gamma)$ in (1)–(3), respectively. Then, we differentiate with respect to λ_P^*, treating λ_D^* as a constant, yielding

$$\frac{\partial S_i^*}{\partial \lambda_P^*} = -\theta_i f_\gamma\left(\gamma_{\text{th}}^*(i)\right)\frac{\partial \gamma_{\text{th}}^*(i)}{\partial \lambda_P^*}\left(c_i P_i^*\left(\gamma_{\text{th}}^*(i)\right) - S_{i+1}^*\right) - \theta_i c_i \frac{\bar{F}_\gamma\left(\gamma_{\text{th}}^*(i)\right)}{\left(\lambda_P^*\right)^2} + \left(1-\theta_i \bar{F}_\gamma\left(\gamma_{\text{th}}^*(i)\right)\right)\frac{\partial S_{i+1}^*}{\partial \lambda_P^*} \tag{41}$$

$$\frac{\partial U_i^*}{\partial \lambda_P^*} = -\theta_i f_\gamma\left(\gamma_{\text{th}}^*(i)\right)\frac{\partial \gamma_{\text{th}}^*(i)}{\partial \lambda_P^*} \times\left[\lambda_P^*\left(c_i P_i^*\left(\gamma_{\text{th}}^*(i)\right) - S_{i+1}^*\right) - \lambda_D^*\left(1-p_{i+1}^*\right)\right] - \theta_i c_i \frac{\bar{F}_\gamma(\gamma_{\text{th}}^*(i))}{\lambda_P^*} + \left(1-\theta_i \bar{F}_\gamma(\gamma_{\text{th}}^*(i))\right)\frac{\partial U_{i+1}^*}{\partial \lambda_P^*} \tag{42}$$

$$\frac{\partial p_i^*}{\partial \lambda_P^*} = -\theta_i f_\gamma\left(\gamma_{\text{th}}^*(i)\right)\frac{\partial \gamma_{\text{th}}^*(i)}{\partial \lambda_P^*}\left(1-p_{i+1}^*\right) \tag{43}$$

$$+\left(1-\theta_i \bar{F}_\gamma\left(\gamma_{\text{th}}^*(i)\right)\right)\frac{\partial p_{i+1}^*}{\partial \lambda_P^*} \tag{44}$$

respectively. Multiplying (41) by $-\lambda_P^*$ and (43) by λ_D^* and then adding them to (42), we can easily show that, for all $i \in \mathcal{M}$

$$\frac{\partial U_i^*}{\partial \lambda_P^*} - \lambda_P \frac{\partial S_i^*}{\partial \lambda_P^*} + \lambda_D \frac{\partial p_i^*}{\partial \lambda_P^*} = 0. \tag{45}$$

We now find the derivative of $\gamma_{\text{th}}^*(i)$ with respect to λ_P^* by differentiating both sides of (7) with respect to λ_P^*, while treating λ_D^* as a constant, then using (45), and then rearranging, we get

$$\frac{\partial \gamma_{\text{th}}^*(i)}{\partial \lambda_P^*} = \frac{c_i P_i^*\left(\gamma_{\text{th}}^*(i)\right) - S_{i+1}^*}{c_i \frac{\lambda_P^*}{\gamma_{\text{th}}^*(i)} P_i^*(\gamma_{\text{th}}^*(i))}. \tag{46}$$

Substituting (46) in (41), we get

$$\frac{\partial S_i^*}{\partial \lambda_P^*} = -\alpha_i\left[c_i P_i^*\left(\gamma_{\text{th}}^*(i)\right) - S_{i+1}^*\right]^2 - \theta_i c_i \frac{\bar{F}_\gamma(\gamma_{\text{th}}^*(i))}{\left(\lambda_P^*\right)^2} + \left(1-\theta_i \bar{F}_\gamma\left(\gamma_{\text{th}}^*(i)\right)\right)\frac{\partial S_{i+1}^*}{\partial \lambda_P^*} \tag{47}$$

where α_i is given by

$$\alpha_i = \frac{\theta_i f_\gamma\left(\gamma_{\text{th}}^*(i)\right)}{c_i \frac{\lambda_P^*}{\gamma_{\text{th}}^*(i)} P_i^*\left(\gamma_{\text{th}}^*(i)\right)} \geq 0. \tag{48}$$

Now, evaluating (47) at $i = M$ and $i = M-1$, we get

$$\frac{\partial S_M^*}{\partial \lambda_P^*} = -\alpha_M\left[c_M P_M^*\left(\gamma_{\text{th}}^*(M)\right)\right]^2 - \theta_M c_M \frac{\bar{F}_\gamma(\gamma_{\text{th}}^*(M))}{\left(\lambda_P^*\right)^2} \tag{49}$$

$$\frac{\partial S_{M-1}^*}{\partial \lambda_P^*} = -\alpha_{M-1}\left[c_{M-1} P_{M-1}^*\left(\gamma_{\text{th}}^*(M-1)\right) - S_M^*\right]^2 - \theta_{M-1} c_{M-1}\frac{\bar{F}_\gamma\left(\gamma_{\text{th}}^*(M-1)\right)}{\left(\lambda_P^*\right)^2} + \left(1-\theta_{M-1}\bar{F}_\gamma\left(\gamma_{\text{th}}^*(M-1)\right)\right)\frac{\partial S_M^*}{\partial \lambda_P^*} \tag{50}$$

respectively. We can see that $(\partial S_M^*/\partial \lambda_P^*) < 0$; hence, $(\partial S_{M-1}^*/\partial \lambda_P^*) < 0$. By induction, let us assume that $(\partial S_{i+1}^*/\partial \lambda_P^*) < 0$. From (47), we get that

$$\frac{\partial S_i^*}{\partial \lambda_P^*} = -\alpha_i\left(c_i P_i^*\left(\gamma_{\text{th}}^*(i)\right) - S_{i+1}^*\right)^2 - \theta_i c_i \frac{\bar{F}_\gamma\left(\gamma_{\text{th}}^*(i)\right)}{\left(\lambda_P^*\right)^2} + \left(1-\theta_i \bar{F}_\gamma\left(\gamma_{\text{th}}^*(i)\right)\right)\frac{\partial S_{i+1}^*}{\partial \lambda_P^*} < 0 \tag{51}$$

since all its terms are negative. Finally, we find that $(\partial S_1^*/\partial \lambda_P^*) < 0$, indicating that S_1^* is monotonically decreasing in λ_P^* given any fixed $\lambda_D^* \geq 0$.

Now, to get an upper bound on λ_P^*, we know that

$$S_i^* = \theta_i c_i \int_{\gamma_{\text{th}}^*(i)}^{\infty}\left(\frac{1}{\lambda_P^*} - \frac{1}{\gamma}\right) f_\gamma(\gamma)\, d\gamma + \left[1-\theta_i \bar{F}_\gamma\left(\gamma_{\text{th}}^*(i)\right)\right] S_{i+1}^*. \tag{52}$$

We can upper bound the first term in (52) by $\theta_i c_i/\lambda_P^*$, while $[1-\theta_i \bar{F}_\gamma(\gamma_{\text{th}}^*(i))] < 1$. Using these two bounds, we can write $S_1^* < \sum_{i=1}^M \theta_i c_i/\lambda_P^*$. However, since $S_1^* = P_{\text{avg}}$, the upper bound on λ_P^*, which was mentioned in Theorem 1, follows. □

APPENDIX C
PROOF OF LEMMA 2

We provide a proof sketch for this bound. We know that, at the optimal point, $p_1^* = (1/\bar{D}_{\max})$ and that $p_1^* = \theta_1 \bar{F}_\gamma(\gamma_{\text{th}}^*(1)) + (1-\theta_1 \bar{F}_\gamma(\gamma_{\text{th}}^*(1)))p_2^*$. However, since the second term in the latter equation is always positive, then

$$\theta_1 \bar{F}_\gamma\left(\gamma_{\text{th}}^*(1)\right) < \frac{1}{\bar{D}_{\max}}. \tag{53}$$

Substituting (12) in (53) and rearranging, we can upper bound λ_D^* by

$$\frac{c_1\left(\log\left(\frac{\lambda_P^*}{\bar{F}_\gamma^{-1}\left(\frac{1}{\theta_1 \bar{D}_{\max}}\right)}\right) - \frac{\lambda_P^*}{\bar{F}_\gamma^{-1}\left(\frac{1}{\theta_1 \bar{D}_{\max}}\right)} + 1\right) + U_2^* - \lambda_P^* S_2^*}{1-p_2^*}.$$

We can easily upper bound $\log(\lambda_P^*/\bar{F}_\gamma^{-1}(1/(\theta_1 \bar{D}_{\max}))) - \lambda_P^*/\bar{F}_\gamma^{-1}(1/(\theta_1 \bar{D}_{\max}))$ by substituting $\lambda_P^{\max}$ for λ_P^* when $\lambda_P^* < \bar{F}_\gamma^{-1}(1/(\theta_1 \bar{D}_{\max}))$ and by 1 otherwise. Moreover, it can also be shown that $U_2^* < U_2^{\max}$, $p_2^* < p_2^{\max}$ and that $\lambda_P^* S_2^* > 0$, and from Theorem 1, we have $\lambda_P^* < \lambda_P^{\max}$; the proof then follows. □

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