Introduction

- The ability to monitor, inform, and assess the quality of everyday movements plays an important role in several health-care applications.
- Recent studies suggest that the most efficient movement between two poses, in certain well defined cases, is often the geodesic path in the pose-space [1, 2].
- We propose an unsupervised method to model the deviation of a given trajectory w.r.t. an ideal path on a pose-space as a measure of quality.

Two different pose-spaces were used to demonstrate this idea:

1. \( S^1 \times S^1 \) to model the interaction of two joint angles
2. \( \mathcal{SE}(3) \times \mathcal{SE}(3) \) to model the movement of two joints

Mathematical Preliminaries

1. **Body-joint angles on** \( S^1 \times S^1 \):
   - Distance between two body-joint angles \( \theta_1 \) and \( \theta_2 \) on circle \( S^1 \):
     \[
     d_{S^1}(\theta_1, \theta_2) = \arccos(\cos(\theta_1 - \theta_2))
     \]
   - Distance between two points \( p_1 = (\phi_1, \theta_1) \) and \( p_2 = (\phi_2, \theta_2) \) on the torus:
     \[
     d_{S^1 \times S^1}(p_1, p_2) = \sqrt{d_{S^1}(\phi_1, \phi_2)^2 + d_{S^1}(\theta_1, \theta_2)^2}
     \]

2. **Product space of** \( \mathcal{SE}(3) \times \mathcal{SE}(3) \):
   - Each pose is represented as a point on the product space.
   - \( \mathcal{SE}(3) \) is a Lie group, containing the set of all \( 4 \times 4 \) matrices:
     \[
     R \in \text{Rotation matrix}, \quad d \in \text{Translation vector} \]
   - \( I \) is the identity element of the group, and the tangent space of \( \mathcal{SE}(3) \) at \( I \) is called Lie algebra – denoted as \( \mathfrak{se}(3) \) and is identified by \( 4 \times 4 \) matrix:
     \[
     \xi = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \quad \omega \in \mathbb{R}^3
     \]
   - **Exponential map:**
     \[
     \exp \xi = I + \omega \quad \text{and} \quad \exp \xi = \begin{bmatrix} 0 \\ \omega \cdot A T \omega \end{bmatrix} \quad \omega \neq 0
     \]
   - **Inverse exponential map:**
     \[
     \xi = \log \left( R \mid d \right) = \begin{bmatrix} \omega \\ 0 \end{bmatrix}
     \]

Measures of Quality

1. **Deviation from Geodesic Measure (DGM):**
   \[
   q = DTW(\gamma(t), \gamma_0(t)) + DTW(\gamma_0(t), \gamma(t))
   \]
   \( \gamma(t) \) = Original trajectory
   \( \gamma_0(t) \) = Ideal trajectory

2. **Deviation from Mean Trajectory Measure:**
   \[
   v(t) = \log_{\gamma(t)} \gamma(t)
   \]
   \( q(t) = ||v(t)|| \)
   \( g(T) = \sum q(t) \)
   \( q = \) Deviation from ideal path score
   \( DTW = \) Dynamic Time Warping
   \( \gamma(t) \) = Original trajectory
   \( \gamma_0(t) \) = Ideal trajectory

Experiments and Results

1. **Sit-to-stand (STS) quality assessment**
   \[
   \text{Movement Quality Measure} = \frac{\text{DGM score} \times \text{IPC of head speed} \times \text{Minimum hip angle}}{1.0625}
   \]

2. **Reach assessment in stroke rehabilitation**

Reference