How to calculate present values

Back to the future

Chapter 3

Discounted Cash Flow Analysis (Time Value of Money)

- Discounted Cash Flow (DCF) analysis is the foundation of valuation in corporate finance

- To use DCF we need to know three things
  - The size of the expected cash flows
  - The timing of the cash flows
  - The proper discount (interest) rate

- DCF allows us to compare the values of alternative cash flow streams in dollars today (Present Value)

FUTURE VALUE (COMPOUNDING):

What will $100 grow to after 1 year at 10% ?

```
      0          10%          1
       |----------------------|
    -100
interest                      10
end of period value          110
```

$FV_1 = PV_0 (1+r) = 100 (1.1) = 110$
where FV_1 is the future value in period 1
PV_0 is the present value in period 0 (today)

NOTE: When r=10%, $100 received now (t=0) is equivalent to $110 received in one year (t=1).

What will $100 grow to after 2 years at 10% ?

```
      0          10%        1          10%          2
       |----------------------|----------------------|
    100
interest                      10                    11
end of period value          110                   121
```

$FV_2 = PV_0 (1+r) (1+r) = PV_0 (1+r)^2$
$= 100 (1.1)^2 = 100 (1.21) = 121$

NOTE: $100 received now (t=0) is equivalent to $110 received in one year (t=1) which is also equivalent to $121 in 2 years (t=2).

The general formula for future value in year N (FV_N)

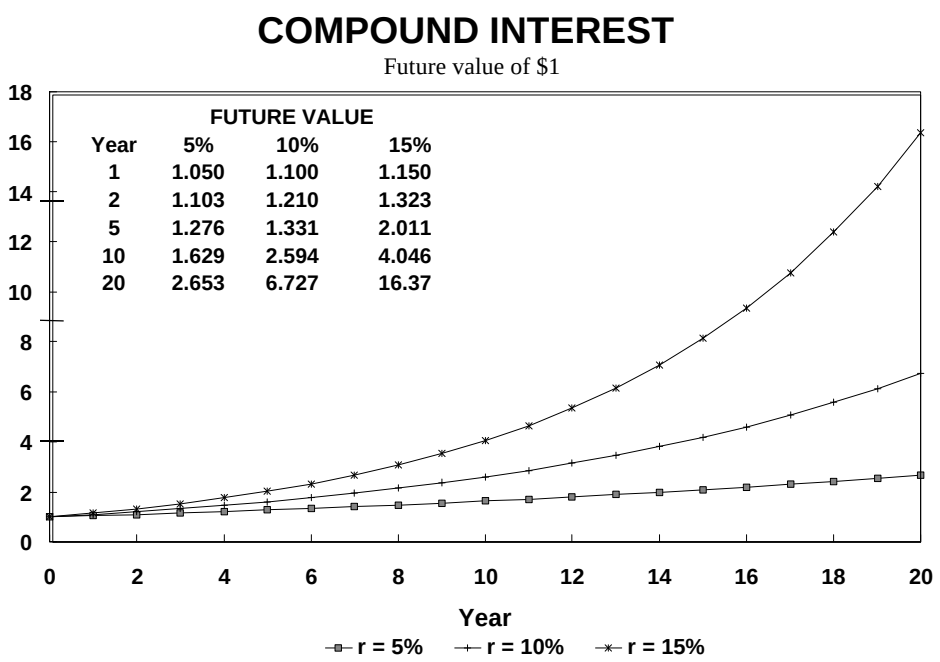
$$FV_N = PV_0 (1+r)^N$$

What will $100 grow to after 8 years at 6% ?

What is the *present value* of $159.40 received in 8 years at 6%?

Or

How much would you have to invest today at 6% in order to have $159.40 in 8 years?



PRESENT VALUE IS THE RECIPROCAL OF FUTURE VALUE:

$$PV_0 = FV_N / (1+r)^N$$

Note: Brealey & Myers refer to $1/(1+r)^N$ as a "discount factor".

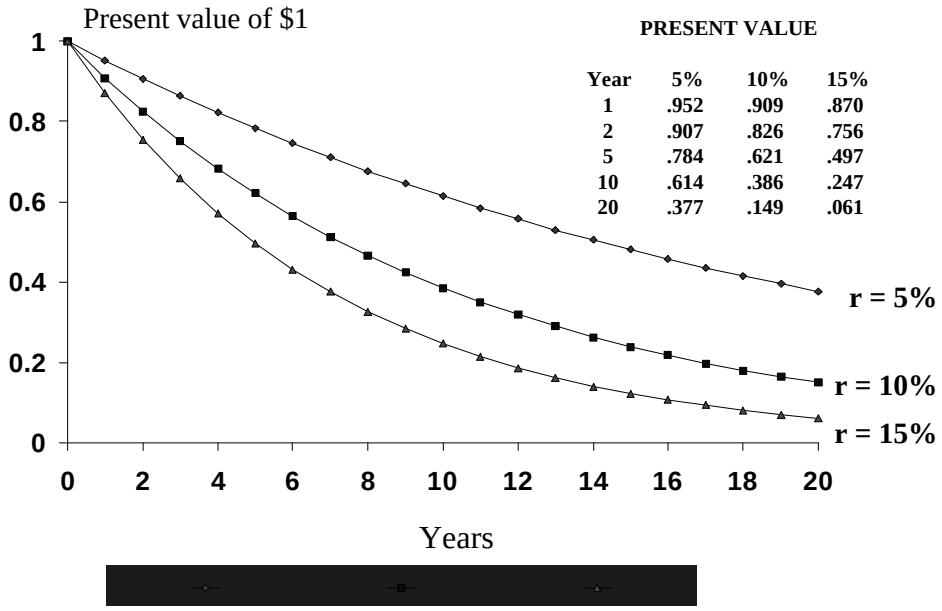
The discount factor for 8 years at 6% is

$$1/(1+.06)^8 = 0.627$$

Thus, the present value of $1.00 in 8 years at 6% is $0.627.

What's the present value of $50 in 8 years?

PRESENT VALUES



PRESENT VALUE

Year	5%	10%	15%
1	.952	.909	.870
2	.907	.826	.756
5	.784	.621	.497
10	.614	.386	.247
20	.377	.149	.061

PRESENT VALUE PROBLEMS

Which would you prefer at r=10%?

$1000 today vs. $2000 in 10 years

There are 4 variables in the analysis
PV, FV, N, and r

Given three, you can always solve
for the other

<u>Four related questions:</u>

2.1. How much must you deposit today to have $1 million in 25 years? (r=.12)

2.2. If a $58,820 investment yields $1 million in 25 years, what is the rate of interest?

2.3. How many years will it take $58,820 to grow to $1 million if r=.12?

2.4. What will $58,820 grow to after 25 years if r=.12?

Present Value Of An Uneven Cash Flow Stream

- In general, the present value of a stream of cash flows can be found using the following general valuation formula.

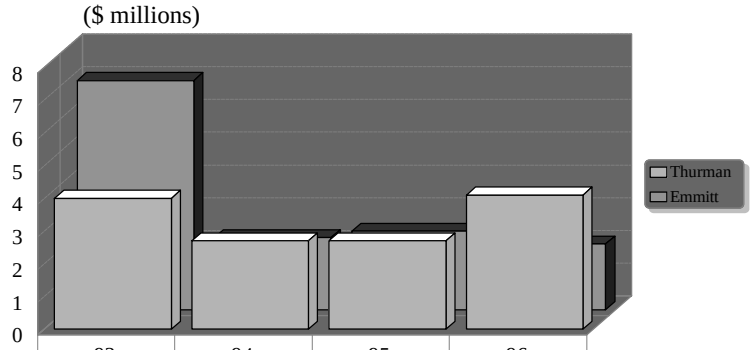
$$PV = \frac{C_1}{(1+r_1)} + \frac{C_2}{(1+r_2)^2} + \frac{C_3}{(1+r_3)^3} + \ldots + \frac{C_N}{(1+r_N)^N}$$
$$= \sum_{t=1}^{N} \frac{C_t}{(1+r_t)^t}$$

- In other words, discount each cash flow back to the present using the appropriate discount rate and then sum the present values.

Example

		r (%)		
		8		
year	A	PV	B	PV
1	100	92.59259	300	277.7778
2	400	342.9355	400	342.9355
3	400	317.5329	400	317.5329
4	400	294.0119	400	294.0119
5	300	204.175	100	68.05832
Present Value		1251.248		1300.316

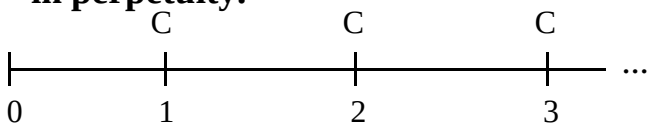
Who got the better contract? Emmitt or Thurman?



	93	94	95	96
Thurman	4	2.7	2.7	4.1
Emmitt	7	2.2	2.4	2

PERPETUITIES

Offer a fixed annual payment (C) each year in perpetuity.



How do you determine present value?

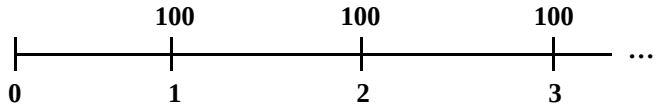
$PV = C/(1+r) + C/(1+r)^2 + C/(1+r)^3 + \ldots$

Fortunately, a simple formula

$$PV_0 \text{ of a perpetuity} = C_1/r$$

An example

Perpetuity: $100 per period forever discounted at 10% per period

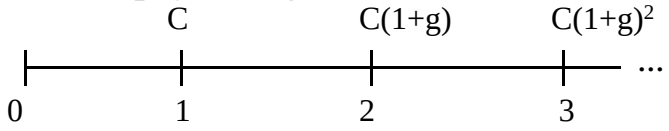


… and some intuition

Consider a $1000 deposit in a bank account that pays 10% per year.

<u>GROWING PERPETUITIES</u>

Annual payment grows at a constant rate, g.



How do you determine present value?

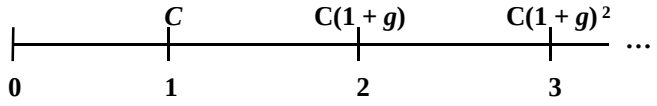
$$PV = C_1/(1+r) + C_2/(1+r)^2 + C_3/(1+r)^3 + \ldots$$

Fortunately, a simple formula

PV_0 of a growing perpetuity = $C_1/(r-g)$

An example

<u>Growing perpetuity:</u> $100 received at time t =1, growing at 2% per period with a discount rate of 10%



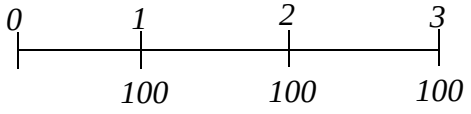
An example

An investment in a growing perpetuity costs $5000 and is expected to pay $200 next year. If the interest rate is 10%, what is the growth rate of the annual payment?

Annuities

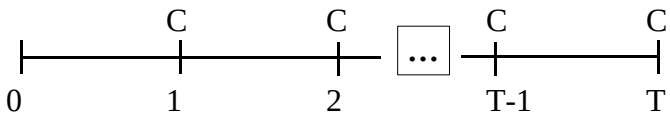
- An annuity is a series of equal payments (PMT on your calculator) made at fixed intervals for a specified number of periods
  - e.g., $100 at the end of each of the next three years
- If payments occur at the end of each period it is an ordinary annuity--(This is most common)
- If payments occur at the beginning of each period it is an annuity due

Ordinary Annuity



Annuities

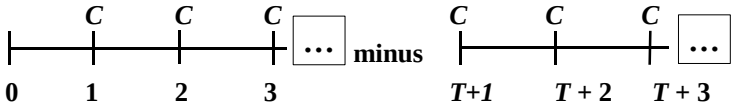
- The present value of an ordinary annuity that pays a cash flow of C per period for T periods when the discount rate is r is



$$\boldsymbol{PV = C\left(\frac{1}{r} - \frac{1}{r(1+r)^T}\right)}$$

Annuities

- **A T-period annuity is equivalent to the difference between two perpetuities. One beginning at time zero, and one with first payment at time T+1.**



- **This implies that**

$$PV = \frac{C}{r} - \frac{C}{r}\left(\frac{1}{(1+r)^T}\right) = C\left(\frac{1}{r} - \frac{1}{r(1+r)^T}\right)$$

Example

- Compute the present value of a 3 year ordinary annuity with payments of $100 at r=10%
- Answer:

$$PVA_3 = 100\frac{1}{1.1} + 100\frac{1}{1.1^2} + 100\frac{1}{1.1^3} = \$248.68$$

Or

$$PVA_3 = 100\left(\frac{1}{0.1} - \frac{1}{0.1(1.1)^3}\right) = \$248.68$$

What is the relation between a lump sum cash flow and an annuity?

- What is the present value of an annuity that promises $2000 per year for 5 years at r=5%?

year	PMT	PV (t=0)
1	2,000.00	1,904.76
2	2,000.00	1,814.06
3	2,000.00	1,727.68
4	2,000.00	1,645.40
5	2,000.00	1,567.05
		_____
		8,658.95

$$PVA_3 = 2000\left(\frac{1}{0.05} - \frac{1}{0.05(1.05)^5}\right) = \$8658.95$$

- Alternatively, suppose you were given $8,658.95 today instead of the annuity

year	principal	interest	PMT	Ending Bal
1	$ 8,658.95	$ 432.95	$ (2,000.00)	$ 7,091.90
2	$ 7,091.90	$ 354.60	$ (2,000.00)	$ 5,446.50
3	$ 5,446.50	$ 272.32	$ (2,000.00)	$ 3,718.82
4	$ 3,718.82	$ 185.94	$ (2,000.00)	$ 1,904.76
5	$ 1,904.76	$ 95.24	$ (2,000.00)	$ 0.00

- Notice that you can duplicate the cash flows from the annuity by investing your money from the lump sum to earn the required rate of return (5% in this example).

A Net Present Value Problem

What is the value <u>today</u> of a 10-year annuity that pays $300 a year (at year-end) if the annuity's first cash flow starts at the end of year 6 and the interest rate is 10%?

Other Compounding Intervals

- Cash flows are often compounded over periods other than annually
  - Consumer loans are compounded monthly
  - Bond coupons are received semiannually

Compounding

```
Annual:                                                    1
 |-------------------------------------------------------------|
100                                                      110.00
Semi-annual:
                                .5                          1
|-------------------------------|-------------------------------|


Quarterly:
                .25             .5              .75             1
  |-------------|---------------|---------------|--------------|
```

Example

- Find the PV of $500 received in the future under the following conditions.
- 12% nominal rate, semiannual compounding, 5 years

$$PV = \frac{500}{\left(1 + \frac{0.12}{2}\right)^{10}} = \$279.20$$

- 12% nominal rate, quarterly compounding, 5 years

$$PV = \frac{500}{\left(1 + \frac{0.12}{4}\right)^{20}} = \$276.84$$

Future value of $1.00 in N years when interest is compounded M times per year

$$FV_N = (1 + r/M)^{MN}$$

<u>*Continuous* compounding :</u>

As M approaches infinity...

… $(1 + r/M)^{MN}$ approaches e^{rN}
where e = 2.718

<u>Example</u>: The future value of $100 continuously compounded at 10% for one year is

$$100\, e^{.10} = 110.52$$

Summary

- Discounted cash flow analysis is the foundation for valuing assets
- To use DCF you need to know three things
  - Size of expected cash flows
  - Timing of cash flows
  - Discount rate (reflects the risk of cash flows)
- When valuing a stream of cash flows, search for components such as annuities that can be easily valued
- Compare different streams of cash flows in common units using present value