How to calculate present values

Back to the future
Chapter 3

Discounted Cash Flow Analysis
(Time Value of Money)

- Discounted Cash Flow (DCF) analysis is the foundation of valuation in corporate finance

- To use DCF we need to know three things
  - The size of the expected cash flows
  - The timing of the cash flows
  - The proper discount (interest) rate

- DCF allows us to compare the values of alternative cash flow streams in dollars today (Present Value)
FUTURE VALUE
(COMPOUNDING):
What will $100 grow to after 1 year at 10% ?

<table>
<thead>
<tr>
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<tbody>
<tr>
<td></td>
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<tr>
<td>-100</td>
</tr>
</tbody>
</table>

interest          10
end of period value 110

\[ FV_1 = PV_0 (1+r) = 100 (1.1) = 110 \]
where \( FV_1 \) is the future value in period 1
\( PV_0 \) is the present value in period 0 (today)

NOTE: When \( r=10\% \), $100 received now (t=0) is
equivalent to $110 received in one year (t=1).

What will $100 grow to after 2 years at 10% ?

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<tbody>
<tr>
<td>100</td>
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</tbody>
</table>

interest          10          11
end of period value 110          121

\[ FV_2 = PV_0 (1+r) (1+r) = PV_0 (1+r)^2 \]
\[ = 100 (1.1)^2 = 100 (1.21) = 121 \]

NOTE: $100 received now (t=0) is equivalent to
$110 received in one year (t=1) which is also
equivalent to $121 in 2 years (t=2).
The general formula for future value in year N ($F_{VN}$)

$$F_{VN} = PV_0 (1+r)^N$$

What will $100 grow to after 8 years at 6% ?

What is the present value of $159.40 received in 8 years at 6%?

Or

How much would you have to invest today at 6% in order to have $159.40 in 8 years?

### COMPOUND INTEREST

#### Future value of $1

<table>
<thead>
<tr>
<th>Year</th>
<th>5%</th>
<th>10%</th>
<th>15%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.050</td>
<td>1.100</td>
<td>1.150</td>
</tr>
<tr>
<td>2</td>
<td>1.103</td>
<td>1.210</td>
<td>1.323</td>
</tr>
<tr>
<td>5</td>
<td>1.276</td>
<td>1.331</td>
<td>2.011</td>
</tr>
<tr>
<td>10</td>
<td>1.629</td>
<td>2.594</td>
<td>4.046</td>
</tr>
<tr>
<td>20</td>
<td>2.653</td>
<td>6.727</td>
<td>16.37</td>
</tr>
</tbody>
</table>
PRESENT VALUE IS THE RECIPROCAL OF FUTURE VALUE:

\[ PV_0 = \frac{FV_N}{(1+r)^N} \]

Note: Brealey & Myers refer to \(1/(1+r)^N\) as a “discount factor”.

The discount factor for 8 years at 6% is

\[ \frac{1}{(1+.06)^8} = 0.627 \]

Thus, the present value of $1.00 in 8 years at 6% is $0.627.

What’s the present value of $50 in 8 years?
PRESENT VALUE PROBLEMS
Which would you prefer at r=10%?
$1000 today vs. $2000 in 10 years

There are 4 variables in the analysis
PV, FV, N, and r

Given three, you can always solve
for the other
Four related questions:

2.1. How much must you deposit today to have $1 million in 25 years? (r=.12)

2.2. If a $58,820 investment yields $1 million in 25 years, what is the rate of interest?

2.3. How many years will it take $58,820 to grow to $1 million if r=.12?

2.4. What will $58,820 grow to after 25 years if r=.12?

Present Value Of An Uneven Cash Flow Stream

• In general, the present value of a stream of cash flows can be found using the following general valuation formula.

\[
PV = \frac{C_1}{(1+r_1)} + \frac{C_2}{(1+r_2)^2} + \frac{C_3}{(1+r_3)^3} + \ldots + \frac{C_N}{(1+r_N)^N} \\
= \sum_{i=1}^{N} \frac{C_i}{(1+r_i)^t}
\]

• In other words, discount each cash flow back to the present using the appropriate discount rate and then sum the present values.
**Example**

<table>
<thead>
<tr>
<th>year</th>
<th>A</th>
<th>PV</th>
<th>B</th>
<th>PV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>92.59259</td>
<td>300</td>
<td>277.7778</td>
</tr>
<tr>
<td>2</td>
<td>400</td>
<td>342.9355</td>
<td>400</td>
<td>342.9355</td>
</tr>
<tr>
<td>3</td>
<td>400</td>
<td>317.5329</td>
<td>400</td>
<td>317.5329</td>
</tr>
<tr>
<td>4</td>
<td>400</td>
<td>294.0119</td>
<td>400</td>
<td>294.0119</td>
</tr>
<tr>
<td>5</td>
<td>300</td>
<td>204.175</td>
<td>100</td>
<td>68.05832</td>
</tr>
</tbody>
</table>

Present Value

1251.248  1300.316

**Who got the better contract? Emmitt or Thurman?**

<table>
<thead>
<tr>
<th></th>
<th>93</th>
<th>94</th>
<th>95</th>
<th>96</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thurman</td>
<td>4</td>
<td>2.7</td>
<td>2.7</td>
<td>4.1</td>
</tr>
<tr>
<td>Emmitt</td>
<td>7</td>
<td>2.2</td>
<td>2.4</td>
<td>2</td>
</tr>
</tbody>
</table>
PERPETUITIES
Offer a fixed annual payment (C) each year in perpetuity.

$\begin{array}{cccc}
& C & C & C \\
0 & 1 & 2 & 3 \\
\end{array}$

How do you determine present value?

$PV = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots$

Fortunately, a simple formula

$PV_0$ of a perpetuity $= \frac{C}{r}$

An example

Perpetuity: $100$ per period forever discounted at $10\%$ per period

$\begin{array}{cccc}
& 100 & 100 & 100 \\
0 & 1 & 2 & 3 \\
\end{array}$

… and some intuition

Consider a $1000$ deposit in a bank account that pays $10\%$ per year.
GROWING PERPETUITIES
Annual payment grows at a constant rate, g.

\[
\begin{array}{c|c|c|c|c}
& C & C(1+g) & C(1+g)^2 & \\
0 & 1 & 2 & 3 & \\
\end{array}
\]

How do you determine present value?

\[PV = \frac{C_1}{(r-g)} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \ldots\]

Fortunately, a simple formula

\[PV_0 \text{ of a growing perpetuity} = \frac{C_1}{(r-g)}\]

An example

Growing perpetuity: $100 received at time \(t=1\),
growing at 2\% per period with a discount rate of 10\%
An example

An investment in a growing perpetuity costs $5000 and is expected to pay $200 next year. If the interest rate is 10%, what is the growth rate of the annual payment?

Annuities

- An annuity is a series of equal payments (PMT on your calculator) made at fixed intervals for a specified number of periods
  - e.g., $100 at the end of each of the next three years
- If payments occur at the end of each period it is an ordinary annuity--(This is most common)
- If payments occur at the beginning of each period it is an annuity due

![Diagram of an ordinary annuity]

Ordinary Annuity

\[0 \quad 1 \quad 2 \quad 3\]

\[100 \quad 100 \quad 100\]
Annuities

- The present value of an ordinary annuity that pays a cash flow of $C$ per period for $T$ periods when the discount rate is $r$ is

$$PV = C \left( \frac{1}{r} - \frac{1}{r(1+r)^T} \right)$$

Annuities

- A $T$-period annuity is equivalent to the difference between two perpetuities. One beginning at time zero, and one with first payment at time $T+1$.

$$PV = \frac{C}{r} - \frac{C}{r} \left( \frac{1}{(1+r)^T} \right) = C \left( \frac{1}{r} - \frac{1}{r(1+r)^T} \right)$$
Example

- Compute the present value of a 3 year ordinary annuity with payments of $100 at r=10%
- Answer:

$$PVA_3 = 100 \left( \frac{1}{1.1} + \frac{1}{1.1^2} + \frac{1}{1.1^3} \right) = $248.68$$

Or

$$PVA_3 = 100 \left( \frac{1}{0.1} \cdot \frac{1}{0.1(1.1)^3} \right) = $248.68$$

What is the relation between a lump sum cash flow and an annuity?

- What is the present value of an annuity that promises $2000 per year for 5 years at r=5%?

<table>
<thead>
<tr>
<th>year</th>
<th>PMT</th>
<th>PV (t=0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2,000.00</td>
<td>1,904.76</td>
</tr>
<tr>
<td>2</td>
<td>2,000.00</td>
<td>1,814.06</td>
</tr>
<tr>
<td>3</td>
<td>2,000.00</td>
<td>1,727.68</td>
</tr>
<tr>
<td>4</td>
<td>2,000.00</td>
<td>1,645.40</td>
</tr>
<tr>
<td>5</td>
<td>2,000.00</td>
<td>1,567.05</td>
</tr>
</tbody>
</table>

$$PVA_5 = 2000 \left( \frac{1}{0.05} \cdot \frac{1}{0.05(1.05)^5} \right) = $8658.95$$
• Alternatively, suppose you were given $8,658.95 today instead of the annuity

<table>
<thead>
<tr>
<th>year</th>
<th>principal</th>
<th>interest</th>
<th>PMT</th>
<th>Ending Bal</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$ 8,658.95</td>
<td>$ 432.95</td>
<td>$ (2,000.00)</td>
<td>$ 7,091.90</td>
</tr>
<tr>
<td>2</td>
<td>$ 7,091.90</td>
<td>$ 354.60</td>
<td>$ (2,000.00)</td>
<td>$ 5,446.50</td>
</tr>
<tr>
<td>3</td>
<td>$ 5,446.50</td>
<td>$ 272.32</td>
<td>$ (2,000.00)</td>
<td>$ 3,718.82</td>
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<tr>
<td>4</td>
<td>$ 3,718.82</td>
<td>$ 185.94</td>
<td>$ (2,000.00)</td>
<td>$ 1,904.76</td>
</tr>
<tr>
<td>5</td>
<td>$ 1,904.76</td>
<td>$ 95.24</td>
<td>$ (2,000.00)</td>
<td>$ 0.00</td>
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</table>

• Notice that you can duplicate the cash flows from the annuity by investing your money from the lump sum to earn the required rate of return (5% in this example).

A Net Present Value Problem
What is the value today of a 10-year annuity that pays $300 a year (at year-end) if the annuity’s first cash flow starts at the end of year 6 and the interest rate is 10%?
Other Compounding Intervals

Cash flows are often compounded over periods other than annually

- Consumer loans are compounded monthly
- Bond coupons are received semiannually

<table>
<thead>
<tr>
<th>Compounding</th>
<th>Annual</th>
<th>Semi-annual</th>
<th>Quarterly</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>.5</td>
<td>.25</td>
</tr>
<tr>
<td>100</td>
<td>110.00</td>
<td>1</td>
<td>.5</td>
</tr>
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<td>--------</td>
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Example

• Find the PV of $500 received in the future under the following conditions.
  • 12% nominal rate, semiannual compounding, 5 years

\[
PV = \frac{500}{\left(1 + \frac{0.12}{2}\right)^{10}} = $279.20
\]

• 12% nominal rate, quarterly compounding, 5 years

\[
PV = \frac{500}{\left(1 + \frac{0.12}{4}\right)^{20}} = $276.84
\]

Future value of $1.00 in N years when interest is compounded M times per year

\[
FV_N = (1 + \frac{r}{M})^{MN}
\]

Continuous compounding:
As M approaches infinity...

\[
\ldots (1 + \frac{r}{M})^{MN} \text{ approaches } e^{rN}
\]
where \( e = 2.718 \)

Example: The future value of $100 continuously compounded at 10% for one year is

\[
100 e^{.10} = 110.52
\]
Summary

- Discounted cash flow analysis is the foundation for valuing assets
- To use DCF you need to know three things
  - Size of expected cash flows
  - Timing of cash flows
  - Discount rate (reflects the risk of cash flows)
- When valuing a stream of cash flows, search for components such as annuities that can be easily valued
- Compare different streams of cash flows in common units using present value