TIME CONSISTENT COLLECTION OF OPTIMAL SEIGNIORAGE: A UNIFYING FRAMEWORK

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Abstract. This paper reviews the existing literature on the time consistency problem of seigniorage collection when monetary policy is determined by optimal taxation considerations. It develops a unifying accounting framework and suggests a general measure for seigniorage, which encompasses the standard measures employed in the literature. In addition, the ex ante optimal solution to the optimal taxation problem is derived and interpreted in relation to the Ramsey principle. We show that the different recommendations of the public finance literature, i.e. the Friedman rule of optimal deflation, [moderately] positive inflation, and seigniorage maximizing inflation, are specific solutions to the optimal taxation problem. The paper continues with a formal illustration of the time consistency problem of the ex ante optimal policy and the characterization of the time consistent solution under discretion. As possible solutions to the time consistency problem, we consider reputational forces, institutional reforms that establish central bank independence, and specific ways of asset and debt management. In particular, it is formally shown that a modified version of the asset and debt management scheme suggested by Persson, Persson, and Svensson is not only necessary but also sufficient for optimality, although this did not hold in their model.

Keywords. Asset and debt management; inflation tax; optimal taxation; seigniorage; time consistency.

1. Introduction
Seigniorage, or the public sector's revenues from the creation of money, has attracted a great deal of attention in monetary economics. The recent European interest in the issue has certainly been related to the forthcoming establishment of a European central bank during the process of European monetary integration; see, for instance, Dornbusch (1988), Drazen (1989), Grilli (1989), Canzoneri and Rogers (1990), Végh and Guidotti (1990), or Alesina and Grilli (1991). The main problem is that the revenues from the creation of money played very different budgetary roles across the member countries of the European union: in contrast to the countries in Northern Europe, the Southern European members of the European Union have used seigniorage to a relatively large extent, in order to finance government expenditure. Drazen (1989), for example, estimated the share of seigniorage in tax revenues for the period 1979–1986 as 5.9 percent in Spain,
6.2 percent in Italy, 9.1 percent in Greece, and 11.9 percent in Portugal. Other calculations of seigniorage for Southern European countries include Bruni, Penati and Porta (1989), Grilli (1989), Repullo (1991), and Gros (1993). Seigniorage in these studies is reported as a share of GDP or GNP and varies between 2 and 4 percent.¹

The budgetary problems of the Eastern European states after the structural break in 1989 have also drawn attention to the importance of seigniorage as a revenue instrument; see, for example, Hochreiter, Rovelli and Winckler (1996). As is the case for many developing countries, most of the emerging democracies are highly dependent on seigniorage, because they have poorly developed excise and income taxation systems, together with significant black market activities, rather underdeveloped domestic capital markets and only limited access to international capital markets. The results of Oblath and Valentinyi (1994) indicate quantitatively how important seigniorage is in Eastern Europe. For Hungary, which is among the more stable Eastern European countries, they value the average share of the revenues from money creation at 3.5 percent of GDP over the period of 1989–1992, with a peak of 4.8 percent in 1991. Measured as the average share of Hungarian tax revenues during this period, the seigniorage transferred to the government amounted to around 10 percent. The scale of these figures does not appear to be unrealistic in comparison to estimates for Southern European countries, which appear to have more fertile alternative revenue sources.

The theoretical literature on seigniorage has started from the two trail-blazing, formal contributions of Bailey (1956) and Cagan (1956).² However, it was Phelps (1973) who provided the theoretical framework used in the modern literature on the optimal collection of seigniorage from a public finance viewpoint. Essentially, Phelps integrated the revenues from the creation of money into a public finance framework of optimal taxation, which calls for an optimal combination of all possible revenue sources. In this context, the term ‘optimality’ is used in a normative sense, and means that a combination of the different revenue instruments raises the required funds at the lowest possible welfare cost. The literature departing from Phelps’ work has shown that optimal inflation rates may be either positive or negative, depending on the microfoundations of money and on the efficiency of alternative revenue instruments, such as output taxes. The only consensus in monetary theory seems to be that, in general, the optimal inflation rate is not the seigniorage-maximizing one.

Another strand of the literature has focussed on the inherent time consistency problem, which has been well recognized since Calvo (1978a,b) applied the classic ideas exposed in Kydland and Prescott (1977) to monetary economics. The question of time consistency arises here because, after individuals have determined their inflation expectations, the policy maker may choose to create unexpected inflation in order to reduce the real value of nominal, public liabilities vis-a-vis the private sector, that is, nominal balances or non-indexed government bonds. This is welfare-improving from the point of view of the policy maker because unexpected inflation does not distort individuals’ past decisions and, thus, causes lower social cost than expected inflation. Moreover, the revenue per unit of

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unexpected inflation is generally larger than that per unit of expected inflation, due to the fact that individuals can only take expected inflation into account when making their decisions. For this reason, creating unexpected inflation has often been viewed as a capital levy.

Irrespective of its importance, however, the time consistency problem is often assumed away in the public finance literature on optimal seigniorage collection by restricting attention to situations in which the policy maker is precommitted. Existing reviews also tend to be focused either on the optimal taxation problem or on the time consistency problem of optimal monetary policy making. In contrast, the present paper discusses the collection of seigniorage both in the contexts of optimal taxation and of time consistency. It may, therefore, be viewed as a supplement to existing discussions, rather than as a substitute for them.

The paper is organized as follows. Section 2 contains an extension of the model suggested by Barro (1983). Emphasis is put on the derivation of a general measure of seigniorage from the consolidated public sector flow budget constraint and on the discussion of the social loss from inflation. The resulting framework encompasses the most commonly used measures of seigniorage. Section 3 contains the formal derivation of the ex ante optimal solution to the optimal taxation problem and its interpretation in relation to the Ramsey (1927) principle of optimal taxation. In particular, the different recommendations of the public finance literature on optimal inflation, notably the Friedman (1969) rule, [moderately] positive inflation, or even seigniorage-maximizing inflation rates, are shown to be solutions to the optimal taxation problem in specific cases.

In section 4, the time inconsistency of the ex ante optimal solution is proved and the time consistent solution is characterized and discussed. The next section discusses three possible solutions to the time consistency problem, namely, reputational effects [in subsection 5.1], institutional reforms that establish central bank independence [in subsection 5.2] and specific ways of managing public assets and debt as proposed by Persson, Persson and Svensson (1987) [in subsection 5.3]. It is shown that a modified version of their scheme is not only necessary but also sufficient to ensure the time consistency of the ex ante optimal policy under discretion, though this did not hold true in their more general model [Calvo and Obstfeld (1990)]. Finally, the implications of the results are discussed in section 6 and a list of used symbols is given at the end of the paper.

2. The optimal taxation problem

In this section, a unifying framework is developed by extending Barro (1983)'s seigniorage model. In subsection 2.1, a simple endowment economy with money is outlined, in which prices and inflation can be determined in a meaningful way. The consolidated flow budget constraint of the public sector is then derived [in subsection 2.2] and a detailed discussion of seigniorage is provided [in subsection 2.3]. Subsection 2.4 contains the motivation and specification of the social loss from the different revenue instruments. The section ends with a formal statement of the optimal taxation problem in subsection 2.5.
2.1 A model economy

Consider a deterministic, closed economy in which the realizations of all exogenous variables other than the policy instruments are known with certainty. To concentrate on the issues related to seigniorage, assume that the stream \(y_0, y_1, y_2, \ldots\) of the homogeneous real output or endowment good is exogenously given. For convenience, real and nominal variables will be denoted by lower and upper case letters respectively. For instance, if \(P_t\) is the price level in period \(t\), this convention implies that nominal output can be expressed as \(Y_t = y_tP_t\).

Prices in the endowment economy are supposed to be perfectly flexible. In order to determine the price level, it is assumed that individuals want to hold nominal balances with a real value \(m_t\) during period \(t\), implying that nominal money demand equals \(m_tP_t\). Denoting the nominal money supply by \(M_t\), the price level follows from equilibrium in the money market, i.e. from equality between nominal money supply and money demand,

\[
M_t = m_tP_t \Rightarrow P_t = M_t/m_t. \tag{1}
\]

The inflation rate between the periods \(t-1\) and \(t\) is defined as

\[
\pi_t = \frac{P_t - P_{t-1}}{P_t}. \tag{2}
\]

This backward-looking definition of inflation is different from the more common, forward-looking measure \((P_t - P_{t-1})/P_{t-1}\). However, specification (2) is appropriate in the present context because the inflation rate will be interpreted as a tax rate on real money holdings. This tax rate ought to converge to one if the price level at \(t\) goes to infinity, provided that the price level at \(t-1\) is finite; see Obstfeld (1991a,b) for a similar definition.6

Bonds may be issued either by private individuals or by the government. All bonds are assumed to pay the same ex ante nominal interest rate \(i_t\) in any period \(t\), that is, for simplicity, private and government bonds are taken to be perfect substitutes. Since rational individuals require interest payments that fully compensate for expected inflation, \(i_t\) is determined by the Fisher relation

\[
1 + i_t = \frac{P_t^e}{P_{t-1}} \frac{1 + r_t}{1 - \pi_t^e} \Rightarrow i_t = \frac{r_t + \pi_t^e}{1 - \pi_t^e}, \tag{3}
\]

where \(P_t^e\) and \(\pi_t^e\) are the period \(t\) price level and inflation rate, as expected in period \(t-1\). To avoid confusion, we note here that this version of the Fisher relation differs from the standard version \(1 + i_t = (1 + r_t)(1 + \pi_t)\), due to the different definition of inflation chosen. However, this difference is not crucial for small inflation rates; taking logarithms, both specifications imply the same familiar approximation \(i_t \approx r_t + \pi_t^e\).

In order to determine the demand for real balances, we postulate the existence of a stable aggregate money demand function. As is standard in the literature, the
demand $m_t$ for real balances in period $t$ is taken to depend negatively and strictly convexly on the real opportunity cost $\rho_t$ of holding money instead of interest bearing bonds. $\rho_t$ is given by the expected present discounted, real value of next period’s interest payments

$$\rho_t = \frac{1}{(1 + r_{t+1})} P_t \frac{P_t}{P_{t+1}} i_{t+1} = \frac{(1 - P_{t+1})i_{t+1}}{1 + r_{t+1}}. \quad (4)$$

Using (3), one may also write $\rho_t$ as $\rho_t = (r_{t+1} + \pi_{t+1})/(1 + r_{t+1})$, which shows that

$$\frac{\partial \rho_t}{\partial \pi_{t+1}} = \frac{1}{1 + r_{t+1}} > 0. \quad (5)$$

Inequality (5) implies that real balances depend negatively and strictly convexly on $\pi_{t+1}$. Assuming that the real interest rate is exogenously given, one may therefore express real balances as a function of the expected inflation rate,

$$m_t = m_t(\pi_{t+1}), \quad m_t'(\pi_{t+1}) \equiv \frac{\partial m_t(\pi_{t+1})}{\partial \pi_{t+1}} < 0, \quad m_t''(\pi_{t+1}) > 0. \quad (6)$$

The index $t$ on $m_t$ indicates that the demand for real balances depends on the real interest rate and, possibly, on additional time-varying factors such as changes in financial technology. It should be mentioned that this specification of the demand for real balances can be derived from individual utility maximization when money is modelled as an argument of a separable utility function; see Obstfeld (1991b). Moreover, Cagan (1956) money demand functions of the type $\alpha_0 \exp(-\alpha_1 \pi_{t+1})$ [$\alpha_0, \alpha_1$ being constants], which are often used in monetary economics, are a special case of (6).

Eventually, by substituting (6) into (1) and changing from levels into growth rates, the realized inflation rate follows as a function of the rates of change of supplied base money and demanded real balances:

$$\pi_t = \frac{\Delta M_t}{M_t} - \frac{\Delta m_t(\pi_{t+1})}{m_t(\pi_{t+1})}. \quad (7)$$

Since in equilibrium the authority knows the rational individual inflation expectation, one can take the realized inflation rate, instead of the nominal money supply, as the policy instrument under the control of the authority. This is is analytically more convenient.\(^7\)

2.2 The consolidated budget constraint of the public sector

We assume for simplicity that real government expenditures, $g_t$, are exogenously given.\(^5\) In order to finance the nominal value of these expenditures, $G_t$, the government may collect output taxes, $T_t$, use the profit of the central bank, $X_t$, or raise funds through increasing the total stock $B_t^{\text{tot}}$ of government bonds. The
stylized government flow budget constraint in nominal terms, which requires equality between nominal expenditures and nominal revenues at the end of any period, is therefore given by

\[ G_t + (1 + i_t)B_t^{\text{tot}} = B_t^{\text{tot}} + T_t + X_t. \]  

(8)

Nominal tax revenues in (8) equal the product of the average tax rate \( \tau \) and nominal output \( Y_t \),

\[ T_t = \tau_t Y_t. \]  

(9)

Furthermore, nominal central bank profit, \( X_t \), is determined by the interest receipts on the previous period’s central bank portfolio:

\[ X_t = i_t(B_t^{\text{cb}} + C_t^{\text{cb}}). \]  

(10)

In equation (10), \( B_t^{\text{cb}} \) and \( C_t^{\text{cb}} \) denote the stock of government bonds and privately issued bonds in the central bank portfolio respectively. To write \( X_t \) in this form, two simplifying assumptions must be made. Firstly, the nominal interest rate needed to induce the central bank to hold privately issued assets must be the same as the one the private sector requires to hold government bonds. This excludes the possibility of implicit central bank subsidies to the private sector through accepting interest rates on \( C_t^{\text{cb}} \) that are below the market rate; compare Klein and Neumann (1990) for further discussion. Secondly, the operating cost of the central bank have been set equal to zero. This is not to say that these cost can be neglected in practice. For Germany, for instance, Klein and Neumann (1990) estimated them at around 10% of the total seigniorage in the 1980s. Given the present interest, however, the operating cost are assumed to be exogenous and can thus be suppressed.

Throughout most of the paper, the central bank is modelled as totally dependent on the government; the entity comprising both of them is synonymously referred to as the authority, the policy maker or the public sector. Substituting (10) into (8) and using the fact that changes in base money must result from open market operations, i.e. \( \Delta M_t = \Delta(B_t^{\text{cb}} + C_t) \), we obtain the consolidated, nominal flow budget constraint of the public sector,

\[ G_t + (1 + i_t)(B_t^{\text{cb}} + C_t^{\text{cb}}) = (B_t - C_t) + T_t + \Delta M_t. \]  

(11)

where \( B_t \) denotes the stock of government bonds not held by the central bank, i.e. \( B_t = B_t^{\text{tot}} - B_t^{\text{cb}} \). Government bonds held by the central bank cancel when consolidating the budget constraint, because they are an asset of the central bank and a liability of the government.

Since it will be needed below when the Persson, Persson and Svensson (1987) solution is discussed, we allow for the possibility that a share \( (1 - \omega_t) \) of government bonds is indexed to realized inflation, \( \omega_t \in [0, 1] \). While both types of bonds pay the same ex ante nominal interest rate, the ex post nominal interest rate factor on indexed bonds is \( (1 + r_t)/(1 - \pi_t) \), as opposed to \( (1 + r_t)/(1 - \pi_t') \) on non-indexed bonds. Dividing equation (11) by the price level and using the fact that \( P_{t-1}/P_t = 1 - \pi_t \), an expression for the consolidated public flow
constraint in real terms is obtained:

\[ g_t + (1 + r_t)(1 - \omega_{t-1})b_{t-1} + \frac{1 - \pi_t}{1 - \pi_t^*} (1 + r_t) (\omega_{t-1} b_{t-1} - c_{t-1}) \]

\[= (b_t - c_t) + \tau_t y_t + \Delta m_t + \pi_t m_{t-1}. \quad (12)\]

Adding \((1 + r_t)(\omega_{t-1} b_{t-1} - c_{t-1})\) on both sides of (12) and rearranging gives the following version of the consolidated flow constraint in real terms:

\[ \Delta (b_t - c_t) = [g_t + r_t(b_{t-1} - c_{t-1})] 
- [\tau_t y_t + \Delta m_t + \pi_t m_{t-1} + (\pi_t - \pi_t^*)(1 + i_t)(\omega_{t-1} b_{t-1} - c_{t-1})]. \quad (13)\]

Equation (13) highlights the fact that the public sector must increase the real value of its net interest-bearing liabilities [left hand side] when its total real expenditures [first sum in brackets on the right hand side] exceed its total real revenues [second sum in brackets on the right hand side].

2.3 The revenues from the creation of money

2.3.1 Total seigniorage

In this subsection, an expression for total [real] seigniorage, \(s_t\), is derived and interpreted. Defining seigniorage in the broadest possible sense as the sum of all public sector revenues resulting from the monopoly power to issue money, \(s_t\) comprises all terms in (13) that would be absent in a non-monetary barter economy. In a barter economy, there would be no central bank assets \(c_t\) and no real balances \(m_t\), i.e. \(c_t = m_t = 0\). Moreover, all government bonds would be like indexed bonds, implying \(\omega_t = 0\). Hence, the flow budget constraint of the public sector in a barter economy would read as simply

\[ \Delta b_t = (g_t + r_t b_{t-1}) - \tau_t y_t. \quad (14)\]

One may, therefore, write the consolidated real flow constraint in a monetary economy as

\[ \Delta b_t = (g_t + r_t b_{t-1}) - (\tau_t y_t + s_t). \quad (15)\]

Subtracting (13) from (15) gives a formal expression for seigniorage,

\[ s_t = \Delta [m_t(\pi_t^* - c_t) + \pi_t m_{t-1}(\pi_t^* + r_t c_{t-1}) + (\pi_t - \pi_t^*)(1 + i_t)(\omega_{t-1} b_{t-1} - c_{t-1})], \quad (16)\]

which comprises the following components:

1. The real revenue from those increases in desired real balances that are not used to purchase additional central bank assets, i.e. \(\Delta (m_t - c_t)\).
2. The reduction of the real value of nominal money holdings due to realized inflation, i.e. \(\pi_t m_{t-1}\). This term is usually referred to as the inflation tax, taking the realized inflation rate \(\pi_t\) as the tax rate and real balances \(m_{t-1}\) as the tax base.
3. The real interest payments $r_c t_{t-1}$ on the previous period’s stock of privately issued assets held by the central bank.

4. The revenue from unexpected devaluations of interest bearing, non-indexed, net public liabilities, i.e. $(\pi_t - \pi_t^n)(1 + i_t)(\omega_{t-1} b_{t-1} - c_{t-1})$.

Notice that government debt enters the expression for seigniorage differently from privately issued assets held by the central bank. All changes of the stock of privately issued assets in the central bank portfolio as well as the real value of interest payments on those assets affect real seigniorage. However, only the changes of the real value of government bonds resulting from unexpected inflation are part of seigniorage. This comes from the fact that all other budgetary effects of government bonds would also occur in a barter economy, and can thus not be related to the revenues from the government’s monopoly to issue money.

2.3.2 Seigniorage from expected inflation

In order to understand the components of seigniorage more deeply, we distinguish between [real] seigniorage from expected and from unexpected inflation.

Consider first a situation in which all inflation is fully anticipated and denote the seigniorage from expected inflation by $s^e_t$. Setting $\pi_t = \pi_t^n$ in (16), one obtains

$$s^e_t = \Delta m_t (\pi_t^n + 1) - c_t + r_c t_{t-1}. \quad (17)$$

It should be pointed out that $s^e_t$ may well comprise components that were not expected in period $t-1$. An example would be the effect of an unexpected change in financial technology on $s^e_t$. However, since, in general, such an unexpected change is not caused by the creation of surprise inflation, it is correctly included in $s^e_t$, rather than in the revenues from unexpected inflation to be defined below.

Two extreme cases may shed some additional light on the expression for $s^e_t$:12

1. The central bank implements monetary policy solely through open-market operations in government bonds:
   In this case, central bank credit is extended exclusively to the government, and $c_t = 0$ in every period. Thus, the seigniorage from expected inflation follows from (2) and (17):
   $$s^e_t = \Delta m_t + \pi_t^n m_{t-1} (\pi_t^n) + r_c t_{t-1}. \quad (18)$$
   Since $s^e_t$ is now given by the real value of changes in nominal base money, (18) has at times been called the cash-flow or the monetary measure of seigniorage; see Repullo (1991) and Klein and Neumann (1990) respectively.

2. The central bank implements monetary policy solely through open market operations in privately issued bonds:
   In this case, central bank credit is exclusively extended to the private
sector, and \( m_t = c_t \). Using (3) and (4), expression (17) then becomes:

\[
s_t^e = (\pi_t + r_t) m_{t-1} = \frac{i_t}{1 + i_t} m_{t-1} = (1 + r_t) \rho_{t-1} m_{t-1}.
\]  

Hence, the seigniorage from expected inflation equals the value of the nominal interest payments that the private sector foregoes when holding non-interest-bearing money instead of interest-bearing assets. Notice first that since the real opportunity cost of holding money was defined in terms of period \( t - 1 \) output [compare (4)], it needs to be multiplied by the factor \( 1 + r_t \) in order to obtain a period \( t \) value. Secondly, for small rates of real interest and expected inflation, one has the approximation \( s_t^e = i_t m_{t-1} \). This is the opportunity cost measure of seigniorage commonly employed in the literature; compare, for instance, Johnson (1969a,b), Barro (1982) and Gros (1993).

It has often been disputed whether or when it is appropriate to use the cash-flow measure or the opportunity cost measure. Since most real world central banks extend credit to both the government and the private sector, neither measure is appropriate in general for calculating the seigniorage from expected inflation accruing in any single period. Instead, (17) ought to be used. However, in stylized infinite horizon models such as the present one, this is not a problem because the present discounted value \( s_t^e \) of the sum of all future period-by-period seigniorage from expected inflation is unaffected by alternative ways of implementing monetary policy. In order to see this, recall the above assumptions that the real interest rate is exogenous, that the Fisher relation is valid and that the central bank does not subsidize the private sector. In addition, we assume the validity of the standard *transversality condition for the evolution of* \( c_t \),

\[
\lim_{t \to \infty} R_t c_t = 0,
\]

where \( R_t \) denotes the market discount factor,

\[
R_{t-1} = 1 + r_{t-1}, \quad R_0 = 1, \quad R_t = \prod_{i=1}^{t} \frac{1}{1 + r_i}.
\]

The transversality condition (20) precludes cases in which the central bank holds seigniorage back by hoarding assets ad infinitum. Providing the above conditions hold, \( s_t^e \) can be shown to equal

\[
s_t^e \equiv \sum_{i=0}^{\infty} R_i s_i^e = \sum_{i=0}^{\infty} R_i \frac{i_{i+1}}{1 + i_{i+1}} m_i + (1 + r_0) c_{-1} - (1 - \pi_0) m_{-1},
\]

where \( c_{-1} \) and \( m_{-1} \) are the given initial values of privately issued bonds held by the central bank and of real balances respectively.

(22) implies that the present discounted value of seigniorage from expected inflation is invariant with respect to the different ways of implementing monetary
policy. Intuitively, this result arises because an additional purchase of privately issued assets decreases current seigniorage by the same amount as it increases the present value of future seigniorage [through the additional revenues from interest payments and sales of these assets]. The invariance of $s^t$ with respect to the implementation of monetary policy is important for what follows. It means that restrictions on the central bank’s open market operations do not affect the intertemporal budgetary position of the public sector. We will come back to this when considering specific ways of asset management that impose such restrictions in order to cure the time consistency problem of monetary policy.

Before moving on to a discussion of the real seigniorage from surprise inflation, it should be mentioned that the chosen measure of real seigniorage matters crucially in empirical studies, which inevitably work with a finite horizon. This is true, because, over a finite horizon, additional central bank purchases of privately issued assets may well affect the present value of real seigniorage from expected inflation; see Klein and Neumann (1990). Although rarely appreciated, the empirically correct measure [in the absence of unexpected inflation] is (17), possibly with some slight modifications reflecting institutional peculiarities of the country under consideration. This was stressed in a series of empirical studies by Klein and Neumann (1990) and Neumann (1992, 1995). In contrast, Barro (1982) or Fischer (1982) are examples of empirical work that crudely use the opportunity cost measure (19), or the cash flow measure (18) respectively. The resulting differences between the figures for real seigniorage can be remarkably large. For example, over the period from 1960 to 1973, Klein and Neumann (1990) estimated for Germany that the government received on average only around 13% of the seigniorage that Fischer (1982) found by using the cash-flow measure.

2.3.3 Seigniorage from surprise inflation

Expression (16) for total seigniorage shows that unexpected inflation yields seigniorage too. Subtracting (17) from (16), the seigniorage from surprise inflation amounts to

$$s^t_t = (\pi_t - \pi^e_t)[m_{t-1}(\pi^e_t) + (1 + i_t)(\omega_{t-1}b_{t-1} - c_{t-1})].$$

(23)

Surprise inflation yields revenues for two reasons: first, nominal balances carried over from the previous to the current period are determined on the basis of the expected inflation rate $\pi^e_t$; compare (6). The public sector can thus reduce the real value of nominal balances through inflating more than was expected by the private sector. Second, the interest bearing net liabilities of the public sector that are not indexed are only protected against expected inflation. Surprise inflation, therefore, reduces the real value of public net liabilities as long as the non-indexed part is positive, i.e. $\omega_{t-1}b_{t-1} - c_{t-1} > 0.13$

2.4 The social losses from taxation and inflation

The previous discussion has shown that the public sector may finance its expenditures through the revenues from output taxation and seigniorage, where
The present value of the sum of current and future period-by-period social loss, \( l_t \), discounted at the market real interest rate is given as:

\[
   l \equiv \sum_{t=0}^{\infty} R_t l_t. \tag{24}
\]

The period \( t \) social loss is assumed to take the additively separable form:

\[
   l_t = l_1(\tau_t) + l_2(i_t) + l_3(\pi_t). \tag{25}
\]

which reflects the following sources of social loss:

1. The social loss from the use of the endowment tax is expressed by the term \( l_1(\tau_t) \). Since output is exogenously given, there are no social cost due to tax distortions in the present model. Instead, the social loss from output taxation is assumed to arise from tax collection or enforcement cost, as in Barro (1979, 1983) and Mankiw (1987). These collection cost do not show up in the public sector budget constraint, because it is implicitly assumed that private agents have to pay them in addition to their tax payments.

   One may motivate tax collection cost by the assumption that individuals try to avoid taxes because taxes reduce their consumption. If utility has the standard convexity properties, these activities will be intensified more than proportionally with increasing average taxes. Hence, we assume that the collection cost are monotonically and strictly convexly increasing in the average tax rate.

2. The term \( l_2(i_t) \) captures the social loss due to expected inflation. Expected inflation leads to a positive opportunity cost of holding money and consequently to lower holdings of real balances than are socially optimal. Avoidable banking and shopping activities then become necessary, giving rise to higher transaction cost. These well known ‘shoe-leather cost’ increase in the expected inflation rate and are minimized when the nominal interest rate is zero and the satiation level of real balances is held. Given the standard convexity properties of utility, these cost are also strictly convex in expected inflation and the larger they are, the more sensitively real balances react to expected inflation [i.e. the higher is the interest elasticity of real balances].

3. In contrast to expected inflation, actual inflation does not cause social loss from suboptimally low holdings of real balances. This holds true because private agents have already determined their desired holdings of nominal balances when the authority decides whether or not to create more inflation than is expected. The social cost of actual inflation, which are represented by the term \( l_3(\pi_t) \), come instead from menu cost, in that
costly adjustments of all non-indexed nominal contracts are required. These costs are also assumed to increase strictly convexly in the actual inflation rate.

It should be mentioned that there is an additional possibility by which actual inflation may cause social loss. If actual inflation exceeds expected inflation and if the population is heterogeneous, then the resulting unexpected inflation may give rise to an unwanted redistribution among private agents. For example, if a positive share of privately issued bonds is not indexed [which appears to be a realistic supposition], then unexpected inflation transfers real resources from creditors to debtors. The resulting social cost could be captured by including a term of the form \( l_{\text{d}}(\pi - \pi^e) \) in (25), which has been suggested by Grossman (1990). However, this would not change the results of the subsequent analysis in an important way, as long as the loss term \( l_{\text{d}}(\pi) \) is preserved.\(^{17}\)

After having motivated the different terms in (25), we need to impose some additional structure on the social loss function. So far, specification (25) only requires separability between the social losses from different revenue instruments, which is a standard, simplifying, but not innocuous assumption. For example, it precludes the possibility that realized inflation affects the social loss from output taxation. Higher inflation could, for example, increase the collection cost of the output tax when there are significant collection lags. The real value of tax revenues would then be reduced until they were collected; see Tanzi (1977) for the classic argument on collection cost due to inflation, and Dixit (1991) or Mourmouras and Tijerina (1994) for reconsiderations. While this may have interesting implications, it is not of primary interest here.

In addition to separability, the previous discussion of the different components of \( l_{\text{s}} \) suggested that social loss must satisfy the following standard requirements:

(i) Social loss depends positively and strictly convexly on each of the different policy instruments.

(ii) Not using an instrument implies that the minimum [marginal] social loss of zero.

(iii) The social loss, as well as the marginal loss, go to infinity if \( \tau, \pi^e \) or \( \pi \) approach one. [This is to say that infinite social loss and infinite marginal social loss are caused by tax rates of one hundred percent.]

In order to ensure that all of the above properties are satisfied, the following functional form of social loss, similar to the one suggested by Barro (1983), is used from now on:

\[
l_i = \kappa_{i,1} \left[ \exp\left(\frac{\tau_i^2}{1 - \tau_i^2}\right) - 1 \right] + \kappa_{i,2} \left[ \exp\left(i_i^2\right) - 1 \right] + \kappa_{i,3} \left[ \exp\left(\frac{\pi_i^2}{1 - \pi_i^2}\right) - 1 \right]. \tag{26}
\]

A little thought reveals that (26) satisfies the properties (i), (ii) and (iii).
2.5 The statement of the optimal taxation problem

We now state formally the optimal taxation problem of finding a sequence of tax rates and inflation rates that minimizes social loss while satisfying the consolidated flow budget constraint in every period. To this end, the Lagrangian $\mathcal{L}$ is defined as the sum of the present discounted value of the period-by-period social loss, and the product of a Lagrange multiplier $\lambda_t$ and the consolidated real flow constraint (13),

$$\mathcal{L} \equiv \sum_{t=0}^{\infty} R_t \left[ l_{t1}(\tau_t) + l_{t2}(i_t) + l_{t3}(\pi_t) \right] - \lambda_t \left[ \Delta b_t - g_t - r_t b_{t-1} + \tau_t y_t + s^c_t + s^s_t \right], \quad (27)$$

where $l_t$, $s^c_t$ and $s^s_t$ are given by (26), (17) and (23) respectively. The optimal taxation problem is to minimize the Lagrangian $\mathcal{L}$ with respect to $b_t$, $\tau_t$ and $\pi_t$ ($t = 0, 1, 2, \ldots$).

In order to ensure that the optimal taxation problem is well defined, some additional assumptions need to be made. To begin with, the monetary authority is supposed to implement base money changes by a policy that uniquely determines $c_t$ for any chosen inflation rate. Possible examples of such a policy have been described in subsection 2.3. Moreover, $\omega_t$ is treated as exogenously given for the moment. Below, it will be shown that one can impose restrictions on $c_t$ and $\omega_t$ such that the time consistency problem is solved.

Furthermore, in addition to the transversality condition (20) on $c_t$, the fulfilment of a transversality condition on the evolution of $b_t$ is required,

$$\lim_{t \to \infty} R_t b_t = 0, \quad (28)$$

This is the well known solvency or no Ponzi-game condition that the growth rate of government debt be smaller than the real rate of interest. Without this condition the optimal taxation problem would not be well defined because the public sector could cover its expenditure by issuing debt at an accelerating rate. We therefore assume from now on that the initial debt level and the exogenously given stream of government expenditure are such that the public sector is solvent, that is, that the flow constraint can be met without violating (28).

Finally, the authority is implicitly assumed to honour its debt, implying that debt repudiation is excluded as a possibility of reducing government liabilities. The conventional justification for this assumption is that a debt default destroys the authority’s reputation for being a reliable debtor. This leads to future cost that presumably out-weight the benefit of defaulting. Such cost, for instance, may arise from subsequent borrowing difficulties. However, Chari and Kehoe (1993a,b) have recently challenged this argument by showing for a closed economy that even the most severe reputational consequences of debt repudiation may not be sufficient to prevent an optimizing government from defaulting. Hence, there is still a controversial theoretical question of why most real world governments do honour their debt [at least in ‘normal times’]. Despite this fact, we have only limited space and restrict attention to a government that does not consider defaulting on its debt as a feasible policy option.
3. The ex ante optimal policy

We have now extended Barro’s (1983) model to a framework rich enough to analyze the optimal taxation problem. As a benchmark case, we will first study the optimal tax policy under precommitment. The term precommitment means that the authority credibly binds itself to follow an announced policy. Many authors, who wanted to concentrate on the public finance aspects of the optimal tax package, have restricted their attention to precommitment; see, for example, Drazen (1979), Kimbrough (1986a), Mankiw (1987), Grilli (1989), Végh (1989b), Yashiv (1989), Dixit (1991), or Calvo and Leiderman (1992). In section 4, we will also characterize the solution of the optimal taxation problem under discretion, which is relevant because most policy makers in the real world are not precommitted. Since the discretionary solution turns out to be welfare inferior compared to the precommitment solution, section 5 provides a discussion of how the precommitment outcome can be achieved.

The plan for the remainder of the present section is as follows: in subsection 3.1, we formally derive the solution to the optimal taxation problem under precommitment. Subsection 3.2 provides an interpretation of the optimality conditions in the light of the Ramsey (1927) principle. The section ends with subsection 3.3, which is concerned with the issue of whether or not our results generalize to equilibrium models of optimal taxation.

3.1 The solution to the optimal taxation problem under precommitment

When the policy maker is precommitted within a deterministic environment, unexpected inflation is not available as a policy instrument and the realized inflation rate must equal the expected inflation rate. Consequently, the inflation expectation is one of the choice variables and the policy maker can optimize in a similar way to a Stackelberg leader. Taking account of the follower’s reaction function, i.e. of the private sector’s demand for real balances, the policy maker chooses the optimal sequence of future stocks of government bonds, tax rates and inflation rates, and precommits to stick to it. Substituting $\pi_t = \pi^*_t$ and $s^*_t = 0$ ($t = 0, 1, 2, \ldots$) into (27), the Lagrangian under precommitment is found to be

$$\mathcal{L}^{\text{pre}} = \sum_{t=0}^{\infty} \sum_{i=0}^{\pi_t} \left[ l_{t1}(\tau_t) + l_{t2}(i_t) + l_{t3}(\pi_t) \right] - \lambda_t \left[ \Delta b_t - g_t - r_t b_{t-1} + \tau_t y_t + s^*_t \right]$$

The optimal taxation problem under precommitment is to minimize (29) with respect to $b_t$, $\tau_t$, and $\pi_t$ ($t = 0, 1, 2, \ldots$). This minimization problem is well defined because the loss function is strictly convex and the constraints are concave functions in the tax rate and in the inflation rate. That $s^*_t$ is a concave function in $\pi_t$ is ensured by the assumption that $m_t$ is convexly decreasing in expected inflation; compare (6).
By the Kuhn-Tucker theorem, the following first order conditions are necessary conditions for an interior solution, \( \{ b_t^{\text{prec}}, \tau_t^{\text{prec}}, \pi_t^{\text{prec}} \} \), to the optimal taxation problem. First, the optimal policy has to satisfy the flow constraint (13) with \( \pi^*_t = \pi_t^{\text{prec}} \) in each period. Also, the first-order condition for \( b_t^{\text{prec}} \) is

\[
\frac{\partial G^{\text{prec}}}{\partial b_t} = 0 \Rightarrow \lambda^{\text{prec}} = \lambda^{\text{prec}}_{t-1}.
\]  

(30)

We can see that the Lagrange multiplier does not change over time, \( \lambda^{\text{prec}} = \lambda^{\text{prec}}_{t-1} \). In order to find the additional two first-order conditions for \( \tau_t^{\text{prec}} \) and \( \pi_t^{\text{prec}} \), we must first recall that output was assumed to be exogenously given and thus does not depend on the average tax rate. Moreover, since \( \pi^*_t = \pi_t^{\text{prec}} \) under precommitment, the Fisher relation (3) implies that the nominal interest rate is affected by the choice of \( \pi_t^{\text{prec}} \),

\[
i_t^{\text{prec}} = \frac{r_t + \pi_t^{\text{prec}}}{1 - \pi_t^{\text{prec}}} \Rightarrow \partial i_t^{\text{prec}}/\partial \pi_t = \frac{1 + i_t^{\text{prec}}}{1 - \pi_t^{\text{prec}}}.
\]  

(31)

Hence, for the two additional first-order conditions, one obtains

\[
\frac{\partial G^{\text{prec}}}{\partial \tau_t} = 0 \Rightarrow \frac{\partial}{\partial \tau_t} l_1(\tau_t^{\text{prec}}) = \lambda^{\text{prec}} y_t,
\]  

(32a)

\[
\frac{\partial G^{\text{prec}}}{\partial \pi_t} = 0 \Rightarrow \frac{\partial}{\partial \pi_t} \left[l_2(i_t^{\text{prec}}) + l_3(\pi_t^{\text{prec}})\right] = \lambda^{\text{prec}} [1 - \varepsilon_{t-1}(\pi_t^{\text{prec}})] m_{t-1}(\pi_t^{\text{prec}}),
\]  

(32b)

where

\[
\varepsilon_{t-1}(\pi_t^{\text{prec}}) \equiv -\frac{r_t + \pi_t^{\text{prec}}}{m_{t-1}} \frac{\partial m_{t-1}}{\partial \pi_t} (t = 0, 1, 2, \ldots).
\]  

(33)

\( \varepsilon_{t-1}(\pi_t^{\text{prec}}) \) can be interpreted as the [absolute-value] elasticity of real balances in period \( t-1 \), with respect to the opportunity cost \( \rho_t^{\text{prec}} \) of holding money. This follows from (4):

\[
\varepsilon_{t-1}(\pi_t^{\text{prec}}) = -\frac{r_t + \pi_t^{\text{prec}}}{m_{t-1}} \frac{\partial m_{t-1}}{\partial \rho_{t-1}} = -\frac{\rho_{t-1}^{\text{prec}}}{m_{t-1}} \frac{\partial m_{t-1}}{\partial \rho_{t-1}}.
\]  

(34)

In summary, the first-order conditions for a solution to the optimal tax problem under precommitment are (13), (30), (32a) and (32b). By the Kuhn-Tucker Theorem, we also know that if there exists a sequence \( \{ b_t^{\text{prec}}, \tau_t^{\text{prec}}, \pi_t^{\text{prec}} \} \), satisfying these first-order conditions, it does indeed solve the optimal taxation problem, which ensures sufficiency. That such a sequence exists can be seen from the following two facts: on one hand, if neither revenue instrument is used, its marginal social loss equals zero; on the other hand, a tax rate or inflation rate of one results in an infinite marginal social loss. Because the marginal social gain of
either instrument [right hand side of the first-order conditions] is positive and remains finite, continuity implies existence. Furthermore, since there can be at most one solution to a problem of minimizing a strictly convex function over a convex set, the uniqueness of this solution is also ensured. Hence, the optimal taxation problem under precommitment has a unique interior solution. Since a first-best policy of financing all expenditure from a loss-free instrument is not available when all revenue instruments give rise to social loss, this solution results in the highest achievable social welfare. For this reason, it is often referred to as the [best] second-best or the ex ante optimal policy.

3.2 The Ramsey principle of optimal taxation

Intuitively, the first-order conditions (32a) and (32b) require, for each revenue instrument, that the marginal social loss from an incremental increase of that instrument be equal to the marginal social gain. As is standard, the marginal gain is expressed in units of social loss. The conversion of units of output into units of social loss is achieved by means of multiplication by the shadow price, $\lambda^{\text{pre}}$, of public revenues. More precisely, $\lambda^{\text{pre}}$ is the marginal decrease of social loss that would result in an optimum if the government had one additional unit of revenues available.23 In order to interpret the ex ante optimal policy further, it is useful to express the first-order conditions (32a) and (32b) in a different way. Employing the definition for the present discounted values of seigniorage from expected inflation, and that of social loss, (22) and (24), it is straightforward to verify that (32a) and (32b) can, equivalently, be written as

$$\frac{\partial l}{\partial \tau_t} = \lambda^{\text{pre}} R_t y_t, \quad (35a)$$

$$\frac{\partial l}{\partial \pi_t} = \lambda^{\text{pre}} \frac{\partial s^e}{\partial \pi_t} (t = 0, 1, 2, \ldots). \quad (35b)$$

The combination of these two equations allows the elimination of the Lagrange multiplier:

$$\frac{\partial l/\partial \tau_t}{R_t y_t} = \frac{\partial l/\partial \pi_t}{\partial s^e/\partial \pi_t}, \quad (36a)$$

$$\frac{\partial l/\partial \tau_t}{R_t y_t} = \frac{\partial l/\partial \pi_{t+1}}{R_{t+1} y_{t+1}}, \quad (36b)$$

$$\frac{\partial l/\partial \pi_t}{\partial s^e/\partial \pi_t} = \frac{\partial l/\partial \pi_{t+1}}{\partial s^e/\partial \pi_{t+1}} (t = 0, 1, 2, \ldots). \quad (36c)$$

This version of the necessary conditions brings out clearly that the ex ante optimal policy must satisfy the Ramsey (1927) principle of optimal taxation:24 equation (36a) is the static first-order condition, requiring equality between the ratios of the
marginal social loss to marginal revenue for both instruments in any period. The
two following equations (36b) and (36c) are the intertemporal first-order
conditions [Euler equations]. They imply that taxes are to be smoothed over time
because, in an optimum, the ratios of the marginal social loss to the marginal
revenue of either instrument must be equalized between periods. If the economy is
in a steady state, in which the marginal losses and revenues from either instrument
grow at the same rate, then the first-order conditions lead to constant output taxes
and constant inflation rates. In this case, temporary changes in expenditure are to
be financed by bond issues, which do not have real effects in a Neo-Ricardian
world.

It is now shown that the different recommendations of the public finance
literature on optimal inflation rates follow from the Ramsey principle under
specific conditions.

(i). The ratio of the marginal social loss to marginal revenue is much larger for
output taxation than for seigniorage

It is optimal to rely mainly on seigniorage to finance public expenditure. Rearranging (36a) shows that for very large marginal social losses from output
taxation, it is optimal to choose the inflation rate so as to maximize the present
value of seigniorage,

$$\frac{\partial l}{\partial \tau_t} = \frac{\partial s^e}{\partial \pi_t} = 0. \quad (37)$$

We can observe that the seigniorage-maximizing inflation rate is the solution to a
standard monopoly problem, notably that of choosing an inflation rate such that
the marginal revenue from the ‘production of money’ equals the marginal cost,
which are zero. From (32b) and (34) it emerges that the marginal seigniorage
revenue is zero if a one-percent increase in the opportunity cost $\rho_{t-1}$ of holding
money leads to a one-percent decrease of real balances,

$$1 = e_{t-1}(\pi_t^{pre}) = \frac{\rho_{t-1}^{pre}}{m_{t-1}} \frac{\partial \pi_t}{\partial \rho_{t-1}}. \quad (38)$$

This is a discrete time version of Auernheimer’s (1974) unit-elasticity rule for
seigniorage-maximizing inflation.

Notice that if attention had been restricted to a stationary state with constant
inflation and constant real balances, and if the cash flow measure (18) had been
used, then the present discounted value of [the ex ante optimal] seigniorage would
have been equal to the present discounted value of the [constant] inflation tax,

$$s^{pre} = \left[ \frac{(1 + r)}{r} \right] \pi^{pre} m(\pi^{pre}).$$

In this case, the seigniorage-maximizing inflation rate is characterized by Cagan’s (1956) version of the unit-elasticity rule,

$$1 = \frac{\pi^{pre}}{m} \frac{\partial m}{\partial \pi}. \quad (39)$$

However, as the previous discussion has shown, using this formula is not
appropriate in general, because it ignores the seigniorage due to changes in real balances; see also the discussion in Auernheimer (1974).


(ii). The ratio of the marginal social loss to marginal revenue is much smaller for output taxation than for seigniorage

The optimal policy implies that a major share of the revenues is collected through output taxes and only a small share through seigniorage. Now, if using the output tax causes almost no rise in social cost, optimality requires that the social losses from inflation ought to be minimized,

$$\frac{\partial l}{\partial \pi_t} = 0 \Rightarrow \frac{\partial l}{\partial \pi_t} = \frac{1}{y_i} \frac{\partial l}{\partial \pi_t} \frac{\partial s^e}{\partial \pi_t} = 0.$$  (40)

Also, if the marginal social loss from a non-zero nominal interest rate is weighted much more heavily than that from actual inflation \(\kappa_2 \gg \kappa_3\) then the opportunity cost of holding money should be driven to zero, i.e. \(i_t^{\text{pre}} = 0\). By the Fisher relation (3) this is equivalent to

$$i_t^{\text{pre}} = -r_t,$$  (41)

i.e. the optimal inflation rate is the negative of the real interest rate. This is the Friedman (1969) rule of optimal deflation, which has at times also been called the Chicago rule or the optimum quantity of money rule; see also Marty (1968) and Brock (1975). It implies a contraction of the money supply in any period, resulting in negative seigniorage revenues, which are financed through output taxation. Friedman’s result arises under the assumption that output taxation is almost a loss-free revenue source, similar to a lump sum tax, through which all expenditure can be financed at zero social cost. Since money is produced without cost, it should then be supplied until the satiation of real balances is reached. This parallels the principle that it is welfare-optimal to supply whatever quantity is demanded of a free good.

It should be stressed that, even if output taxes do not give rise to significant social loss, the Friedman rule is only optimal if the cost of deflation are negligible relative to the social cost of expected inflation.

(iii). For both revenue instruments, the ratios of the marginal social loss to marginal revenue are of the same order of magnitude

According to the Ramsey principle, expenditures are in this case optimally financed from both instruments, and positive inflation rates are part of the optimal
policy. In particular, optimality implies that the higher the marginal social loss from an instrument relative to its marginal revenue, the less revenue should be collected from it.

In summary, positive inflation rates have been shown to be optimal if the ratio of the marginal social loss to the marginal revenue of output taxation is not negligible. The following features may even lead to the optimality of high inflation: first, tax enforcing and collecting authorities work inefficiently or tax evasion is significant. This is captured in our model by large marginal collection cost for output taxes. Second, the interest elasticity of real balances is small, so that high expected inflation rates lead to small reductions in real balances and, thus, to small marginal social losses from expected inflation. This corresponds to the intuition that more inelastic tax bases ought to be taxed more heavily. And third, desired real balances are large for any given inflation rate, offering a large tax base for inflation. This would be the case if the financial sector is relatively underdeveloped and the bulk of transactions is facilitated in cash. Note that, for these reasons, an active black market or shadow economy tends to increase the optimal inflation rate, because illegal transactions are mostly facilitated in cash, in order to avoid taxes.

In light of these arguments it seems worthwhile to devote some attention to the effects of financial innovations on the optimal inflation rates and on social welfare. This issue is important, for instance, when the welfare effects of the introduction of new financial technologies in developing countries or in Eastern Europe are to be judged. For each given rate of expected inflation, a financial innovation presumably reduces the desired holdings of real balances, leading to lower marginal social cost and lower marginal seigniorage from expected inflation. The net effect on the optimal rate of inflation is thus ambiguous: if the marginal social cost decrease by more [less] than the marginal revenue, higher [lower] optimal inflation rates would result.25 Similarly, it is not clear how a financial innovation affects social welfare. Ceteris paribus, of course, it increases welfare. However, the public sector also has to increase the tax rate and the rate of inflation when agents hold fewer real balances and seigniorage revenues decrease. These higher tax rates lower social welfare. If this effect is large enough, it may even overcompensate the initial welfare increase, thereby resulting in a negative net effect on social welfare. This is more likely to happen, the more important seigniorage is as a source of public revenue. If individuals reduce their desired holdings of real balances sharply, for example, because credit cards become an accepted means of payment or an efficient cheque-clearing system is installed, large increases in inflation may be inevitable to meet the revenue requirements.26 Authorities confronted with this problem thus ought to be advised to improve the efficiency of their tax system, in order to have alternative revenue sources at hand when seigniorage decreases.

3.3 Possible generalizations to equilibrium models of optimal taxation

Having discussed the optimal policy, the question remains as to whether the results found are a consequence of the simplicity of our model. Needless to say,
the use of an equilibrium framework would be more satisfactory. In a more complete analysis, our tax on exogenous output would be substituted by a *distortionary tax* on labor income or on consumption expenditure. Two basic cases can then be distinguished. First, if collecting taxes is costly, as in cases 1 and 3 of the last subsection, then our results can also be obtained in general equilibrium frameworks; see Aizenman (1983, 1987) and Végh (1989a). Second, in the absence of collection cost, the only clear-cut theoretical result is that positive domestic inflation is optimal in small open economies with currency substitution when the foreign nominal interest rate is positive; see Végh (1989b, 1995) and Guidotti and Végh (1993a). Otherwise, whether the Friedman rule is optimal depends on the way in which money is introduced into the model. There are three main alternatives: (i) money holdings enter the utility function directly; (ii) individuals face a cash-in-advance constraint; (iii) money is an input factor in a transactions technology.

For a long time, there has been agreement that positive inflation is optimal even in the absence of collection costs, if holding money yields direct utility or if individuals face a cash-in-advance constraint. The first approach was used by, among others, Phelps (1973), Marty (1978), Chamley (1985), Persson et al. (1987), and Aizenman (1993), whereas Braun (1994) employed the second one. In contrast to their results, a recent contribution by Chari, Christiano and Kehoe (1996) shows that the Friedman rule remains optimal in both cases, if the utility function is separable in leisure and if the subutility function over consumption goods is homothetic. Note that these are the conditions for the optimality of uniform taxation in the public finance literature [Atkinson and Stiglitz (1972)].

If the demand for money is derived from a transaction technology, conflicting results have been obtained. Initially, Drazen (1979) and Leach (1983) claimed that a positive inflation rate is optimal, but Kimbrough (1986a,b) proved them to be wrong. Later authors, such as Guidotti and Végh (1993b), thought that Kimbrough’s result depends critically on the assumption of a transactions technology that is homogeneous of degree of one. However, recently Correia and Teles (1996) have shown that this is incorrect and that, indeed, the Friedman rule is optimal for transactions technologies that are homogeneous of any degree.

In sum, it appears that the recommendations of equilibrium models without collection cost are not entirely robust. Hence, I agree with Guidotti and Végh (1993, p. 203) in concluding that ‘… given that the microfoundations of money demand still remain a controversial and, to a large extent, unexplored territory, … we feel that until … the profession reaches something of a consensus, no definite conclusions should be drawn’.

### 4. The time consistency problem

In this section, we show that generally the ex ante optimal policy is not time consistent when the authority has discretion [subsection 4.1]. A policy is called *time consistent* when it is ex ante and ex post optimal. Put differently, time consistency means that the planned policy determined as optimal at the initial date
is still found to be optimal at any later date. Subsection 4.2 contains the characterization of the time-consistent policy under discretion, and a discussion of its properties.

4.1 The time inconsistency of the ex ante optimal inflation policy

While Strotz (1955) initially recognized the possible time inconsistency in economic decision making, Auernheimer (1974) was the first to point out its potential importance in monetary economics. He, however, assumed the problem away by restricting attention to an 'honest government’ that sticks to its promises independently of whether this is optimal or not from a later point of view. Following Kydland and Prescott’s (1977) initial analysis, Calvo (1978a) proved that the inflation pattern optimal from a public finance point of view is in general not time consistent under discretion. The source of the time consistency problem in Calvo’s model is exactly the same as here: individuals determine their inflation expectation and their demand for real balances before the authority chooses the actual inflation rate. For this reason, the amount of nominal balances held in any period adjusts only to expected inflation and is inelastically given when actual inflation is chosen.

To see formally that the ex ante optimal policy is time inconsistent under discretion we proceed indirectly. For an arbitrary period t, it is shown that if individuals expect the precommitment solution $\pi^{prec}$, then the policy maker will, in general, choose a different inflation rate when he reoptimizes in period t, while taking the expectation $\pi^{prec}$ as given. Notice that the nominal interest rate is given when inflation expectations are given.

Taking the partial derivative of the Lagrangian (27) with respect to $\pi_t$, while treating $\pi_t^e$ and $i_t$ as given, one finds the first-order condition for inflation under discretion,

$$\frac{\partial}{\partial \pi_t} l_3(\pi_t) = \lambda [m_{t-1}(\pi_t^e) + (1 + i_t) (\omega_{t-1} b_{t-1} - c_{t-1})].$$

If the ex ante optimal inflation policy $\pi_t^{prec}$ were time consistent, it would have to satisfy both (32b) and this first-order condition, 

$$\frac{\partial}{\partial \pi_t} l_3(\pi_t^{prec}) = \lambda^{prec} [m_{t-1}(\pi_t^{prec}) + (1 + i_t^{prec}) (\omega_{t-1} b_{t-1}^{prec} - c_{t-1}^{prec})].$$

In general, (43) fails to hold, the reasons being the following: first, the left hand side of (43) is smaller than the left hand side of (32b). This comes about because, differently from the case of expected inflation, an increase in actual inflation does not distort the holdings of real balances, because they have already been determined. Second, the right hand side of (43) must be larger than the right hand side of (32b), because actual inflation reduces the real value of the given, non-indexed, net public liabilities. And finally since (32b) holds with equality, the two previous inequalities imply that for $\pi_t^{prec}$ the marginal social loss from inflation

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[left hand side of (43)] is smaller than the marginal social gain [right hand side]. Consequently, under discretion, the policy maker would find it optimal to raise actual inflation beyond expected inflation if individuals expected $\pi_t^{prec}$. For this reason, the ex ante optimal policy is not time consistent.

4.2 The solution to the optimal taxation problem under discretion

Since rational private agents understand that the ex ante optimal solution is time inconsistent under discretion, they will not base their decisions on the expectation of $\pi_t^{prec}$ when the policy maker is not precommitted. Instead, under discretion, individuals expect an inflation rate that does not leave an incentive to deviate. Hence, the policy maker loses his Stackelberg leadership and has to take $\pi_t^e$ as given when optimizing.

In order to characterize the equilibrium under discretion, we solve the optimal taxation problem for an arbitrary inflation expectation, $\pi_t^e$, that the policy maker takes as given. The flow constraint (13) and the two first-order conditions (30) and (32a) are not affected by this change. In contrast, the first-order condition for inflation now becomes (42). In equilibrium, expectations must be self-fulfilling, i.e. $\pi_t^{disc} = (\pi_t^{disc})^e$. In general, this fixed-point problem might not have a solution. However, for the functional form (26), a solution does exist. This can be seen as follows: first, recall that for $\pi_t^{prec}$ the left hand side of (42) is smaller than the right hand side. If the left hand side becomes larger than the right hand side for some finite inflation rate, then continuity implies the existence of at least one finite inflation rate satisfying (42). Since, as before, the optimal tax problem amounts to the minimization of a strictly convex loss function subject to a concave constraint, the Kuhn-Tucker Theorem guarantees that (42) is sufficient and that the solution is unique.

All we therefore need to prove is that an inflation rate exists for which the left hand side of (42) exceeds the right hand side. Substituting both (26) and $\pi_t^{disc} = (\pi_t^{disc})^e$ into the first condition (42) and multiplying the resulting equation by $[1 - (\pi_t^{disc})^2]$, this turns out to be equivalent to

$$2\kappa_1 \frac{\pi_t^{disc}}{1 - (\pi_t^{disc})^2} \exp \left( \frac{(\pi_t^{disc})^2}{1 - (\pi_t^{disc})^2} \right) > \lambda^{disc} \left[ [1 - (\pi_t^{disc})^2] m_{t-1}(\pi_t^{disc}) + [1 + \pi_t^{disc}][1 + r_t]\left[\omega_{t+1} b_{t+1}^{disc} - e_{t+1}^{disc}\right]\right].$$

Since the left hand side of (44) goes to infinity, while the right hand side remains bounded when $\pi_t^{disc}$ approaches one, the left hand side must be larger than the right hand side for some finite inflation rate.

Comparing (32b) and (42), we observe that the equilibrium inflation rate under discretion is larger than under precommitment, $\pi_t^{disc} > \pi_t^{prec}$. An inflationary bias therefore results from the time consistency problem. This bias is larger, the smaller the marginal cost and the larger the marginal revenue from actual inflation. In particular, it increases with the period t stock of non-indexed, net
nominal liabilities of the public sector. We also note that the inflationary bias results in a reduction of social welfare compared to that under precommitment; this must be the case since the discretionary inflation rate, though feasible, is not chosen under precommitment. Similar results are standard in the literature; see, for example, Kydland and Prescott (1977), Fischer (1980) and Barro and Gordon (1983a,b), who used models different from the present one, or Barro (1983) and de Kock and Grilli (1993), who worked with a similar framework.

5. Solutions to the time consistency problem

Having realized the welfare inferiority of the outcome under discretion, it is natural to look for solutions to the time consistency problem. This section discusses three different possibilities, namely, reputational forces [in subsection 5.1], institutional reforms that establish central bank independence [in subsection 5.2], and specific ways of asset and debt management [in subsection 5.3].

5.1 Reputational effects

In order to sketch the idea of reputational effects, we must first observe that so far the equilibrium in any period has been entirely determined by whether or not the authority can precommit and by the inherited initial values of the state variables, i.e. $b_{-1}$, $c_{-1}$, and $m_{-1}$. In particular, any possible effect of reputation on the formation of individual expectations has been ignored.

5.1.1 Reputation in infinitely repeated policy games

Probably the simplest way of modelling an effect of reputation is as follows. Suppose that at the beginning of each period the authority announces that it will follow the ex ante optimal policy during the next period. Without accounting for reputation, this would not be an equilibrium under discretion, because rational private agents understand the time inconsistency inherent in the ex ante optimal policy plan. Therefore, one needs to make the additional assumption that, as long as the authority has not deviated from the announced policy in the last period, it has built up a reputation for not creating surprise inflation. Private agents are then assumed to expect that there will not be an inflation surprise in the next period either. In contrast, if the authority deviates from its announcement and creates surprise inflation in the current period, then its reputation is lost by assumption and individuals expect the discretionary policy for at least a finite number of future periods.

The expectation formation mechanism just described is called a trigger strategy. It was invented by Friedman (1971a) in the context of oligopoly models and was discussed by Barro and Gordon (1983b) as a possible solution to the time consistency problem of monetary policy. Confronted with a trigger strategy, the policy maker faces the following trade-off: a one period deviation from the ex ante optimal policy allows it to collect revenues at a lower social cost than is
possible under the ex ante optimal policy plan; compare the discussion in subsection 4.1. Ceteris paribus, this improves social welfare. However, afterwards, individuals will expect the discretionary inflation rate for at least a finite number of future periods. These periods may be viewed as a punishment phase for the creation of surprise inflation, during which the discretionary equilibrium results and social loss is higher than under the ex ante optimal solution. Hence, the present discounted value of the net gain from a deviation may either be positive or negative.

The ex ante optimal policy is implementable as a subgame-perfect reputational equilibrium through a trigger strategy, if a [possibly infinite] number of punishment periods exists such that the present discounted value of social loss from the ex ante optimal policy is smaller than the one after a deviation from it. In this case, a deviation does not yield a net decrease of the present discounted value of social loss. Note that this is more likely to be the case for small discount rates, the reason being that the authority then values more highly the future social loss during the punishment phase. In contrast, if the public discount rate is too high, then the present value of future loss is smaller than the current gain from an inflation surprise and trigger strategies cannot support the ex ante optimal policy as a subgame-perfect equilibrium.

Although trigger strategies are a fairly simple and straightforward way of modelling a reputational effect, they have serious problems. Most importantly, the folk theorem for infinitely repeated games implies the existence of a multiplicity of possible equilibria. As pointed out by, among others, Backus and Driffill (1985) and Rogoff (1987), it is not clear how price taking individuals in a competitive environment can coordinate on any of the possible equilibrium strategies. In particular, it appears to be a “heroic” assumption that individuals can agree to expect the most favorable equilibrium, i.e. the ex ante optimal policy.

Two solutions to the coordination problem in infinitely repeated games have been suggested in the literature. First, in many countries, there exists a centralized trade union, which coordinates the individual expectation formation process. However, renegotiation of the trigger strategy then becomes an issue. This comes about because, as opposed to a large number of price taking individuals, a trade union is a single player that presumably takes into account the adverse effects of higher inflation on its utility during the punishment phase. If the policy maker deviates by creating a possibly small rate of surprise inflation, it may be optimal for the union to renegotiate and not to play the trigger strategy, because the resulting punishment period may hurt it more than the small inflation surprise. Alternatively, Minford (1995) has suggested that the punishment may take the form of withdrawal of votes in a democratic election. Since voting possibilities are few and clearly specified, coordinating individual expectations no longer appears to be a severe problem. However, it seems doubtful that monetary policy issues completely dominate individual voting decisions. Hence, there is the possibility that the creation of surprise inflation will not be of major concern at some future election day and will thus not be punished.
5.1.2 Reputation in finitely repeated policy games with incomplete information

Interestingly, the above coordination problem is absent in finitely repeated signalling games of the form that was, in this context, first suggested by Backus and Driffill (1985). Their idea, essentially, was that restricting attention to a finite number of periods may result in a unique equilibrium, since the folk theorem no longer holds. However, as it stands, the unique equilibrium of the finitely repeated game is the discretionary policy outcome in any period of the game because of the well known ‘backwards unraveling effect’. Despite this fact it is possible to introduce reputational forces. This is usually done through the assumption that there exist [at least] two different possible types of policy maker, notably, a strong and a weak one. Backus and Driffill (1985) assumed that the strong type is precommitted to the ex ante optimal policy, whereas the weak type has discretion to choose the policy that maximizes utility ex post.

At the beginning of the signalling game, individuals are assumed not to know which type they are facing, and to hold a prior belief in the form of the probability with which the strong policy maker is in office. After any period, each individual observes the inflation rate chosen by the policy maker in office and, by means of Bayes law, updates the previous period’s probability that the strong type is in office. From this updated probability, inflationary expectations for the next period result. For certain parameter values of the model, a possible equilibrium of this signalling game is a so-called pooling equilibrium, in which the weak policy maker imitates the strong one by choosing the ex ante optimal inflation policy. The outcome in a pooling equilibrium may be interpreted as arising from a reputational effect too, because imitating the strong type allows the weak policy maker to maintain the reputation for being strong. Provided that the discount rate is not too small, this may yield a smaller present value of social loss than the creation of surprise inflation, because surprise inflation not only leads to an additional current gain, but also to a future loss. The latter holds true because, given that the strong policy maker is precommitted to zero inflation, surprise inflation reveals that the weak type is in office and the suboptimal discretionary outcome materializes for the rest of the game instead of the ex ante optimal one. Hence, the consequences of surprise inflation in the signalling game are similar to those in the infinitely repeated game when individuals play a trigger strategy.

It should be stressed that the coordination problem discussed above is absent in the finitely repeated signalling game, because it is rational for each individual to form inflation expectations according to Bayes rule. Hence, no coordination of the individual expectation formation mechanisms is needed. While this is an improvement compared to infinitely repeated games in which reputation is modelled by trigger strategies, signalling games have a different problem: since no future disadvantages arise after the creation of surprise inflation in the last period of the game, the weak policy maker will chose the discretionary inflation rate at least in the last period. Hence, reputational effects can at best explain why the weak type may abstain from the creation of surprise inflation in all but the last period of the game. An additional problem arises since the equilibrium strategy...
may involve randomization between the ex ante optimal policy and the discretionary policy. As most observers of politics would probably agree, such mixed strategies are not a terribly convincing description of policymaking.

In summary, reputational effects may explain why the ex ante optimal outcome, instead of the discretionary one, can be achieved even though the policy maker has discretion. A necessary condition for this to be true is that the policy maker values the adverse future effects from a loss of reputation highly enough, that is, that his discount rate is not too large. In situations in which the future is discounted highly, there may, for this reason, be a need for other solutions to the time consistency problem.

5.2 Establishing central bank independence through an institutional reform

So far, we have assumed that the central bank is institutionally dependent on the government, which is the case in many countries. However, some central banks are fairly independent, the most prominent example probably being the Deutsche Bundesbank. Recent empirical work on central bank independence has shown that, for industrialized countries, central bank independence tends to be associated with lower average inflation; see Cukierman (1992,1994), Walsh (1993) and Eijffinger and de Haan (1995). Consequently, it has been argued that central bank independence is a key element of institutional solutions to the time consistency problem.35

In this subsection, we consider three alternative solutions to the time consistency problem, which all involve establishing the institutional independence of the central bank from the government. They differ in the way in which the central bank’s policy objective is determined: (i) the central bank is precommitted by law to follow a policy rule; (ii) monetary policy is delegated to a ‘conservative central banker’; (iii) a performance contract for the central banker is legislated. It should be noted that since its policy objective is not chosen by the central bank itself, institutional independence means instrument independence [from the government in the daily conduct of monetary policy], but not goal independence [Fischer (1995)].

5.2.1 Precommitment by adopting a policy rule

Perhaps the simplest way to specify the central bank’s objective is to pass a law, requiring the central bank to follow a certain policy rule. This has sometimes been called the legislative approach. Within the deterministic model of the present paper, the preferable policy rule is clearly the ex ante optimal policy, characterized in subsection 3.1.

The practical problem with legislating the ex ante optimal policy rule is that informational and computational requirements may prevent its feasibility in practice. The reason is that only in special cases does the ex ante optimal policy turn out to be fairly simple, for example, when the Friedman rule is found optimal, implying that the inflation rate is to be set equal to the real interest rate;
compare the discussion in subsection 3.2. In general, though, the implementation of the ex ante optimal policy presupposes agreement on the social losses from taxation and from inflation, and on the interest elasticity of the demand for real balances. Reacting to this problem, the literature has focussed on simpler policy rules, which deviate from the ex ante optimal one, but are easier to implement. The simplest conceivable option appears to be a rule requiring a constant inflation rate, or money growth rate, as was suggested by Friedman (1960). This inflation rate could, for example, be the average of the ex ante optimal inflation rates over a certain period of time. In countries [for example Germany] in which the ex ante optimal inflation rate is close to zero, one may, alternatively, legislate a mandate for price stability; see Neumann (1991) for a further discussion.

A different, simple policy rule has played a crucial role in the literature on the disinflation properties of the European Monetary System [EMS]. For an open economy, Giavazzi and Pagano (1988) interpreted fixing a currency’s nominal exchange rate vis-a-vis the German Mark as an indirect employment of a [foreign] precommitted central banker, namely, the president of the German Bundesbank. Since German inflation is thereby imported [at least in the long run], the inflationary bias is reduced to the extent to which Germany has succeeded in reducing it. Hence, fixing the exchange rate vis-a-vis a foreign, low inflation currency may be an interesting option for countries that have not succeeded in finding the parliamentary support to reform the institutional status of their own central bank. However, apart from reducing inflation, fixing the exchange rate involves a cost whenever the ex ante optimal inflation rates are different for the two countries, because the imported foreign inflation rate is then suboptimal from a domestic point of view; compare Dornbusch (1988), Gros (1988), de Kock and Grilli (1993), and Herrendorf (1995a).

5.2.2 Delegation of monetary policy to a conservative central banker

Rogoff (1985) was the first to suggest that part of the inflationary bias arising under discretion can be eliminated when the government appoints a central bank governor who is more inflation averse than the government. This has sometimes been coined the preference-based approach. More specifically, such a ‘conservative central banker’ is an individual who puts a larger weight, $\kappa_t$, on the cost from actual inflation than the government does. The discussion in subsection 4.2 has shown that the higher the degree of conservativeness, $\kappa_t$, the lower is the equilibrium inflation rate under discretion. Hence, if a conservative central bank governor is vested with institutional independence from the government, he can conduct monetary policy according to his preferences, and the inflationary bias will be reduced; see van der Ploeg (1995) for a formal evaluation in a public finance model of monetary policy making.

Unfortunately, there exist some problems with this suggestion of appointing a conservative central banker. To begin with, for a multisector economy, in which the sectors are differently affected by monetary policy, Waller (1992) pointed out that each sector will prefer a different ‘degree of conservativeness of the central
banker’. Consequently, the selection of the central banker may well be subject to partisan influences, which might lead to a suboptimal outcome. Moreover, since the marginal gain from surprise inflation may depend on the state variables of the model [compare the discussion in subsection 4.1], the optimally chosen weight $\kappa_3$ will be time variant too. For this reason, employing a central banker with well defined constant preferences does not fully solve the time consistency problem. Finally, the preferences of the central banker are not observable in reality, implying the possibility of wrong appointments. Therefore, we now turn to a third suggestion.

5.2.3 Designing an optimal contract for the central banker

Walsh (1995b) has recently argued that, instead of precommitting the central bank to implementing mechanically some policy rule, or choosing a central banker with certain preferences, the government may be able to *legislate a performance contract for the central banker* that sets the right incentives.\(^{39}\) The essential idea is to influence the central banker’s decision problem through the introduction of a monetary transfer [or bonus payment], which depends negatively on the deviations of the realized inflation rate from the ex ante optimal inflation rate. Walsh has shown that, since a deviation from the ex ante optimal inflation rate does not only reduce social loss, but also the received transfer, an appropriately chosen transfer scheme can eliminate the incentive to create surprise inflation.

As is happens, the optimal transfer has a relatively simple functional form: in Walsh’s static framework with a quadratic loss function, it is linear in the difference between realized inflation and ex ante optimal inflation, and exhibits a constant first derivative with respect to realized inflation. One possibility of approximating such a transfer scheme lies in fixing the central bank’s budget in nominal terms. Alternatively, it can be shown that the optimal incentive structure may also be generated through either of the following two possibilities: (i) to implement a properly chosen dismissal rule with the threat to fire the central banker under certain conditions [Walsh (1995a)]; (ii) to implement an inflation targeting procedure, which grants independence to the central bank in the execution of monetary policy, but not in the choice of the inflation target [Svensson (1995); see also Herrendorf (1996)].

While the linearity of the optimal contract survives in dynamic models with a linear-quadratic structure, recent contributions have shown that the optimal contract may become quite complicated; see, for example, Lockwood (1996) and Herrendorf and Lockwood (1996). In particular, its coefficients will not be constant when the incentive to create surprise inflation varies over time, as it does in the present model since the stock of non-indexed, net public liabilities is not constant in general. Hence, feasibility issues of the same nature as discussed above for legislated policy rules will arise: the optimal contract may easily become too complicated to be implementable. A topic for further research, therefore, is to compare different *feasible* solutions, which will all be suboptimal.\(^{40}\)
In summary, we have seen, in the last two sections, that reputational effects or an institutional reform may mitigate the adverse consequences from the time consistency problem of monetary policy making, which arises when the policy maker has discretion. However, both approaches have been found to be subject to some qualifications. In the first case, a necessary condition for reputational forces to have bite is that the discount rate of the policy maker is not too large. As for the second option, it might be difficult to mobilize sufficient democratic support for an institutional reform that establishes central bank independence and, even if this is not the issue, it is not entirely clear how the objective of the independent central bank should be optimally specified. For these reasons, we will now discuss an additional solution to the time consistency problem, i.e. a specific way of asset and debt management. As will become clear below, this solution has the advantage of achieving the ex ante optimal outcome independently of both the public sector’s discount factor and the institutional status of the central bank.

5.3 Asset and debt management

5.3.1 The general idea of asset and debt management

The idea of curing a time consistency problem through specific ways of asset and debt management was first thought of by Lucas and Stokey (1983), who considered a closed barter economy without capital. In their model, the time consistency problem arises because the intertemporal consumption-leisure decision reacts to expected changes of a distortionary consumption tax, but cannot be changed once a future period is entered. Lucas and Stokey’s essential insight was that future optimization problems are affected by the current choice of the structure of public debt. In particular, they showed that the choice of a specific maturity structure of [indexed] government bonds in the current period can solve the time consistency problem of their model. This comes about because today’s policy maker can shift debt repayments into periods in which the future policy maker would otherwise prefer higher real interest rates than those which are optimal from today’s point of view. An intuitive interpretation of the implied debt structure in terms of the public sector cash flow was provided by Persson and Svensson (1984).

The literature originated by Lucas and Stokey (1983) has studied whether the fundamental idea carries over to more general models, for example, with heterogeneous agents or with additional time consistency problems, which could, for instance, arise from a labor income tax. The basic answer to this question appears to be affirmative, provided that the policy maker has sufficiently many debt instruments available to influence future decision problems; see Rogers (1989) and Faig (1994), who maintain the closed economy assumption, or, alternatively, Persson and Svensson (1986), Faig (1991b), and Huber (1992), who analysed an open economy. For a more complete account of this literature, the reader is referred to the book by Persson and Tabellini (1990) and the survey by Missale (1995).

In the present context, the interest lies in using asset and debt management as a
cure for the time consistency problem arising from the existence of money in a closed economy. Persson et al. (1987) proposed that the authority manage its assets and debt so as to ensure zero net, non-indexed public liabilities vis-à-vis the private sector. Unexpected inflation does not then generate net public revenues. Persson et al. (1987) therefore concluded that there is no longer an incentive to create unexpected inflation. However, as Calvo and Obstfeld (1990) pointed out, the resulting policy satisfies only the first-order condition for optimality, but always violates the second-order conditions. While this does not necessarily create a problem, Calvo and Obstfeld (1990) showed that Persson et al. (1987) did in fact characterize ‘welfare saddlepoints’ instead of welfare maxima. Without providing a formal proof, Persson, Persson and Svensson (1989) suggested in their reply to Calvo and Obstfeld that this problem might be avoided when one accounts for the social loss of actual inflation, whereas they only modelled the social loss from expected inflation in their 1987 paper.

We will take up this question now because it is still open. An additional reason for doing so is that the policy implications of the Persson et al. (1987) device have not been intensively discussed in the literature. Apart from the unresolved technical problem, this might be due to the fact that working with a ‘full blown’ general equilibrium model, while appropriate, makes it hard to grasp the very intuitive and appealing underlying idea. The model presented in the previous sections of this paper is more specific than the set-up used by Persson et al. (1987). In particular, the real rate of interest is given and the endowment tax is assumed to relate to an exogenously given tax base. The time consistency problem of labor taxation is thus not present by assumption. For this reason, it is fairly easy to illustrate the idea and to prove its sufficiency in our framework.

5.3.2 The solution of Persson, Persson and Svensson

Remembering the source of the time consistency problem as discussed in subsection 4.1, it was argued there that the ex ante optimal policy must fulfill the additional first-order condition (43) to be time consistent. We have seen that, in general, this is not the case, because the marginal loss from actual inflation, as perceived by the policy maker [left hand side], is smaller than the marginal revenue [right hand side]. To understand the suggestion of Persson et al. (1987), we observe that there are two variables on the right hand side of (43) that are not determined by the optimal tax problem. These are the stock $c_{s+1}^{prec}$ of private non-indexed assets in the central bank portfolio and the share $\omega_{t+1}^{prec}$ of government bonds not indexed to actual inflation. For this reason, one can impose restrictions on the ways assets and debt ought to be managed that ensure equality in (43), and thereby remove the incentive to create unexpected inflation. Since there are ‘two degrees of freedom’, many alternative ways of achieving this exist. We choose to focus on a policy as close as possible to the one originally considered by Persson et al. (1987).

Two cases are to be distinguished. In the first one, the time consistency problem is sufficiently severe. This means that if no government bonds are indexed to
actual inflation, then the marginal revenue from reducing the real value of these bonds exceeds the marginal loss from realized inflation,

$$\frac{\partial}{\partial \pi_t} l_3(\pi^\text{prec}_t) \leq \lambda^\text{prec} (1 + i^\text{prec}_t) b^\text{prec}_{t-1}. \quad (45)$$

In this case, a restriction on the way assets and debt are managed could be:

1. Implement monetary policy so as to ensure that the stock of private assets in the central bank portfolio is determined as

$$C^\text{prec}_{t-1} = \frac{M^\text{prec}_{t-1}}{1 + i^\text{prec}_{t-1}} \Rightarrow m_t^\text{prec} - (1 + i^\text{prec}_t) c^\text{prec}_{t-1} = 0. \quad (46)$$

2. Index a share $1 - \omega^\text{prec}_{t-1}$ of government bonds to actual inflation, where

$$\omega^\text{prec}_{t-1} = \frac{1}{\lambda^\text{prec} (1 + i^\text{prec}_t) b^\text{prec}_{t-1}} \frac{\partial}{\partial \pi_t} l_3(\pi^\text{prec}_t). \quad (47)$$

Notice that (45), together with (47), imply $\omega^\text{prec}_{t-1} \in [0, 1]$.

First, we can observe that if (46) and (47) are satisfied, then the additional first-order condition (43) holds with equality and the ex ante optimal solution is time consistent despite the government’s having discretion. As before, the Kuhn-Tucker theorem guarantees that the ex ante optimal policy sequence \{$b^\text{prec}_t, \tau^\text{prec}_t, \pi^\text{prec}_t$\} indeed minimizes social loss. The ‘saddle point problem’ of the analysis of Persson et al. (1987) is thus not present in our simpler model.

In order to get an intuitive understanding, let us take a closer look at the restrictions (46) and (47). Condition (46) requires that in any period $t - 1$, the central bank hold a stock of privately issued assets, such that equality is ensured between the period $t$ nominal values of these assets and the stock of base money issued in period $t - 1$. If this is done, then any creation of unexpected inflation in period $t$ will reduce the real value of central bank liabilities from nominal balances by exactly the same amount as it reduces the real value of central bank assets. Hence, the marginal revenue gain from surprise inflation is reduced to that resulting from the reduction in the real value of non-indexed government bonds, i.e. $(1 + i^\text{prec}_t) \omega^\text{prec}_{t-1} b^\text{prec}_{t-1}$.

The requirement of indexing a sufficient share of government bonds to actual inflation, (47), ensures that the marginal social gain from unexpected inflation equals the marginal social loss. If we neglected the marginal social loss from actual inflation, as did Persson et al. (1987), then we would have exactly their asset and debt management scheme, i.e. satisfy (46) and index all government bonds. However, if the social loss from actual inflation is accounted for, it is, in general, not appropriate to remove the marginal revenue gain from surprise inflation completely, because there already exists a positive marginal social loss from unexpected inflation. Put differently, setting the marginal revenue from unexpected inflation equal to zero when actual inflation is costly would lead to an
incentive to create surprise deflation and would thus not solve the time consistency problem.

Finally, it remains to explain what the policy maker may do in the second possible case, in which the time consistency problem is not very severe, that is, inequality (45) does not hold,

\[ \frac{\partial}{\partial \pi_t} l_{t+1}(\pi_t^\text{prec}) > \lambda^\text{prec} (1 + i_t^\text{prec}) b_{t-1}^\text{prec}. \]  

(48)

It is straightforward to show in this case that (43) can be ensured, and the problem resolved, if one sets

\[ C_{t-1}^\text{prec} = B_{t-1}^\text{prec} - \frac{1}{1 + i_{t}^\text{prec}} \left[ \frac{P_{t-1}^\text{prec}}{\lambda^\text{prec}} \frac{\partial}{\partial \pi_t} l_{t+1}(\pi_t^\text{prec}) - M_{t-1}^\text{prec} \right] \]  

(49)

and does not index any government bonds.

5.3.3 Policy implications

Some policy implications of conditions (46) and (47) can now be suggested. Although there are many alternative possibilities that can bring about time consistency, it is still instructive to restrict attention to (46) and (47), the reason for this being that alternative restrictions will also require the central bank to hold part of its portfolio in privately issued assets and the government to index a share of government bonds to actual inflation.

First, the consequences of condition (46) are studied. It could be seen as imposing a restriction on the open market operations that the central bank can undertake. Provided that (46) did in fact hold for period \( t - 1 \), it will also hold for period \( t \), if a share \( \frac{1}{1 + i_{t}^\text{prec}} \times 100\% \) of all changes of base money is implemented through purchases of [non-indexed] private bonds. Consequently, a share of \( \frac{i_{t+1}}{1 + i_{t+1}} \times 100\% \) of all base money changes must be realized through direct credit to the government, i.e. purchases of government bonds. Note that the resulting procedure of implementing monetary policy lies between the two extreme cases discussed in subsection 2.3. In addition, such a restriction does not affect the intertemporal real seigniorage revenues of the public sector; compare (22). Hence, budgetary problems do not arise from this way of implementing monetary policy as long as the public sector is solvent.

It should be mentioned for practical purposes that deviations of the daily central bank policy from this procedure are not problematic as long as the validity of (46) is re-ensured by purchases or sales of private assets in the open market in exchange for government bonds. This argument also indicates how a central bank that in the past has mainly transferred the revenues from seigniorage directly to the government and thus holds most of its portfolio in government bonds can meet the requirement (46). It simply has to sell government bonds to the private sector and use the proceeds to buy nominal private assets until (46) is met.

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Thus, we can conclude that it is, in principle, possible for a central bank to change its portfolio composition so as to meet requirement (46). It is interesting to note in this context that the portfolio of the Deutsche Bundesbank seems to be broadly in line with (46); compare the results reported by Klein and Neumann (1990) in a different context. In particular, between 1961 and 1974, the Bundesbank transferred only 13 percent of the revenues from money creation to the government, while this value amounted to 73 percent between 1974 and 1987. Following Klein and Neumann, this phenomenon can be explained by the fact that the Bundesbank had built up a portfolio of bonds not issued by the German government during the first period, which then started to mature. This policy has certainly been a consequence of the legal constraint on the Bundesbank’s monetary policy not to lend directly to the government and to purchase government bonds in the secondary markets only under very restrictive conditions.

Finally, we must address briefly the issue of indexing government bonds to actual inflation as required by (47). It has often been asserted that this possibility has hardly been used in OECD countries, England and Israel being two exceptions. However, many countries have actually indexed quite a share of their government bonds indirectly by denormalizing it in foreign currencies. In order to understand the effect of this, we will depart for a moment from the closed economy assumption of the present model, and define the nominal exchange rate, $E_t$, as the domestic currency price of a foreign currency,

$$E_t = e_t \frac{P_t}{P_t^*},$$

(50)

where $e_t$ denotes the real exchange rate and $P_t^*$ the foreign price level. Suppose now that, until the end of the previous period, the domestic government had issued a stock $B_t^*$ of bonds that were denominated in the foreign currency, paid an interest rate $i_t^*$, and were not held by the domestic central bank. Expressed in the domestic currency, the nominal, period $t$ value of this foreign currency debt amounts to $E_t (1 + i_t^*) B_t^*$. Using (50), and deflating by the domestic price value, its real value in units of the domestic good is found to be $e_t (1 + i_t^*) B_t^* / P_t^*$. Hence, the domestic government can only reduce the real value of its foreign currency debt if it manages to appreciate the real exchange rate. Since the standard open economy macro-models predict that this is not possible through the creation of surprise inflation, a denomination of government debt in a foreign currency protects it from surprise inflation in a similar way to an indexation to domestic inflation; for further discussion, see Bohn (1990), de Fontenay, Milesi-Ferretti and Pill (1995), and the references therein.

6. Conclusion

In this paper, the intertemporal problem of seigniorage collection within an optimal taxation context has been considered. The different recommendations of
the public finance literature on optimal inflation were found to be specific cases of the ex ante optimal solution. In particular, it was shown that, providing collecting output taxes involves cost, the optimal taxation package contains both output and inflation taxation. This replicated the classical result of Phelps (1973). Only in extreme cases, in which the ratio of the marginal social loss of one instrument to its marginal revenue clearly dominates that of the other, does it turn out to be optimal to rely almost exclusively on the revenue source with the lower marginal loss to revenue ratio.

Furthermore, the ex ante optimal solution has been proven to be time inconsistent, and the time-consistent solution has been characterized and shown to be welfare sub-optimal. As possible solutions to the time consistency problem of the ex ante optimal policy, the paper has discussed reputational effects, institutional reforms that establish central bank independence, and specific ways of asset and debt management. It has been argued that a modified version of the asset and debt management scheme, as suggested by Persson, Persson and Svensson (1987), cures the time consistency problem that arises when optimal taxation considerations determine inflation.

What are the policy implications that can be drawn from these results? First, it is important to stress that positive inflation rates of ‘moderate’ magnitude are not necessarily due to the emergence of an inflationary bias in discretionary equilibrium. As has been argued above, moderate positive inflation rates may well arise as the optimal outcome when the output tax system is inefficient. A large underground economy, for instance, strengthens the case for positive inflation, because inflation may be the only way of taxing the shadow economy. Programs with the goal of reducing inflation ought to take account of this fact by putting emphasis on the improvement of the output and excise taxation system. This is likely to be an important issue in most of the states in Eastern Europe.

In this context, it is interesting to note that Dornbusch and Fischer (1993) reported evidence that successful stabilizations from high to moderate inflation rates have often been accompanied by drastic improvements of the efficiency of the tax system. On the other side, Cukierman, Edwards and Tabellini (1992) analysed when an authority finds it in its interest to reform of the tax system. They argued that, in a politically unstable and polarized economy, the gains from such a reform are likely to be inherited by the succeeding administration, which will probably use them for purposes not appreciated by the current one. For this reason, it may not be optimal to devote resources to a reform of the tax system. The casual empirical observation that politically unstable countries are also often the ones with high inflation rates seems to suggest support for this point of view.

As a second policy implication, Persson, Persson and Svensson’s (1987) contribution suggests two possibilities of how the authority may decrease its incentive to resort to inflation surprises. First, it may restrict the direct and indirect borrowing of the government from the central bank. This would allow the central bank to build up a portfolio of nominal private assets, which could, at least partly, help to reduce the net nominal liabilities of the public sector. Second, Persson
et al.’s analysis suggests that the time consistency problem can be mitigated by indexing government bonds to realized inflation, or by denominated them in foreign currencies.

Although the main focus of this review has been theoretical and normative, it should be mentioned that seigniorage models similar to the present one have also been used to test whether real world inflation series satisfy the Ramsey principle. This literature was originated by Mankiw (1987), and includes the work of Grilli (1989), Roubini and Sachs (1989), Poterba and Rotemberg (1990), Trehan and Walsh (1990), Calvo and Leiderman (1992), Goff and Toma (1993), Fukuta and Shibata (1994), Diba and Martin (1995), and Froyen and Waud (1995). It may not come as a surprise that the empirical evidence reported in these studies is mixed. Of course, using the public finance approach to the determination of optimal inflation rates is an oversimplification. First, Ricardian equivalence may be violated, implying that the issue of government bonds, necessary to smooth tax and inflation rates, involves real effects and, therefore, is no longer costless. Interest rate smoothing then becomes an issue; compare Barro (1989), and Froyen and Waud (1995). Second, it may be too much of an abstraction to take the inflation rate as a policy instrument which can be perfectly controlled; see Dornbusch (1989). And finally, the framework used here is extremely monetaristic, in that prices are perfectly flexible and output is not at all affected even by unexpected inflation.

In my opinion, rather than discarding the public finance approach to the determination of optimal inflation ‘as one of the most overstudied areas in monetary theory’ [Summers (1991)], one should interpret the results with some care. Economic models are supposed not only to tell the truth, but also to organize thinking. There is little doubt that the public finance approach has improved the profession’s conceptual understanding, in comparison to the paradigm of seigniorage maximizing inflation that had been widely used prior to the classic paper of Phelps (1973). Furthermore, its predictions appear to be consistent with the experience of many countries with moderately high or high inflation; see, for example, Dornbusch and Fischer (1993). For those countries, the public finance approach offers important policy implications. That the same does not hold true for most low inflation countries, in which seigniorage is relatively unimportant as a revenue source, is neither very surprising nor extremely disturbing.

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Notes
1. For countries outside Europe, in which the revenues from the creation of money are important, Fischer (1982), Fischer and Easterly (1990) and Dornbusch and Fischer (1993) estimated that seigniorage of a similar order of magnitude is sustainable over relatively long time horizons. Moreover, Edwards and Tabellini (1991) reported even higher revenues from money creation during relatively short high-inflation or hyper-inflation periods. They found that at times the inflation tax was around twenty percent of GDP in Argentina [1973–1978, 1983–1987], Chile [1963–1973] and Uganda [1983–1987]. Note that the inflation tax is only a crude estimate for seigniorage; see subsection 2.3.

2. Earlier, largely informal discussions include those of Keynes (1923) and Friedman (1953).


5. Why people hold money can be explained by the fact that money provides liquidity services. For example, larger cash holdings may, to a certain extent, reduce the time devoted to unproductive banking and shopping activities and therefore increase utility. In addition, legal requirements [such as the required payment of public fees in cash] lead to a demand for money.

6. It should be mentioned that the difference between the two specifications is small for small inflation rates. Using the property of the logarithm that log(1 + \eta) \approx \eta for any small \eta, it can be seen that either measure approximately equals \log(P_t) – \log(P_{t-1}). This logarithmic difference yields the same inflation measure, independent of whether one measures forward or backward.

7. Notice that perfect control over the money supply is implicitly assumed.

8. It should be mentioned that Aizenman (1983), Fischer (1983a) and Rogers (1989) analyzed an optimal taxation problem with endogenous government expenditure.

9. This assumption is relaxed in subsection 5.2, which discusses the possibility of solving the time consistency problem through establishing central bank independence.

10. Note that the possibility is ignored that privately issued bonds in the central bank portfolio can be indexed to inflation.

11. Examples of OECD countries in which government bonds have been indexed in the past are England and Israel.

12. Similar results, for a continuous time framework, can be found in Drazen (1985).

13. Notice that surprise inflation devalues all [non-indexed] public liabilities, whereas expected inflation only devalues the [non-interest bearing] nominal balances, showing that interest bearing liabilities compensate for expected inflation when the Fisher relation is valid.

14. For simplicity, the discount rate of the policy maker equals the real interest rate. The generalization of the subsequent analysis to situations in which this is not the case would not be difficult. The crucial issue is that the discount rate does not become too large relative to the real interest rate because this can lead to violations of public solvency [Levine and Pearlman (1993)].

15. Fischer and Modigliani (1979), Fischer (1983b) and Driffill, Mizon and Ulph (1990) all provide a more detailed discussion of the social loss from inflation.
16. Social loss from tax distortions could be incorporated by making output a function of the average tax rate. However, this would not add much insight.

17. If \( l_s(\pi_t) \) were omitted, then the discretionary equilibrium inflation rate to be determined below would be either infinite or equal to the highest technologically feasible inflation rate in a closed economy; see Calvo (1978b). Note that, in an open economy, currency substitution might give rise to a ‘natural upper bound’ on discretionary equilibrium inflation.

18. An empirical test, which accepted the solvency condition for the United States, was conducted by Hamilton and Flavin (1986).


20. As was pointed out by a referee, this need no longer hold true in a stochastic setting with random disturbances: planned inflation may not be completely predictable when the policy maker is precommitted to a contingent rule, calling for some response to forecasts of shocks.

21. A clear and intuitive exposition of the Kuhn-Tucker theorem can be found in Romer (1988). It should be mentioned that the standard formulation of the optimization problem is one of maximizing a concave function subject to concave constraints. Maximization of minus one times our Lagrangian would thus translate into this type of problem. Moreover, note that for simplicity we have assumed the budget constraint to be binding. If we wanted to be more general, we could formulate the flow constraint as

\[
\Delta b_t - g_t - r b_{t-1} + r y_t + s_t = 0.
\]

Since the left hand side is concave and the set for which a concave function is not negative is convex, this formulation would show more clearly that the loss function is minimized over a convex set. Apart from this, however, it would not add anything to the analysis, because the flow constraint must be binding in an optimum.

22. It should be stressed that the reason for this lies in the way the Lagrangian \( L \) has been set up, implying that \( \lambda^{\text{pre}} \) is a current value multiplier here. In contrast, a present value multiplier would not be constant over time.

23. Note that, since \( \lambda^{\text{pre}} \) is a current value multiplier, it is expressed in units of social loss at \( t \).

24. Note that Ramsey (1927) considered the static problem of characterizing the optimal tax rates on a finite number of goods. However, the interpretation of the same good at different dates as different goods directly allows the application of his idea to a dynamic context.


26. In a different context, Fischer and Summers (1989) have discussed a similar phenomenon.

27. A cash-in-advance constraint requires that individuals must accumulate money before they go to the goods market and buy consumption goods [Lucas and Stokey (1983)].

28. Money then yields transactions services and decreases the time spent for unproductive shopping activities [Drazen (1979) and Leach (1983)].

29. See also Faig (1988, 1991a).

30. Subsequent contributions by Fischer (1980), Kydland and Prescott (1980) and Turnovsky and Brock (1980) analysed the time consistency problem in a variety of different contexts.

31. Note that, differently from the present analysis, Barro and Gordon (1983b) analyzed the unemployment-inflation trade off, resulting from a Phillips curve. In contrast, Grossman and Van Huyck (1986), de Kock and Grilli (1993), and Herrendorf (1995a) employed trigger strategies in models in which the incentive to create surprise inflation arises from the problem of seigniorage collection, as is the case here.
32. See al Nowaihi and Levine (1994) for the derivation of a subgame perfect equilibrium that is also resistant to certain ways of renegotiation. They find that, in this case, reputational effects reduce albeit not eliminate the inflationary bias.

33. It should be mentioned that, while there generally still exists a multiplicity of equilibria of a finitely repeated game with incomplete information, one may use equilibrium refinements to obtain uniqueness. This is not dealt with here.

34. This assumption has been subject to quite some discussion in the literature. For example, Cukierman and Liviatan (1991) generalize it to the case in which the strong policy maker is not necessarily precommitted to the ex ante optimal inflation policy. Instead, they assume that he can chose the policy to which he wants to precommit and then announce it to the public. Alternatively, Vickers (1986) considered a game in which both policy makers have discretion but differ in their objective functions. More precisely, the strong type in Vickers’ model attaches a higher coefficient \( \kappa \) to the welfare losses from inflation. For a discussion of these and other generalizations the reader is referred to Driffill (1988,1989), Rogoff (1987,1989) or Cukierman (1992).

35. Note that, for non-industrialized countries, the negative relation between central bank independence and average inflation does not hold. This is probably due to the fact that in these countries legal independence typically does not translate into factual independence.

36. Note that if the central bank was assigned a responsibility of stabilizing the economy in the face of stochastic shocks [which have been not considered here], then the ex ante optimal policy rule would become considerably more complex.

37. It should be mentioned that maintaining a constant exchange rate parity may reduce inflation even when the domestic authority is not legislatively precommitted to this policy, but instead has some discretion to realign the exchange rate parity. The reason for the credibility gain from a pegged, albeit not fixed, exchange rate is that the exchange rate is better observable and more easily controlled than the inflation rate. Hence, a pegged exchange rate is more transparent than a policy of money supply control under a float, thus enhancing the disciplinary effect that reputational effects may have on the domestic policy maker; see Herrendorf (1995b).

38. See Lohmann (1992) and Lockwood, Miller and Zhang (1994) for generalizations.

39. This principal-agent approach has been taken up by, among others, Persson and Tabellini (1993), Fratianni, von Hagen and Waller (1995) and Lockwood (1996).

40. This is particularly relevant in stochastic settings, incorporating a role for stabilization policy; for recent reviews of the literature in this area, the reader is referred to Cukierman (1996).

41. For a review of the literature on Ricardian equivalence, see Seater (1993).

42. The classic articles on optimal monetary policy under imperfect control over the inflation rate or the money supply are Canzoneri (1985) and Cukierman and Meltzer (1986). More recent work includes Basar and Salmon (1990), Cripps (1991) and Herrendorf (1995b).

### List of symbols

The following notation is used [If not mentioned differently upper case letters denote nominal variables while lower case letters represent real variables.]:

- \( b_t \): real value of the stock of government bonds held by individuals in period \( t \).
- \( b_t^{cb} \): real value of the stock of government bonds held by the central bank in period \( t \).
- \( b_t^{tot} \): real value of the total stock of government bonds outstanding in period \( t \).
$B^*_t$: nominal, foreign currency value of the stock of domestic government bonds that are denominated in a foreign currency and not held by the domestic central bank.

c_t: real value of privately issued bonds held by the central bank in period t.

e_t: real exchange rate in period t.

$E_t$: nominal exchange rate in period t.

$g_t$: real government expenditure in period t.

$i_t$: nominal interest rate paid in period t on assets issued in period $t-1$.

$l_t$: present discounted period 0 value of the sum of all future social loss.

$l_{st}$: period t social loss from the use of the different policy instruments.

$l_{og}t$: period t social loss from output taxation.

$l_{og}t$: period t social loss from expected inflation.

$l_{og}t$: period t social loss from actual inflation.

$L_t$: Lagrangian of the public sector’s optimization problem.

$m_t$: real balances private agents want to hold in period t.

$M_t$: supply of nominal base money in period t.

$P_t$: price level in period t.

$P^*_t$: price level in period t as expected in period $t-1$.

$p^*_t$: foreign price level in period t.

$r_t$: real interest rate between period $t-1$ and t.

$R_t$: real discount factor for calculating the period 0 present value of a period t variable.

$s^*_t$: [real] seigniorage from expected inflation in period t.

$s^*_t$: [real] seigniorage from surprise inflation in period t.

$s^*$: total [real] seigniorage in period t.

$s_t$: present discounted period 0 value of the sum of all $s^*_t$.

t_t: time index.

$T_t$: nominal value of output taxes collected in period t.

$X_t$: nominal central bank profit in period t.

$y_t$: real output [or endowment] in period t.

$\kappa_{1t}$, $\kappa_{2t}$ and $\kappa_{3t}$: parameters of the social loss function (26).

$\lambda_t$: current value multiplier of the Lagrangian $L_t$ [shadow price of government revenues].

$\pi^*_t$: inflation rate between period $t-1$ and t as expected in period $t-1$.

$\rho_t$: real opportunity cost of holding money instead of bonds during period t.

$\tau_t$: average output tax rate in period t.

$\omega_t$: share of the government bonds held by individuals that is not indexed to inflation.

References


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