Determinacy through intertemporal capital adjustment costs

Berthold Herrendorf \textsuperscript{a,b,c,\ast} and Ákos Valentinyi \textsuperscript{b,c,d}

\textsuperscript{a} Universidad Carlos III de Madrid, 28903 Getafe (Madrid), Spain
\textsuperscript{b} University of Southampton, Southampton SO17 1BJ, UK
\textsuperscript{c} Centre for Economic Policy Research, London, UK
\textsuperscript{d} Institute of Economics of the Hungarian Academy of Sciences, Budapest, Hungary

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Abstract

It is well known that if there are mild sector-specific externalities, then the steady state of the standard two-sector real business cycle model can become indeterminate and endogenous business cycles can arise. We show that this result is not robust to the introduction of standard intertemporal capital adjustment costs, which may accrue when total capital is adjusted or when each sector’s capital is adjusted. We find for both forms of adjustment costs that the steady state is determinate for all empirically plausible parameter values. We also find that determinacy occurs for a much larger range of parameter values when adjusting each sector’s capital is costly.

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1. Introduction

In their classical paper, Kydland and Prescott (1982) showed that technology shocks can account for a large part of business cycle fluctuations. An alternative view is that business cycle fluctuations are largely accounted for by self-fulfilling shocks to individual beliefs.
For a long time, the major problem with this latter view had been that shocks to individual beliefs can become self-fulfilling only if equilibrium is indeterminate, whereas in the neo-classical growth model with externalities equilibrium is determinate at the steady state as long as labor supply is regular (Benhabib and Farmer, 1994; Boldrin and Rustichini, 1994).\footnote{In this context, determinacy means that equilibrium is isolated or locally unique and indeterminacy means that equilibrium is non-isolated or locally non-unique.} It was therefore important when Boldrin and Rustichini (1994) and Benhabib and Farmer (1996) argued that determinacy is not a generic property of the neo-classical growth model. Specifically, Boldrin and Rustichini (1994) showed that indeterminacy can arise in the two-sector version of the model with sector-specific externalities and regular labor supply; in the bounded version, the steady state can become indeterminate and there can be indeterminate cycles and in the unbounded version, balanced growth rates can be indeterminate and there can be indeterminate growth cycles and indeterminate chaotic growth paths. Benhabib and Farmer (1996) showed that indeterminacy can arise in the bounded version for mild strengths of the externality that can be defended empirically. These results made self-fulfilling business cycles look rather plausible and numerous authors have since studied them in two-sector models; examples include Perli (1998), Weder (1998), and Schmitt-Grohé (2000).

We show that indeterminacy at the steady state is not a generic property of two-sector models. In particular, we show that whether or not there is indeterminacy at the steady state depends critically on whether or not adjusting capital is costly. While Benhabib and Farmer (1996) and Boldrin and Rustichini (1994) assumed that it is not, we assume that it is. Our assumption can be justified by at least three arguments. First, there is substantial empirical evidence in favor of capital adjustment costs at the firm level (see Hammermesh and Pfann, 1996, for a review of the evidence). Second, without capital adjustment costs Tobin’s \(q\) (i.e., the ratio between the price of installed capital and the price of new capital) is constant over the business cycle, which is counterfactual (Jermann, 1998). Third, without capital adjustment costs the allocations of the two-sector model considered here have several counterfactual properties (such as excess investment volatility and countercyclical consumption) that mostly disappear when capital adjustment costs are modeled (Huffman and Wynne, 1999; Boldrin et al., 2001).

We consider capital adjustment costs of the standard intertemporal form employed by Gould (1968), Treadway (1969), Lucas and Prescott (1971), and Mortensen (1973). We distinguish between two possibilities: the adjustment costs can accrue either when the total capital stock is changed (irrespective of by how much each sector’s capital stock changes) or when a sector’s capital stock is changed (irrespective of by how much the total capital stock changes). Our main result is that both specifications eliminate the scope for indeterminacy at the steady for all empirically plausible parameter choices. We also find that adjustment costs on sectoral capital lead to a larger (smaller) parameter range for which determinacy (local indeterminacy) occurs than adjustment costs on total capital.

The intuition for the main result of this paper is similar to that underlying the work by Wen (1998b), Guo and Lansing (2002), and Kim (2003) on the effects of capital adjustment costs on the local stability properties of the one-sector neoclassical growth model. These
authors found that capital adjustment costs have an “offsetting effect.” Given a strength of increasing returns that implies indeterminacy at the steady state, there is a minimum size of intertemporal capital adjustment costs that implies determinacy at the steady state. While this offsetting effect is at work here too, the value added of our paper is to show that it matters for empirically relevant parameter values in the two-sector version of the model. This is not an issue in the one-sector version anyway because indeterminacy at the steady state does not occur for empirically relevant parameter values.

The rest of the paper is organized as follows. Section 2 lays out the economic environment. Section 3 characterizes the competitive equilibrium. Section 4 reports our results. Section 5 concludes the paper.

2. Model economy

Time is continuous and runs forever. There are measures one of identical, infinitely-lived households, of identical firms that produce a perishable consumption good, and of identical firms that produce new capital goods. The representative household is endowed with the initial capital stocks, with the property rights of the representative firms, and with one unit of time at each instant. We assume that installing new capital is costly and that installed capital is sector specific. At each point in time there are then four commodities: a consumption good, a new capital good suitable for the production of consumption goods, a new capital good suitable for the production of new capital goods, and labor. Trade takes place in sequential markets.

The preferences of the representative household are represented by the following utility function:

$$\int_{0}^{\infty} e^{-\rho t} \left( \log c_t - l_{ct} - l_{xt} \right) dt,$$

where $\rho > 0$ is the discount rate, $\log$ is the natural logarithm, $c_t$ denotes the consumption good at time $t$ (which is the numeraire), and $l_{ct}$ and $l_{xt}$ are labor in the consumption- and in the capital-producing sector. The instantaneous utility is separable in consumption and leisure, logarithmic in consumption, and linear in leisure. This functional form is standard in the literature on self-fulfilling business cycles, and it would be consistent with the existence of a balanced growth path if exogenous technological progress were considered. The separability assumption does not affect the possibility of local indeterminacy (Hintermaier, 2003). The linear utility in leisure implies an infinite labor supply elasticity, which can be justified by the lottery argument of Hansen (1985) and Rogerson (1988). Since it is easier to get local indeterminacy the higher is the labor supply elasticity, the determinacy results to be derived for an infinite labor supply elasticity would apply for any finite labor supply elasticity too.

The representative household’s problem depends on the form that capital adjustment costs take. The first form arises when it is costly to change the total capital stock of the model economy, irrespective of by how much each sector’s capital stock changes. In the
first case the household problem, called (HP1), is to choose  
\( c_t, I_{ct}, l_{ct}, x_t, x_{ct}, x_{xt}, k_{ct}, k_{xt} \)
so as to maximize (1) subject to
\[
(k_{ct} + k_{xt}) \psi \left( \frac{x_{ct} + x_{xt}}{k_{ct} + k_{xt}} \right) = x_t 
\]
and
\[
c_t + p_t x_t = \pi_{ct} + \pi_{xt} + w_{ct} l_{ct} + w_{xt} l_{xt} + r_{ct} k_{ct} + r_{xt} k_{xt},
\]
\[
\dot{k}_{ct} = x_{ct} - \delta_c k_{ct},
\]
\[
\dot{k}_{xt} = x_{xt} - \delta_x k_{xt},
\]
\[
k_{c0} = \bar{k}_c > 0 \text{ given},
\]
\[
k_{x0} = \bar{k}_x > 0 \text{ given},
\]
\[
0 \leq c_t, l_{ct}, l_{xt}, x_t, x_{ct}, x_{xt}, k_{ct}, k_{xt},
\]
\[
l_{ct} + l_{xt} \leq 1.
\]
The function \( \psi : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) is twice continuously differentiable, monotonically increasing, and convex. The notation is as follows: \( x_t \) is the composite new capital good produced by the capital-producing sector and \( p_t \) is its relative price in terms of the consumption good. \( \pi_{ct} \) and \( \pi_{xt} \) are the two profits (which will be zero in equilibrium), \( w_{ct} \) and \( w_{xt} \) are the wages in the two sectors, \( k_{ct} \) and \( k_{xt} \) are the capital stocks in the two sectors, \( r_{ct} \) and \( r_{xt} \) are the corresponding real interest rates, \( \delta_c \) and \( \delta_x \) are the corresponding depreciation rates, and \( x_{ct} \) and \( x_{xt} \) are the investments in the two sectors.

The second form of capital adjustment costs arises when it matters by how much the capital stocks of each sector change, irrespective of by how much the total capital stock changes. In the second case the household’s problem, called (HP2), is to choose  
\( c_t, I_{ct}, l_{st}, x_t, x_{ct}, x_{xt}, k_{ct}, k_{xt} \) so as to maximize (1) subject to
\[
(k_{ct} \psi_c \left( \frac{x_{ct}}{k_{ct}} \right) + k_{xt} \psi_x \left( \frac{x_{xt}}{k_{xt}} \right)) = x_t 
\]
and (2b)–(2h). Again, the functions \( \psi_c, \psi_x : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) are twice continuously differentiable, monotonically increasing, and convex.

The functional form for intertemporal capital adjustment costs has a long tradition (see, for example, Gould, 1968; Treadway, 1969; Lucas and Prescott, 1971; and Mortensen, 1973). Our version has external capital adjustment costs that depend on how much new capital is acquired in the market. The alternative is internal capital adjustment costs that would depend on how much the capital stock varies. While the economic interpretations are different (external adjustment costs rely on some market imperfection, internal adjustment costs do not), the results of this paper are not affected by whether adjustment costs are external or internal. The strict convexity of \( \psi, \psi_c, \) and \( \psi_x \) captures the notion that it becomes the more costly to change the capital stock the more quickly this is done. The non-negativity constraints on the two capital goods, \( x_{ct}, x_{xt} \geq 0 \), imply that installed capital is sector specific, which is consistent with the evidence reported by Ramey and Shapiro (2001). Nevertheless the capital stock of a sector can be reduced by not replacing depreciated capital, so in equilibrium the non-negativity constraints will not be binding.
close the steady state and we can ignore them from now on. It should be mentioned that the model version without capital adjustment costs is nested in our set-up for

$$\psi(x_{ct} + x_{xt}) = x_{ct} + x_{xt},$$

$$\psi_j(x_{jt}) = x_{jt}, \quad j \in \{c, x\}.$$  

Note that the model version without capital adjustment costs still has sector-specific capital. This does not, however, matter for the local stability analysis because sector-specific capital alone does not have an effect (Christiano, 1995 and Herrendorf and Valentinyi, 2002).

Denoting by $\lambda_t$ the multiplier attached to (2b) and by $\mu_{ct}$ and $\mu_{xt}$ the current value multipliers attached to the accumulation equations (2c) and (2d), the solution to (HP1) is characterized by (2a)–(2h) and

$$c_t = w_{ct} = w_{xt} = \frac{1}{\lambda_t}, \quad (3a)$$

$$\frac{\mu_{ct}}{\mu_{xt}} = 1, \quad (3b)$$

$$\frac{\mu_{ct}}{\lambda_t p_t} = \psi'\left(\frac{x_{ct} + x_{xt}}{k_{ct} + k_{xt}}\right), \quad (3c)$$

$$\lambda_t r_{ct} - \mu_{ct} \delta_c = \lambda_t r_{xt} - \mu_{xt} \delta_x, \quad (3d)$$

$$-\dot{\mu}_{ct} + \mu_{ct} \rho = r_{ct} \lambda_t + \lambda_t p_t \left[ x_{ct} + x_{xt} \psi'\left(\frac{x_{ct} + x_{xt}}{k_{ct} + k_{xt}}\right) - \psi\left(\frac{x_{ct} + x_{xt}}{k_{ct} + k_{xt}}\right) - \mu_{ct} \delta_c, \quad (3e)\right.$$  

$$\lim_{t \to \infty} \mu_{ct} k_{ct} = \lim_{t \to \infty} \mu_{xt} k_{xt} = 0. \quad (3f)$$

The solution to (HP2) is characterized by (2b)–(2i) and

$$c_t = w_{ct} = w_{xt} = \frac{1}{\lambda_t}, \quad (4a)$$

$$\frac{\mu_{ct}}{\mu_{xt}} = \psi'\left(\frac{x_{ct}}{k_{ct}}\right) \psi'\left(\frac{x_{xt}}{k_{xt}}\right), \quad (4b)$$

$$\frac{\mu_{ct}}{\lambda_t p_t} = \psi'\left(\frac{x_{ct}}{k_{ct}}\right), \quad (4c)$$

$$-\dot{\mu}_{ct} + \mu_{ct} \rho = \lambda_t r_{ct} + \lambda_t p_t \left[ x_{ct} \psi'\left(\frac{x_{ct}}{k_{ct}}\right) - \psi\left(\frac{x_{ct}}{k_{ct}}\right) - \mu_{ct} \delta_c, \quad (4d)\right.$$  

$$\lim_{t \to \infty} \mu_{ct} k_{ct} = \lim_{t \to \infty} \mu_{xt} k_{xt} = 0. \quad (4f)$$
We now turn to the production side of the model economy. The representative firm of the consumption-producing sector solves:

$$\max_{c_t, k_t, l_t} \pi_{ct} \equiv c_t - r_{ct} k_t - w_{ct} l_t \quad \text{s.t.} \quad c_t = k_t^{a_t} l_t^{1-a_t}, \quad c_t, l_t, k_t \geq 0,$$

where \(a \in (0, 1)\) is the constant capital share parameter. The first-order conditions include

$$r_{ct} = a k_t^{a_t-1} l_t^{1-a_t},$$

$$w_{ct} = (1-a) k_t^{a_t} l_t^a.$$

For future reference, note that the production function and (6b) imply that

$$l_t = 1 - a,$$

$$c_t = (1-a)^{1-a} k_t^a.$$

The representative firm of the capital-producing sector solves:

$$\max_{x_t, l_t, k_t} \pi_{xt} \equiv p_t x_t - r_{xt} k_t - w_{xt} l_t \quad \text{s.t.} \quad x_t = B_t k_t^{b_t} l_t^{1-b_t}, \quad x_t, k_t, l_t \geq 0,$$

where \(b \in (0, 1)\) denotes the constant capital share parameter and \(B_t \geq 0\) denotes total factor productivity in the sector, which is given from the representative firm’s perspective. The first-order conditions include

$$r_{xt} = p_t B_t k_t^{b_t} l_t^{1-b_t},$$

$$w_{xt} = p_t (1-b) B_t k_t^{b_t} l_t^{1-b_t}.$$

\(B_t\) is specified so that there are positive externalities at the level of the capital-producing sector. This is consistent with the empirical evidence in favor of positive externalities in the production of manufacturing durables (Basu and Fernald, 1997). The externalities are modeled as is standard in the related literature by assuming that total factor productivity in the capital-producing sector depends on aggregate output there. Recalling that there is a measure one of firms in each sector, this gives

$$B_t = k_t^{\theta b_t} (1-b_t),$$

where \(\theta \in [0, (1-b)/b)\). Substituting (10a) back into the production function, aggregate capital output becomes

$$x_t = k_t^{\beta_1} l_t^{\beta_2},$$

where

$$\beta_1 \equiv (1+\theta)b, \quad \beta_2 \equiv (1+\theta)(1-b).$$

Some features of this specification deserve further comment. First, the upper bound \((1-b)/b\) on \(\theta\) is imposed to exclude the possibility of endogenous growth and to guarantee the stationarity of our model economy. For plausible parameter values it will not be binding. Second, as is standard, the externality is not taken into account by the individual firms in the capital-producing sector, so a competitive equilibrium exists.
Moreover, equilibrium profits are zero (and will be omitted from now on) and the capital and labor shares are the usual ones: \( (r_{xt} k_{xt})/k_t = b \) and \( (w_{xt} l_{xt})/k_t = 1 - b \). Of course, in a competitive equilibrium the \( B_t \) on which the firms base their decisions must be equal to the \( B_t \) that results from these decisions:

**Definition 1** (Competitive equilibrium). A competitive equilibrium are prices \( \{w_{ct}, w_{xt}, r_{ct}, r_{xt}, p_t\}_{t=0}^{\infty} \), allocations \( \{c_t, l_{ct}, l_{xt}, x_t, x_{ct}, x_{xt}, k_{ct}, k_{xt}\}_{t=0}^{\infty} \) and total factor productivity in the capital-producing sector \( \{B_t\}_{t=0}^{\infty} \) such that:

(i) \( \{c_t, l_{ct}, l_{xt}, x_t, x_{ct}, x_{xt}, k_{ct}, k_{xt}\}_{t=0}^{\infty} \) solve the problem of the representative household;

(ii) \( \{c_t, l_{ct}, k_{ct}\}_{t=0}^{\infty} \) solve the problem of the representative firm of the consumption-producing sector, (5);

(iii) \( \{x_t, l_{xt}, k_{xt}\}_{t=0}^{\infty} \) solve the problem of the representative firm of the capital-producing sector, (8);

(iv) \( B_t \) is determined consistently, that is, (10a) holds.

### 3. Equilibrium dynamics

Combining the first-order conditions derived above, it can be shown that with (HP1) the equilibrium dynamics is characterized by the terminal conditions (3f) together with the following eight equations in the eight unknowns \( x_{ct}, x_{xt}, k_{ct}, k_{xt}, \mu_{ct}, l_{ct}, p_t \), and \( \lambda_t \):

\[
\begin{align*}
    k_{ct}^{\beta_1} k_{xt}^{\beta_2} &= (k_{ct} + k_{xt}) \psi \left( \frac{x_{ct} + x_{xt}}{k_{ct} + k_{xt}} \right), & (11a) \\
    \frac{\mu_{ct}}{\lambda_t p_t} &= \psi' \left( \frac{x_{ct} + x_{xt}}{k_{ct} + k_{xt}} \right), & (11b) \\
    1 &= \lambda_t p_t (1 - b) \mu_{ct}^{\beta_1} k_{xt}^{\beta_2 - 1}, & (11c) \\
    (1 - a)(1 - \mu_{ct}) k_{ct} &= p_t (1 - b) k_{ct}^{\beta_1} k_{xt}^{\beta_2 - 1}, & (11d) \\
    a k_{ct} - \mu_{ct} \delta_c &= b \frac{l_{ct}}{1 - b k_{xt}} - \mu_{ct} \delta_x, & (11e) \\
    \dot{\mu}_{ct} &= \mu_{ct} \left[ \rho + \delta_c + \frac{x_{ct} + x_{xt}}{k_{ct} + k_{xt}} - \frac{\psi'((x_{ct} + x_{xt})/(k_{ct} + k_{xt}))}{\psi'((x_{ct} + x_{xt})/(k_{ct} + k_{xt}))} \right] - \frac{a}{k_{ct}}, & (11f) \\
    \dot{k}_{ct} &= x_{ct} - \delta_c k_{ct}, & (11g) \\
    \dot{k}_{xt} &= x_{xt} - \delta_x k_{xt}. & (11h)
\end{align*}
\]

Equation (11a) specifies how the equilibrium supplies of the two new capital goods are related to the output of the capital-producing sector. It follows by substituting the first equation of (10b) into (2a). Equation (11b) restates Eq. (3c). It specifies how the equilibrium demands of the two new capital goods depend on the relative price of installed capital in terms of new capital, \( \mu_{ct}/(\lambda_t p_t) \), which is called Tobin’s \( q \). Note that the
presence of adjustment costs permits variation in Tobin’s \(q\).\(^2\) Equation (11c) says that the marginal utility from leisure is to be equalized to the marginal utility from the increase in the capital-producing sector’s output due to the last unit of labor. It follows by substituting (9b), (10a), and (11b) into (3a). Equation (11d) says that the marginal products of labor are to be equalized in equilibrium. It follows by substituting (6b) and (9b) into (3a). Equation (11e) says that the marginal products of installed capital net of depreciation are to be equalized in equilibrium. Note that with (HP1) this holds in equilibrium because the costs come from adjusting the total capital stock irrespective of how the two sectors’ capital stocks are adjusted. Equation (11e) follows by substituting (6a), (7), the production function of (8), (9a), and (10a) into (3d). Equation (11f) describes the equilibrium law of motion of the shadow price of installed capital. It follows by substituting (3c), (6a), and (7) into (3e). Note that since with (HP1) the shadow prices of the two installed capital goods are equal, only one law of motion is required. Equations (11g) and (11h) are the laws of motion from (2c) and (2d) once more.

Combining the first-order conditions derived above, it can be shown that with (HP2) the equilibrium dynamics is characterized by the terminal conditions (4f) together with the following nine equations in the nine unknowns \(x_{ct}, x_{xt}, k_{ct}, k_{xt}, \mu_{ct}, \mu_{xt}, l_{xt}, p_t\), and \(\lambda_t\):

\[
k_{ct}^{\beta_1}l_{ct}^{\beta_2} = k_{ct} \psi_c \left( \frac{x_{ct}}{k_{ct}} \right) + k_{xt} \psi_x \left( \frac{x_{xt}}{k_{xt}} \right),
\]

\[
\frac{\mu_{ct}}{\lambda_t p_t} = \psi_c' \left( \frac{x_{ct}}{k_{ct}} \right),
\]

\[
\frac{\mu_{xt}}{\lambda_t p_t} = \psi_x' \left( \frac{x_{xt}}{k_{xt}} \right),
\]

\[
1 = \lambda_t p_t (1 - b) k_{ct}^{\beta_1} l_{ct}^{\beta_2},
\]

\[
(1 - a)^{1-a} k_{ct}^a = p_t (1 - b) k_{ct}^{\beta_1} l_{ct}^{\beta_2},
\]

\[
\dot{\mu}_{ct} = \mu_{ct} \left[ \rho + \delta_c + \frac{x_{ct}}{k_{ct}} - \psi_c' \left( \frac{x_{ct}}{k_{ct}} \right) \right] - \frac{a}{k_{ct}} - \frac{\mu_{ct}}{k_{ct}},
\]

\[
\dot{\mu}_{xt} = \mu_{xt} \left[ \rho + \delta_x + \frac{x_{xt}}{k_{xt}} - \psi_x' \left( \frac{x_{xt}}{k_{xt}} \right) \right] - \frac{b}{k_{xt}} - \frac{\mu_{xt}}{k_{xt}},
\]

Equation (12a) specifies how the equilibrium productions of the two new capital goods are related to the output of the capital-producing sector. It follows by substituting the (10b) into (2i). Equations (12b) and (12c) restate Eqs. (4b) and (4c). They specify how the equilibrium demands of the two new capital goods depend on the relative prices of installed capital in terms of new capital, \(\mu_{ct} / (\lambda_t p_t)\) and \(\mu_{xt} / (\lambda_t p_t)\). Equations (12d) and (12e) are Eqs. (11c) and (11d) once more. Equations (12f) and (12g) describe the equilibrium laws

\(^2\) When we speak of Tobin’s \(q\), we mean marginal \(q\). In any case, there is no difference between marginal \(q\) and average \(q\) in our model because the capital adjustment costs used here are linear homogeneous in its arguments (Hayashi, 1982).
of motion of the shadow prices of installed capital. Note that with (HP2) the marginal products of installed capital net of depreciation need no longer be equal because the costs of adjusting the two sectors’ capital stocks drive a wedge between them. (12f) and (12g) follow by substituting (4b) and (4c) into (4d) and (4e), respectively, and plugging (6a) and (7) into the results. Equations (12h) and (12i) are the laws of motion from (2c) and (2d) once more.

In order to guarantee the existence of a steady state, we need to put more structure on \( \psi \), \( \psi_c \), and \( \psi_x \). We adopt the standard assumption that the presence of capital adjustment costs does not affect the steady state of the model economy. Denoting steady state variables by dropping the time index, this gives six restrictions:

\[
\psi \left( \frac{x_c + x_x}{k_c + k_x} \right) = \frac{x_c + x_x}{k_c + k_x}, \quad \psi' \left( \frac{x_c + x_x}{k_c + k_x} \right) = 1, \quad (13a) \\
\psi_c \left( \frac{x_c}{k_c} \right) = \frac{x_c}{k_c}, \quad \psi'_c \left( \frac{x_c}{k_c} \right) = 1, \quad (13b) \\
\psi_x \left( \frac{x_x}{k_x} \right) = \frac{x_x}{k_x}, \quad \psi'_x \left( \frac{x_x}{k_x} \right) = 1. \quad (13c)
\]

Thus, we are left with the three second derivatives as free parameters, which will play a key role for the local stability properties of the steady state.

**Proposition 1 (Steady state).** Given (13), there is a unique steady state. The steady state is the same for (HP1) and (HP2).

**Proof.** Given (13), the steady state versions of (11) and (12) both become:

\[
k^\beta_1 l^\beta_2 = x_c + x_x, \quad (14a) \\
1 = \frac{\mu_c}{\lambda p} = \frac{\mu_x}{\lambda p}, \quad (14b) \\
1 = \lambda p (1 - b) k^\beta_1 l^\beta_2 - 1, \quad (14c) \\
(1 - a)^{-a} k^a_c = p (1 - b) k^\beta_1 l^\beta_2 - 1, \quad (14d) \\
\frac{a}{k_c} - \mu_c \delta_c = \frac{b}{1 - b} l_x - \mu_x \delta_x, \quad (14e) \\
0 = \mu_c (\rho + \delta_c) - a \frac{k_x}{k_c}, \quad (14f) \\
x_c = \delta_c k_c, \quad x_x = \delta_x k_x. \quad (14g)
\]

Consolidating (14), we obtain the following three equations in the three unknowns \( k_c, k_x, \) and \( l_x \):

\[
k^\beta_1 l^\beta_2 = \delta_c k_c + \delta_x k_x, \quad (15a) \\
\rho + \delta_c = \frac{a (1 - b)}{k^\beta_1 l^\beta_2}, \quad (15b) \\
\frac{\rho + \delta_x}{\rho + \delta_c} = \frac{b l_x}{1 - b k_x}. \quad (15c)
\]
That these equations have a unique solution can be seen as follows. First, solve (15c) for \( k_c \) and substitute the result into (15b). This gives \( l_x \) as a function of \( k_x \). Substitute this into (15a) to get \( k_c \) as a linear function of \( k_x \). Substituting this into (15c) gives \( l_x \). Obtaining \( k_x \) and \( k_c \) is then straightforward. □

4. Local stability properties

With (HP1) there is one state, the aggregate capital stock, whereas with (HP2) there are two states, the two sectors’ capital stocks. In any case, the steady state is saddle-path stable if and only if the number of eigenvalues with negative real parts equals the number of states. If the steady state is saddle-path stable, then the equilibrium nearby is determinate, that is, given the initial capital stocks close to the steady state values there are unique initial shadow prices such that the economy converges to the steady state. The steady state is stable if and only if it has more eigenvalues with negative real parts than states. If the steady state is stable, then the equilibrium nearby is indeterminate, that is, given the initial capital stocks close to the steady-state values, there exists a continuum of shadow prices such that the economy converges to the steady state. The steady state is unstable if and only if has more eigenvalues with positive real parts than states.

We will choose a functional form for \( \psi \), \( \psi_c \), and \( \psi_x \) and empirically plausible parameters values and we will then compute the eigenvalues numerically. Specifically, we follow Boldrin et al. (2001) and set

\[
\psi \left( \frac{x_{ct} + x_{xt}}{k_{ct} + k_{xt}} \right) = \kappa_1 + \frac{\kappa_2}{1 + \psi} \left( \frac{x_{ct} + x_{xt}}{k_{ct} + k_{xt}} \right)^{1+\psi},
\]

(16a)

\[
\psi_c \left( \frac{x_{ct}}{k_{ct}} \right) = \kappa_{c1} + \frac{\kappa_{c2}}{1 + \psi} \left( \frac{x_{ct}}{k_{ct}} \right)^{1+\psi},
\]

(16b)

\[
\psi_x \left( \frac{x_{xt}}{k_{xt}} \right) = \kappa_{x1} + \frac{\kappa_{x2}}{1 + \psi} \left( \frac{x_{xt}}{k_{xt}} \right)^{1+\psi},
\]

(16c)

where \( \kappa_1, \kappa_2, \kappa_{c1}, \kappa_{c2}, \kappa_{x1}, \kappa_{x2}, \) and \( \psi \) are constants. The six \( \kappa \)-constants are chosen so as to ensure that the six restrictions listed in (13) are satisfied (i.e., there is no effect of the capital adjustment costs in steady state). The parameter \( \psi \) remains free and it affects the curvature of the capital adjustment costs. In fact, the expressions in (11b), (12b), and (12c) show that \( \psi \) is the inverse of the elasticity of the relevant investment-to-installed-capital ratio with respect to Tobin’s \( q \).

The inverse of the elasticity of the investment-to-installed-capital ratio with respect to Tobin’s \( q \) and the degree of increasing returns in the capital-producing sector are the key parameters that determine the local stability properties of the steady state. We will therefore not choose a benchmark calibration for them but vary them widely. To get an idea about the intervals in which plausible parameter choices can lie, we briefly report the key empirical evidence. Eberly (1997) estimates \( \psi \) from a panel of OECD countries and reports point estimates in \([0.51, 1.54]\), with the USA having 0.82. If one computes the confidence intervals of the different estimates, then the lowest possible realization for the whole sample is 0.34, which comes from the French data, and the lowest possible realization
for the USA is 0.74. Jermann (1998) calibrates \( \varphi \) the US economy and finds a much larger value: 4.35. We remain agnostic and just conclude that values for \( \varphi \) short of 0.34 are unreasonable. The evidence about \( \theta \) is as follows. Hall’s (1988) initially estimated \( \theta \approx 0.5 \). It turned out, however, that this estimate was upward biased. More recent empirical studies instead find estimates between constant returns and much milder increasing returns that at most might reach 0.3; see, e.g., Basu and Fernald (1997). It is generally agreed by now that values for \( \theta \) in excess of 0.3 are unreasonable. 

The remaining parameters to be chosen are \( a, b, \rho, \delta_c, \) and \( \delta_x \). Some recent related studies set them as follows. Benhabib and Farmer (1996) choose \( a = b = 0.3, \rho = 0.0125, \) and \( \delta_c = \delta_x = 0.026. \) Huffman and Wynne (1999) choose \( a = 0.41, b = 0.34, \rho = 0.01, \delta_c = 0.018, \) and \( \delta_x = 0.20. \) Boldrin et al. (2001) choose \( a = b = 0.36, \rho = 0.00001, \delta_c = \delta_x = 0.021. \) Since the parameter choices of Huffman and Wynne (1999) come from a serious calibration of a model that is very close to the one employed here, we will use their values. However, we will also conduct extensive sensitivity checks, which incorporate the other possibilities too, except for the rather awkward choice of \( \rho = 0.00001. \)

The local stability properties for our benchmark calibration are summarized by Fig. 1. In particular, part (a) shows the case of adjustment costs on total capital and part (b) shows the case of adjustment costs on sectoral capital. The main result of this paper is that, in both cases, determinacy of the steady state occurs for \( \varphi \geq 0.34 \) and \( \theta \leq 0.3 \), which, as we argued above, are necessary conditions for reasonable parameter values. This result turns out to be very robust to changes in the other parameter values, which we establish as follows: fix \( \varphi \) and \( \theta \) at the two values that might still be defendable while local indeterminacy results most easily, \( \varphi = 0.34 \) and \( \theta = 0.3 \), thereby “loading the gun in favor of” local indeterminacy; then vary the other parameters: \( \rho, \delta_c, \delta_x \) each in \([0.005, 0.03]\) with step size 0.01, \( a \) and \( b \) each in \([0.25, 0.45]\) with step size 0.01. The result is that there is no parameter combination on these grids for which the steady state is indeterminate; instead it is determinate everywhere.

Two interesting aspects of the above figures deserve further comment: the local indeterminacy range is significantly larger with costly adjustment of total capital whereas the determinacy parameter range is significantly larger with costly adjustment of sectoral capital. One implication of this fact is observed as the adjustment costs parameter \( \varphi \) converges to 0. For the first form of adjustment costs the local stability properties change from determinacy to local indeterminacy at \( \theta = 0.072 \) and from local indeterminacy to instability at \( \theta = 0.198. \) These threshold values are exactly the same as without capital adjustment costs (Herrendorf and Valentinyi, 2002), so the local stability properties change continuously as \( \varphi \) goes to zero. In contrast, for the second form of adjustment costs the local stability properties do not change continuously; instead as \( \varphi \) goes to zero the first threshold disappears and only the second one survives.\(^5\)

\(^3\) The discrepancy can be understood as follows. Jermann calibrates his model such that asset pricing paradoxes can be accounted for. High inverse elasticities are helpful to this end because they allow for persistent deviations of \( q \) from its steady state value.

\(^4\) Note that \( a \) does not matter at all for the local stability properties.

\(^5\) Note that in the second case we could not formally take the limit in the numerical computations. The smallest \( \varphi \) we considered was 0.000000001.
Fig. 1. Local stability with $a = 0.41$, $b = 0.34$, $\rho = 0.01$, $\delta_c = 0.018$, $\delta_x = 0.020$. (a) Costly adjustment of total capital. (b) Costly adjustment of sectoral capital.

The differences between the two forms of adjustment costs are related to the change of dimension of the reduced-form dynamics from four to two, which has important implications for Tobin’s $q$. Specifically, if adjusting total capital is costly, then the equilibrium relative prices of both new capital goods are equal and there is only one Tobin’s $q$. This one relative price affects the consumption/savings decision of the representative household but has no instantaneous effect on the two sectoral investments. So, in this case, capital adjustment costs only affect the local stability because they partly offset the increasing returns. In contrast, if adjusting each sector’s capital is costly, then the
equilibrium relative prices of each sector’s new capital goods are different and there are two Tobin’s $q$’s. These two relative prices do not only affect the consumption/savings decision but also the sectoral investments. For example, if one $q$ rises relative to the other one, then instantaneously more new capital goods are channeled to the sector where $q$ has risen. So, in this case, the capital adjustment costs do not only offset partly the increasing returns but also determine the allocation of new capital goods to the two sectors, thus eliminating one degree of freedom. In a follow-up paper, Herrendorf and Valentinyi (2002), we study this second effect in more detail for intratemporal capital adjustment costs. 6

5. Conclusion

This paper contributes to the debate about whether business cycle fluctuations may be driven by self-fulfilling changes in individual beliefs. A sufficient condition for this to be a plausible is that the steady state is indeterminate for empirically reasonable choices of the parameter values. We have explored this possibility in the standard two-sector version with mild sector-specific externalities, which has been the focus of much of the recent research on self-fulfilling business cycles. We have found that the introduction of standard intertemporal capital adjustment costs excludes indeterminacy of the steady state for all empirically reasonable choices of the parameter values. This finding turns out to be independent of whether the adjustment costs arise from changes of the total capital stock or from changes of each sector’s capital stock.

There are two other possibilities in which self-fulfilling business cycles might arise. The first one is that indeterminate endogenous cycles might exist; see Boldrin and Rustichini (1994) for an example. The second one is that for other functional forms or other versions of the neoclassical growth model the steady state might be indeterminate for reasonable parameter values. Wen (1998a) suggests that this is the case in the one-sector version of the model when capital utilization is variable. Guo and Harrison (2001) confirm this finding for the standard two-sector model. Exploring these possibilities further is an interesting topic, which we leave for future research.

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6 The difference between intertemporal and intratemporal capital adjustment costs is as follows. Intratemporal capital adjustment costs accrue when the allocation of capital across sectors is changed but not when the total capital stock is changed as long as that allocation remains the same. Intertemporal capital adjustment costs accrue in both cases.
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