Abstract

Structural change in developed countries slows down aggregate productivity growth by reallocating production to industries with low productivity growth. We document that this so-called Baumol’s disease considerably reduced productivity growth in the postwar U.S. We capture the effect of Baumol’s disease on productivity growth with a novel model of structural change that disaggregates the service sector into services with low and high productivity growth. The data imply that the two service subsectors are substitutes. We find that this substitutability limits how important services with low productivity growth may become, which bounds the future effect of Baumol’s disease.

Keywords: Baumol’s Disease; Productivity Growth Slowdown; Service Sector; Structural Change.

JEL classification: O41; O47; O51.

This paper incorporates material from an earlier manuscript, which was called “Unbalanced Growth Slowdown”. We are indebted to Richard Rogerson for detailed feedback. For helpful comments and suggestions, we thank Zsófia Bárány, Aspen Gorry, Marti Mestieri, Rachel Ngai, Michael Sposi, and Gustavo Ventura, as well as the audiences of presentations at ASU, the BEA, CEPR’s ESSIM 2017, CERGE–EI Prague, the Christmas Meeting of the German Expat Economists 2016, the Economic Growth and Fluctuations Group of the Barcelona Summer Forum 2017, Erasmus University Rotterdam, the European Monetary Forum 2016 at the Bank of England, the Federal Reserve Banks of Atlanta and St. Louis, the RIDGE Workshop on Growth and Development 2016, the SED Meeting 2017, Southern Methodist University, the Universities of Barcelona, Manchester, Munich, North Carolina at Charlotte, and Western Ontario, the Vienna Macroeconomic Workshop, and the World Bank. Valentinyi thanks the Hungarian National Research, Development and Innovation Office (Project KJS K 124808). All errors are our own.
1 Introduction

Starting with the seminal work of Baumol (1967), it has been recognized that structural change in developed economies slows down (aggregate) productivity growth by reallocating production to industries with low productivity growth; see for example Baumol et al. (1985) and Nordhaus (2008). This phenomenon is often referred to as cost disease or Baumol’s disease. Baumol (1967) drew particular attention to the fact that production is even reallocated to stagnant industries of the service sector that have no productivity growth. Baumol’s observation raises the important question whether the stagnant industries will gradually take over the economy in the future, which would drive aggregate productivity growth down to zero. This question is particularly relevant in the broader context of the slowdown in productivity growth that has occurred since the 1970s in industrialized economies like the U.S. and that led to a lively debate about whether future productivity growth will rebound or decline further; see for example Fernald and Jones (2014) and Fernald (2016).

The goal of this paper is to investigate the effects of Baumol’s disease on past and future U.S. productivity growth. We define productivity as the value added per human–capital–adjusted hours (“efficiency units”), either in a sector or at the aggregate level of the entire economy. Using efficiency units is crucial in the context of Baumol’s disease, because it allows us to think about the implications for productivity of reallocating workers with different levels of human capital across sectors. Our focus in this paper is on how changes in the sectoral composition of the economy affect productivity growth, taking as exogenously given the processes for total efficiency units and sectoral productivity. In other words, we will not address the related questions of how human capital or sectoral productivities may change in response to changes in the sectoral composition; see Herrendorf and Valentinyi (2015) for an analysis of the latter question.

We first document that Baumol’s disease considerably reduced productivity growth in the postwar U.S. Our preferred measure implies that Baumol’s disease accounts for a third of the observed productivity slowdown. We then build a model that captures the effect of Baumol’s disease on past productivity growth and that is a natural starting point for investigating how important Baumol’s disease will be in the future. The novel feature of our model is that it disaggregates the service sector into services with high productivity growth and services with low productivity growth. In contrast, the literature on structural change typically abstracts from the heterogeneity among the service industries and considers just one broad service sector. Connecting our model to the postwar U.S. economy implies that services with high and low productivity growth are substitutes. We find that the substitutability between the two service subsectors implies that the service sector with low productivity growth will not gradually take

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1To be clear about terminology, a sector is a collection of industries and an industry is a group of establishments that produce similar products or provide similar services.
over the economy in the future. As a result, our model predicts that in the next 60 years the effect of Baumol’s disease on productivity growth will be less than half of what it has been in the last 60 years.

Turning now to the details of our analysis, we first establish that Baumol’s disease considerably reduced productivity growth in the post-war U.S. by comparing the average annual growth rates of productivity during the twenty-year periods 1947–1967 and 1987–2007. We take 20–year averages to smooth out business cycle fluctuations and we stop in 2007 because this paper is not about the Great Recession. Our data source is WORLD KLEMS because it has quality–adjusted hours (“efficiency units”). We find that in the postwar U.S., annual productivity growth during 1947–1967 was 0.9 percentage points higher than during 1987–2007. If the 1947 sectoral composition had remained unchanged while the sectoral productivity growth rates had been the observed ones, then the difference between the growth rates in the two periods would have been 0.6 instead of 0.9 percentage points. This means that Baumol’s disease caused a 0.3 percentage points decline in productivity growth, which is a third of the total decline. We conclude from this that Baumol’s disease has had a sizeable effect on postwar U.S. productivity growth that is too large to ignore.

Looking at the past importance of Baumol’s disease also reveals that in recent years virtually all of the productivity effects of Baumol’s disease have been due to structural change from the goods sector to the service sector and within the service sector, instead of due to structural change within the goods sector. This makes intuitive sense because the U.S. economy already went through most of the structural change within the goods sector (mainly from agriculture to manufacturing). Moreover, the service sector is rather heterogeneous, containing industries that have very high productivity growth (e.g. air transportation) as well as industries that have very low productivity growth (e.g. food services and drinking places). The implication of this observation is that the usual splits in the structural change literature, which treat services as one broad sector and abstract from structural change within the service sector, are not useful in our context.\(^2\) A natural first step for taking into account the structural change within the service sector is to distinguish between service industries with high and low productivity growth, defined as above and below the average productivity growth in the service sector. We establish that this way of disaggregating services captures almost all of the past productivity effects resulting from structural change within the service sector.

In light of the evidence for the effect of Baumol’s disease on past productivity growth, it is rather surprising that the literature on structural change has paid little attention to the productivity slowdown that results from Baumol’s disease. For example, the review article of Herrendorf et al. (2014) does not discuss it at all. The likely reason for this is that having an

\(^2\)An exception is Duarte and Restuccia (2016), who focused on final expenditure and split services into modern and traditional ones. Unfortunately, this split is not applicable in our context in which industries produce value added.
aggregate balanced growth path simplifies the analytical characterization of the equilibrium. In many models of structural change, an aggregate balanced growth path exists if GDP in each period is expressed in terms of a numeraire from this period. Since productivity growth is constant along aggregate balanced growth paths, it seems natural to conclude that Baumol’s disease is not an issue in standard models of structural change. In a related paper, Duerniecker et al. (2017), we show that this conclusion is misleading: if we measure GDP through Fisher indexes as it is done in the National Income and Product Accounts (NIPA), instead of in terms of a numeraire from the current period, then Baumol’s disease can play an important role in otherwise standard models of structural change. In the current paper, we take this result to heart and quantify the effects of Baumol’s disease on future productivity growth in a novel model of structural change in which GDP is measured as in the data.

Our model has three sectors that produce goods, services with high productivity growth, and services with low productivity growth. The broad goods sector lumps agriculture together with construction, manufacturing, mining, and utilities, because agriculture now plays a secondary role only. Our model generates the usual structural change between the broad sectors of goods and services as well as the little studied structural change within the service sector. We keep things as simple as possible and assume that sectoral value added is produced from labor under constant returns and that sectoral productivity and total efficiency units grow exogenously. We then study how changes in the sectoral composition affect the growth of aggregate productivity. Since our model does not have capital, all sectoral value added is consumed and the utility function governs how production is reallocated in response to changes in the exogenous variables. Rogerson (2008) argued that this is a useful first step for understanding the aggregate implications of structural change if the utility function captures well the observed reallocation of production. We choose a specification that meets this criterion. Herrendorf et al. (2017) study structural change within the investment sector and derive conditions under which this is possible.

We model preferences through a nested version of the non–homothetic CES function proposed by Comin, Mestieri, and Lashkari (2015). The outer layer aggregates the value added of the broad sectors goods and services into an aggregate consumption index. The inner layer aggregates the value added of the two service subsectors into the broad service sector. Our nested specification allows the elasticities of substitution and the income effects to differ by layer. The parameter values of the elasticities and the income effects will be crucial for capturing the observed reallocation of production in response to exogenous productivity growth at the sector level. Consistent with the evidence provided by Boppart (2016) and Comin et al. (2015), our version of the non–homothetic CES function implies persistent income effects that do not disappear in the limit. This will be crucial when we make prediction about the distant future.

We calibrate our model to capture salient features of structural change and nominal and
real sectoral productivity growth in the postwar U.S. economy. For the outer layer of our utility function, the calibration yields that goods are necessities, services are luxuries, and goods and services are complements. These features of our utility function are standard; see for example Herrendorf et al. (2013). They imply that the sector with the low productivity growth – the service sector – will take over our economy in the limit. We emphasize that this does not mean that the service subsector with low productivity growth will take over, because the inner layer of our utility function generates structural change within the service sector. The calibration for that inner layer yields the novel features that the services with high productivity growth are necessities, the services with low productivity growth are luxuries, and the two service subsectors are substitutes. The substitutability between the two service subsectors is going to be critical for the future importance of Baumol’s disease, because it works against the income effects and limits how important the service subsector with low productivity growth can become.

Assuming that the recent growth of sectoral labor productivity and total efficiency units of labor continues into the future and simulating our model forward, we find that the future productivity effects of Baumol’s disease will be limited. Specifically, over the next 60 years, the reduction in productivity growth that results from Baumol’s disease is less than half of what it has been in the postwar period. The reason for this is that the service sector with low productivity growth does not take over the entire economy, because the substitutability between the two service subsectors constrains the future slowdown in productivity growth. If the relative price of services with low productivity growth increases without bound, then households increasingly substitute away from them. As a result, the economy reallocates less and less production to the service industries that have no productivity growth. This feature of our model is very robust to changing the details of the forward simulations and it markedly differs from the canonical model of structural transformation which abstract from the substitutability within the service sector.3

In the next Section, we discuss the closely related literature. Section 3 presents evidence that Baumol’s disease has contributed to the slowdown in labor productivity in the postwar U.S. In Section 4, we develop our model. In Section 5, we calibrate our model and use it to predict how much future slowdown in productivity growth will result from Baumol’s disease. Section 6 concludes. An Appendix contains background information for the data work and the proofs of our results.

3Strictly speaking, the previous statement refers only to models of structural change without capital. In contrast, models of structural change with capital usually do not feature structural change within the investment sector, implying that, in the limit of these models, the sector with the slowest productivity growth takes over the consumption sector, instead of the whole economy. An exception to this statement is the model of Herrendorf et al. (2017) which captures structural change in the investment sector and the consumption sector.
2 Related Literature

Our work is closely related to a recent literature that wonders whether the past slowdown in productivity growth is a temporary or a permanent phenomenon; see Antolin-Diaz et al. (2017) for a statistical analysis. For example, Gordon (2016) argued that we picked the “low–hanging fruit” (e.g., railroads, cars, and airplanes) during the “special century 1870–1970” and that more recent innovations pale in comparison. Bloom et al. (2016) provided evidence that supports this view. Fernald and Jones (2014) pointed out that the engines of economic growth like education or research and development require the input of time which cannot be increased ad infinitum. The tendency in this literature is to conclude that low productivity growth rates may be the future norm. Our work adds an additional reason for why future productivity growth rates are not likely to return to past ones: Baumol’s disease.

Our work is also related to attempts to measured cross–country gaps in sectoral TFP or labor productivity; see for example Duarte and Restuccia (2010), Herrendorf and Valentinyi (2012), and Duarte and Restuccia (2016). The most closely related paper to ours is Duarte and Restuccia (2016), who used the 2005 cross section of the International Comparisons Program of the Penn World Table. Assuming that sectors produce final goods and distinguishing between traditional and modern services, they found that the largest cross–country productivity gaps are in goods and modern services and the smallest cross–country productivity gaps are in traditional services. The main differences to our study are that we are interested in the U.S. time series, assume that sectors produce value added, and distinguish between services with high and low productivity growth. Nonetheless, if we used our model to generate a cross section of countries with different levels of aggregate productivity, then it would generate a result that has the same flavor as that of Duarte and Restuccia (2016): the cross–country productivity differences in the goods and the service sector with high productivity growth are larger than those in the service sector with low productivity growth.

Lastly, our work is related to several papers that argued the usual three–sector split into agriculture, manufacturing, and services is not useful anymore in a rich country like the U.S. where most of production is in the service sector. Since productivity growth is rather heterogenous within the service sector, with some industries showing strong productivity growth while others showing no productivity growth, the effects of reallocation within the service sector are crucial when one seeks to understand the aggregate implications of structural change in rich countries [Baumol et al. (1985), Jorgenson and Timmer (2011), and Buera and Kaboski (2012)]. Since studying Baumol’s disease is just one example where this point is central, our model is likely to be useful beyond the narrow focus on Baumol’s disease.
3 Evidence on Structural Change within Services and Baumol’s Disease

We use data from the March 2017 version of WORLD KLEMS, which offers information about quality–adjusted hours that take into account differences in human capital. We call these efficiency units of labor. Jorgenson et al. (2013) describe how the data are constructed. Since we want to study counterfactuals that reallocate workers with potentially different levels of human capital across sectors, conducting the analysis in terms of efficiency units is crucial. The BEA does not provide that information.

3.1 Baumol’s disease

To establish that Baumol’s disease importantly contributed to the productivity growth slowdown in the postwar U.S., we start with the obvious exercise that compares the actual growth rates of productivity with the counterfactual growth rates assuming that there had not been structural change among the 65 industries of WORLD KLEMS. As always in this paper, productivity is defined as real value added per efficiency unit. For the counterfactual, we fix the sectoral composition of 1947/8 and calculate the aggregate productivity growth rate that would have resulted from the actual growth rates of sectoral productivity with the unchanged sectoral composition from 1947/8. Calculating the counterfactual growth rates is not as straightforward as one might think because WORLD KLEMS is built around Törnqvist indexes that are not additive. In Appendix A below, we develop a productivity accounting framework that is similar to that of Nordhaus (2002) and addresses the resulting complications. One implication of this method is that to be consistent with the way in which the Törnqvist indexes are constructed, we must fix the sectoral composition in two adjacent years. That is the reason why we wrote 1947/8 above.

The upper panel of Table 1 reports the results. With the actual changes in the sectoral composition, the annual growth rates of productivity fell by 0.86 percentage points from 2.31% during 1947–1967 to 1.45% during 1987–2007. In contrast, with the sectoral composition held fixed at the 1947/8 values, the annual growth rates of productivity would have fallen by only 0.58 percentage points from 2.45% to 1.87%. The 0.28 percentage point difference between 0.86 and 0.58 implies that Baumol’s disease was responsible for one third of the total slowdown in productivity growth. Note that we stopped our analysis in 2007 to avoid the effects of the Great Recession. While all results go through when we include the Great Recession, the productivity slowdown becomes a lot larger. Since that has little to do with structural change, we do not include it here.

Nordhaus (2008) proposed an alternative way to measure the effects of structural change on productivity growth by comparing the counterfactual average growth rates over the period 1947–2007 when the sectoral composition is fixed at its 1947/8 and 2006/7 values while the
sectoral productivity growth rates are kept at what they were in the data. In addition to being useful on its own, Nordhaus way of looking at the data naturally conveys additional information about what level of disaggregation does best at capturing the effects of Baumol’s disease at the beginning and the end of the sample. We first calculate the Nordhaus counterfactuals for the 65 industries from the data. The results are contained the lower panel of Table 1. The table confirms the conclusion of the previous paragraph that structural transformation affected aggregate productivity growth in a quantitatively important way: if the composition among the 65 industries of WORLD KLEMS had been fixed at the 1947/8 (2006/7) values, then the average aggregate productivity growth rate would have been 2.03 (1.44) percent instead of the actual 1.74 percent in the data.

Given that we build on the productivity accounting method of Nordhaus, it is natural to compare our results with his. Nordhaus (2008) finds that if the sector composition had not changed, then the annual average growth rate would have been 0.37 percentage points higher; the average growth rate with the fixed initial sector composition is 0.64 percentage points larger than the average growth rate with the fixed final sector composition. It is reassuring that his numbers are in the same ballpark as our numbers. His numbers should not be expected to be exactly the same as our numbers because there are two important differences between the two studies. Whereas we focus on 1947–2007 and work with WORLD KLEMS data, Nordhaus focuses on the shorter time period 1948–2001 and worked with BEA data. Whereas we focus on productivity per efficiency unit, Nordhaus focuses on total factor productivity.

3.2 Towards a new three-sector split

So far, we have measured the effect of Baumol’s disease at the level of 65 disaggregate industries. Now we develop a new three-sector split that captures the essence of the past effects of Baumol disease on aggregate productivity in an analytically tractable way and that forms the foundation of a model of structural change that can predict the future effects of Baumol’s disease on productivity.

Consistent with the literature, we split the economy into the standard broad sectors of goods

Table 1: Actual and counterfactual average annual productivity growth (in %)

<table>
<thead>
<tr>
<th>Period</th>
<th>Sector composition from data</th>
<th>1947/8</th>
<th>2006/7</th>
<th>Baumol’s disease</th>
</tr>
</thead>
<tbody>
<tr>
<td>1947–1967</td>
<td>2.31%</td>
<td>2.45%</td>
<td>0.86</td>
<td>2.03 – 1.44 = 0.59</td>
</tr>
<tr>
<td>1987–2007</td>
<td>1.45%</td>
<td>1.87%</td>
<td>0.58</td>
<td></td>
</tr>
<tr>
<td>difference</td>
<td></td>
<td>0.58</td>
<td>0.58</td>
<td></td>
</tr>
</tbody>
</table>

Nordhaus’s disease
Table 2: Counterfactual U.S. Productivity Growth for Different Sector Splits in %

<table>
<thead>
<tr>
<th>Industry composition from</th>
<th>65 industries goods &amp; services</th>
<th>26 goods &amp; services</th>
<th>goods &amp; 39 services</th>
<th>goods &amp; high/low services</th>
<th>goods &amp; non/market services</th>
<th>goods &amp; un/skilled services</th>
</tr>
</thead>
<tbody>
<tr>
<td>1947/8</td>
<td>2.03</td>
<td>1.88</td>
<td>1.99</td>
<td>1.92</td>
<td>1.97</td>
<td>1.91</td>
</tr>
<tr>
<td>2006/7</td>
<td>1.44</td>
<td>1.58</td>
<td>1.56</td>
<td>1.46</td>
<td>1.49</td>
<td>1.53</td>
</tr>
<tr>
<td>difference</td>
<td>0.59</td>
<td>0.30</td>
<td>0.43</td>
<td>0.46</td>
<td>0.48</td>
<td>0.38</td>
</tr>
</tbody>
</table>

and services. The goods sector comprises agriculture, construction, manufacturing, mining, and utilities and the service sector comprises the remaining industries. Table 2 shows that this standard split does not work well in our context. For the purpose of predicting the future effects of Baumol’s disease on productivity growth, it is of first–order importance to disaggregate the service sector, but it is only of second–order importance to disaggregate the goods sector. To see why we are saying this, we start with the third column. It shows that for the usual two–sector split between goods and services the effect of Baumol’s disease is only 0.30 compared to 0.59 for all 65 industries. This implies that the usual two–sector split is not disaggregated enough for our purpose here. The fourth column shows that the 26 goods industries together with the broad service sector do better than the usual two–sector split: the effect of Baumol’s disease is now 0.43. Moreover, the number for the counterfactual productivity growth rate for the 1947/8 sector composition is fairly close to that for all 65 industries (1.99 versus 2.03), but the number for the 2006/7 sector composition is off (1.56 versus 1.44). The fifth column shows that the broad goods sector together with the 39 service industries from the data does better than the previous two splits: the effect of Baumol’s disease is now 0.46, which gets closer to 0.59 for all 65 industries. Moreover, the number for the 2007 sector composition is now close (1.46 versus 1.44), but the number for the 1947 sector composition is off (1.92 versus 2.03). These findings suggest that at the beginning of the sample, the reallocation within the goods sector was crucial for the effects of Baumol’s disease whereas at the end of the sample the reallocation within the service sector was crucial.

The previous findings confirm that if the focus is on the future productivity effects of Baumol’s disease, then we need to find an appropriate disaggregation of service sector, because that matters most towards the end of the sample. We consider three natural two–sector splits of services: services with high versus low productivity growth; market versus non–market services; skilled versus unskilled as suggested by Buera et al. (2015). The service subsector with high (low) productivity growth comprises the service industries that have productivity growth above (below) the average of 1.40% in the service sector. Market (non–market) services are defined by the guidelines of the System of National Accounts. Skilled (unskilled) service industries as service industries that pay a higher (lower) share of labor compensation to college graduate
than all service industries do on average. Table 3 lists all service industries in declining order of their average productivity growth rates and in which other two-sector splits these service industries belong. Table 9 in Appendix D lists the average annual productivity growth rates, confirming the observation of Baumol et al. (1985) that the service sector “contains some of the economies most progressive activities as well as its most stagnant”.

Table 2 shows that goods and services with high and low productivity growth provide a very good approximation to what we get for goods and the 39 service industries from the data, whereas the two-sector split into market versus non-market services is fairly off. Coincidentally, the two-sector split into skilled and unskilled services generates nearly the same numbers as that into high and low productivity growth. We conclude from this that using goods together either with services with high and low productivity growth or with services with high and low skills would work for studying the future effects of Baumol’s disease on productivity growth. In what follows, we use the former because it seems more natural in our context to construct the disaggregation of services according to the differences in sectoral productivity growth. Another two-sector split that has been considered by Duarte and Restuccia (2016) distinguishes between modern vs. traditional services. We do not pursue this two-sector split here because it is defined only for final expenditure whereas we are focused on value added.

Figure 1 shows the behavior of our three sectors in the postwar U.S. economy. All figures use efficiency units. The left panel plots the standard disaggregation between goods and services while the right panel plots the new disaggregation between services with high and low productivity growth. The figures in the upper panel plot ratios of sectoral efficiency units and sectoral nominal expenditures. The upper-left figure shows the usual patterns for the ratios of services to goods, which have both increased. The upper-right figure shows the novel pattern for the ratios of services with low to high productivity growth, which both increased until the 1970s and then flattened out. The figures in the lower panel plot relative labor productivities and relative prices. The lower-left figure shows the usual increases in the labor productivity of goods relative to services and in the price of services relative to goods. The lower-right panel shows the novel patterns that the labor productivity of services with high relative to low productivity growth and the price of services with low relative to high productivity growth increased somewhat until the 1970s and strongly afterwards.

There is some debate about whether the low productivity growth of many service industries comes in part from the fact that quality improvements in services are underestimated. Triplett and Bosworth (2003), for example, wrote that “perhaps the service industries were never sick, it was just, as Griliches (1994) has suggested, that the measuring thermometer was wrong”. In

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4 We use the BEA–BLS Industry Level Production Account, 1998–2015, for this calculation, which yields 0.50% as the average labour compensation share paid to college graduates in the service sector.

5 We have conducted the analysis that follows also for the other two-sector splits. The results are qualitatively similar and are available upon request.
Table 3: Definitions of Different Two–Sector Splits

<table>
<thead>
<tr>
<th>Service Industries in WORLD KLEMS</th>
<th>Market (1) vs. Non–market (2)</th>
<th>Unskilled (1) vs. Skilled (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(in declining order of productivity)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Service industries with high productivity growth**

- Pipeline Transportation: 1 1
- Air Transportation: 1 1
- Broadcasting and Telecommunications: 1 2
- Wholesale Trade: 1 1
- Securities, Commodity Contracts, and Investment: 1 2
- Waste Management and Remediation Services: 1 1
- Social Assistance: 2 1
- Railroad Transportation: 1 1
- Publishing Industries (includes Software): 1 2
- Water Transportation: 1 1
- Administrative and Support Services: 1 1
- Rental and Leasing Services and Lessors of Intangible Assets: 1 1
- Truck Transportation: 1 1
- Retail Trade: 1 1
- Insurance Carriers and Related Activities: 1 2
- Performing Arts, Spectator Sports, Museums, and Related Activities: 1 2
- Warehousing and Storage: 1 1
- Motion Picture and Sound Recording Industries: 1 2

**Service industries with low productivity growth**

- Management of Companies, and Enterprises: 1 2
- Miscellaneous Professional, Scientific, and Technical Services: 1 2
- Accommodation: 1 1
- Federal General Government: 2 1
- Federal Reserve Banks, Credit Intermediation, and Related Activities: 1 2
- Educational Services: 2 2
- Real Estate: 2 2
- Legal Services: 1 2
- Federal Government Enterprises: 2 2
- Ambulatory Health Care Services: 2 2
- Computer Systems Design and Related Services: 1 2
- Hospitals, Nursing, and Residential Care Facilities: 2 2
- Other Transportation and Support Activities: 1 1
- State and Local Government Enterprises: 2 2
- Amusements, Gambling, and Recreation Industries: 1 1
- Information and Data Processing Services: 1 2
- Food Services and Drinking Places: 1 1
- Transit and Ground Passenger Transportation: 1 1
- State and Local General Government: 2 2
- Other Services, except Government: 1 1
- Funds, Trusts, and other Financial Vehicles: 1 2
contrast, Byrne et al. (2016) and Syverson (2016) argued that underestimated quality improvements are not likely to have played an important in the recent productivity growth slowdown. Be that as it may, it is undeniable that if there were underestimated quality improvements in the postwar U.S., then they did translate into an exaggerated slowdown of aggregate productivity growth. We will nonetheless take the numbers from WORLD KLEMS at face value and proceed under the assumption that there were no issues with unmeasured quality improvements. This way of proceeding is informative in our context because, as we will show below in Subsection 5.3, our estimates then provide an upper bound of the effects of Baumol’s disease on productivity growth. Since our key finding is that the future effects of Baumol’s disease will remain limited, unmeasured quality improvements cannot overturn the main message of our findings.

**Figure 1: Postwar U.S. Structural Transformation**

3.3 Discussion

Having established that in the postwar U.S., structural change within the service sector led to Baumol’s disease and important reductions in productivity growth, the question arises why the literature on structural change has paid little to no attention to Baumol’s disease. In Duernecker
et al. (2017), we argue that the likely reason for this is that the literature looks for an aggregate balanced growth path because that simplifies dramatically the analytical characterization of the equilibrium path. In many models of structural change, an aggregate balanced growth path exists if GDP growth is measured in a particular way (in terms of numeraires that change from period to period). Since productivity growth is constant along any aggregate balanced growth path, it seems natural to conclude that Baumol’s disease is not an issue in the standard models of structural change. In Durnecker et al. (2017), we show that this conclusion is misleading: if we measure GDP growth as it is done in NIPA through a chain index, then Baumol’s disease does play a potentially important role along the balanced growth path of the standard models.\(^6\)

In what follows, we will therefore make sure to measure GDP in the exact same way as it is done in the data so as to give Baumol’s disease a chance to affect productivity growth.\(^7\) We start by building a simple model to capture the effects of Baumol’s disease on productivity growth. Afterwards, we will calibrate the model and use it to assess by how much Baumol’s disease will reduce future productivity growth.

### 4 Model

#### 4.1 Environment

There are three sectors producing goods, services with high productivity growth, and services with low productivity growth. We index the sectors by \(g, h,\) and \(l\). In each sector, value added is produced from labor:

\[
Y_{it} = A_{it}H_{it}, \quad i = g, h, l, \tag{1}
\]

where \(Y_i, A_i,\) and \(H_i\) denote value added, total factor productivity, and efficiency units of labor of sector \(i\). The linear functional form implies that labor productivity equals TFP, that is, \(Y_{it}/H_{it} = A_{it}\). We use it because it is simple and because it captures well the essence of the role that technological progress plays for structural transformation [Herrendorf et al. (2015)].

There is a measure one of identical households. Each household is endowed with a finite number of efficiency units of labor that are inelastically supplied and can be used in all sectors. We will refer to GDP per efficiency unit as labor productivity or productivity for short.

Utility is described by two nested, non–homothetic CES utility functions. The utility from

\(^6\)In independent work, Leon-Ledesma and Moro (2017) also made some of these points.

\(^7\)In our application that means to measure GDP growth through a Törnqvist index because we use data from WORLD KLEMS. Similar statements as for chain indexes apply.
the consumption of goods and (aggregate) services, \( C_g \) and \( C_{st} \), is given by:

\[
C_t = \left( \frac{1}{\alpha_g} C_t^{\sigma_c} C_g^{\sigma_{c_g}} + \frac{1}{\alpha_s} C_t^{\sigma_s} C_s^{\sigma_{s_s}} \right)^{\frac{1}{\sigma_c - 1}}. \tag{2a}
\]

Services are given by a non–homothetic CES aggregator of the consumption from the two service sub–sectors, \( C_{ht} \) and \( C_{lt} \):

\[
C_{st} = \left( \frac{1}{\alpha_h} C_t^{\sigma_h} C_{ht}^{\sigma_{h_t}} + \frac{1}{\alpha_l} C_t^{\sigma_l} C_{lt}^{\sigma_{l_t}} \right)^{\frac{1}{\sigma_s - 1}}. \tag{2b}
\]

\( \alpha_g, \alpha_s, \alpha_h, \alpha_l \geq 0 \) are weights; \( \sigma_c, \sigma_s \geq 0 \) are the elasticities of substitution; \( \varepsilon_g, \varepsilon_s, \varepsilon_h, \varepsilon_l > 0 \) capture income effects.

The non–homothetic CES utility function (2) goes back to the work of Hanoch (1975) and Sato (1975) on implicitly additive utility and production functions. It has recently been introduced to the literature on structural transformation by Comin et al. (2015). For \( \varepsilon_i = 1 \), the expressions in (2) reduce to the standard CES utility that implies homothetic demand functions for each consumption good. For \( \varepsilon_i \neq 1 \), the level of consumption affects the relative weight that is attached to the consumption goods. Although in this case there is no closed–form solution for utility as a function of the consumption goods, it turns out that the implied non–homothetic demand functions remain tractable.

In our context, the most important feature of the non–homothetic CES utility function (2) is that the implied income effects do not disappear in the limit as consumption grows without bound. Boppart (2016) and Comin et al. (2015) established that this is consistent with the available evidence for the broad sectors of goods and services that are usually considered in the structural transformation literature. Figure 2 establishes that this is also consistent with the evidence for the two service subsectors that we consider here. Specifically, the figure establishes that the income effect on the relative demand of the two service sectors does not systematically decline as the economy grows. Modelling persistent income effects is important in our context in which we are interested in the limit behavior of productivity. A standard Stone–Geary utility specification would not capture persistent income effects because, as consumption grows without bound, it converges to a homothetic utility function.

The non–homothetic CES aggregators make economic sense only if they satisfy the usual regularity conditions. To ensure that this is the case, we restrict the parameters as follows:

**Assumption 1**

- \( \sigma_c < \min\{\varepsilon_g, \varepsilon_s\} \) or \( \max\{\varepsilon_g, \varepsilon_s\} < \sigma_c \).
- \( \sigma_s < \min\{\varepsilon_h, \varepsilon_l\} \) or \( \max\{\varepsilon_h, \varepsilon_l\} < \sigma_s \).
Figure 2: Persistent Income Effects within Services

Partial correlation between relative value added within services and aggregate value added per efficiency unit

Source: WORLD KLEMS, March 2017 release and own calculations
Note: Residuals on the y-axis are from regressing the log difference of nominal value added of services with low and high productivity growth on the corresponding log difference of prices. Residuals on the x-axis are from regressing the log of aggregate real value added per efficiency unit on the same log differences of prices.

Hanoch (1971, 1975) showed that this assumption guarantees that the utility function is monotonically increasing in each of its arguments and quasi–concave.

We complete the description of the environment with the resource constraints:

\[ C_{it} \leq Y_{it}, \quad i = g, h, l, \quad (3a) \]
\[ H_{gt} + H_{st} = H_{gt} + H_{ht} + H_{lt} \leq H_t. \quad (3b) \]

4.2 Competitive equilibrium

In the data, the nominal labor productivities per efficiency unit are not equalized across sectors. It is potentially important to capture the resulting level differences in productivity in our context, because the effects of structural transformation on aggregate productivity depend on the differences in both the growth rates and the levels of sectoral productivity. A simple way of capturing them is to introduce a sector–specific tax \( \tau_{it} \) that firms pay per unit of wage payments and that is lump–sum rebated through a transfer \( T_t = \sum_{i=g,h,l} \tau_{it} w_t H_{it} \) to the households.

With the tax, the problem of firm \( i = g, h, l \) becomes:

\[
\max_{H_{it}} P_{it} A_{it} H_{it} - (1 + \tau_{it}) w_t H_{it}.
\]

The first–order conditions imply that

\[
\frac{P_{jt}}{P_{gt}} = \frac{(1 + \tau_{jt}) A_{gt}}{(1 + \tau_{gt}) A_{jt}}, \quad j = h, l.
\]

Using this and the specification of the production function in (1), we obtain that the relative
sectoral labor productivities equal the relative taxes:
\[
P_{jt} C_{jt} / H_{jt} = \frac{1 + \tau_{jt}}{1 + \tau_{gt}}, \quad j = h, l.
\]  
(5)

As intended, the taxes drive a wedge between the nominal sectoral labor productivities which will allow us to match differences in them.

To solve the household problem, we split it into two layers. The “inner layer” of the problem is about allocating a given quantity of service consumption between the consumption of the two services:

\[
\min_{C_{ht}, C_{lt}} P_{ht} C_{ht} + P_{lt} C_{lt} \quad \text{s.t.} \quad \left( \frac{1}{\alpha_h} \frac{\sigma_{et} - 1}{\sigma_{eh}} C_{ht}^{\frac{\sigma_{eh}}{\sigma_{et}}} + \frac{1}{\alpha_l} \frac{\sigma_{et} - 1}{\sigma_{el}} C_{lt}^{\frac{\sigma_{el}}{\sigma_{et}}} \right) \geq C_{st}.
\]

Appendix B shows that the first–order conditions imply that

\[
P_{ht} C_{ht} = \frac{\alpha_l}{\alpha_h} \left( \frac{P_{ht}}{P_{ht}} \right)^{1-\sigma_e} C_{ht}^{\frac{1}{1-\sigma_e}}, \quad (6a)
\]

\[
P_{lt} = \left( \alpha_h C_{ht}^{\sigma_e - 1} P_{ht}^{1-\sigma_e} + \alpha_l C_{lt}^{\sigma_e - 1} P_{ht}^{1-\sigma_e} \right)^{1-\sigma_e}, \quad (6b)
\]

where \( P_{st} \) is price index of services. The “outer layer” of the problem is about allocating a given quantity of total consumption between the consumption of goods and services:

\[
\min_{C_{gt}, C_{st}} P_{gt} C_{gt} + P_{st} C_{st} \quad \text{s.t.} \quad \left( \frac{1}{\alpha_g} \frac{\sigma_{et} - 1}{\sigma_{eg}} C_{gt}^{\frac{\sigma_{eg}}{\sigma_{et}}} + \frac{1}{\alpha_s} \frac{\sigma_{et} - 1}{\sigma_{es}} C_{st}^{\frac{\sigma_{es}}{\sigma_{et}}} \right) \geq C_{st}.
\]

Appendix B shows that the first–order conditions imply

\[
P_{gt} C_{st} = \frac{\alpha_s}{\alpha_g} \left( \frac{P_{st}}{P_{gt}} \right)^{1-\sigma_e} C_{st}^{\frac{1}{1-\sigma_e}}, \quad (7a)
\]

\[
P_t = \left( \alpha_g C_{st}^{\sigma_e - 1} P_{lt}^{1-\sigma_e} + \alpha_s C_{st}^{\sigma_e - 1} P_{st}^{1-\sigma_e} \right)^{1-\sigma_e}, \quad (7b)
\]

where \( P_t \) is the aggregate price index and \( P_t C_t = \sum_{l=e,g,h,l} P_t C_{lt} \).

---

The given quantity of total consumption is determined by the endowment of labor and the technology: \( C_t = A_{gt} / P_t \). A particular household takes \( A_{gt} / P_t \) as given because \( A_{gt} \) is exogenous and \( P_t \) is the aggregate price index that is independent of its actions.
4.3 Equilibrium properties

We now turn to the equilibrium properties of our model. Since it is impossible to solve for the equilibrium in closed form, we will resort to numerical simulations to characterize the precise behavior of our model. Before we do so, it is nonetheless useful to give the reader some idea about how the model behaves qualitatively.

We begin with the structural transformation between goods and services. Dividing (7a) for periods $t + 1$ and $t$ by each other and denoting growth factors by “hats”, we obtain:

$$\left( \frac{P_{st}C_{st}}{P_{gt}C_{gt}} \right)^{1-\sigma} = \left( \frac{P_{st}}{P_{gt}} \right)^{\epsilon_s - \epsilon_g}. \quad (8)$$

The first term on the right-hand side is the relative price effect and the second term is the income effect. We make the standard assumption that goods and aggregate services are complements, goods are necessities, and services are luxuries:

**Assumption 2** $0 < \sigma < 1$ and $\epsilon_g < \epsilon_s$.

Our calibration below will generate parameter values that are consistent with these assumptions. Expression (8) shows that our model then generates the observed structural transformation from goods to services if $P_{st}/P_{gt}$ and $C_t$ both grow. While in general this may or may not be the case for the non-homothetic utility function (2), we will establish below that it is the case for the numerically relevant parameters that our calibration generates.

Before turning our attention to the structural change within the service sector, it is useful to draw the attention to a feature of (8) that is going to become important in the quantitative analysis following below. The right-hand side of the demand system depends only on the difference of $\epsilon_s - \epsilon_g$, implying that the two $\epsilon_i$ will not be separately identified. We therefore need to normalize one of them.

We continue with the structural transformation between the two service subsectors. Combining Equations (5) and (6a), we obtain:

$$\left( \frac{P_{lt}C_{lt}}{P_{ht}C_{ht}} \right)^{1-\sigma_s} = \left( \frac{P_{lt}}{P_{ht}} \right)^{\epsilon_l - \epsilon_h}. \quad (9)$$

To fit the data, we will need to assume that the two service subsectors are substitutes, services with high productivity growth are a necessity, and services with low productivity growth are a luxury:

**Assumption 3** $1 < \sigma_s$ and $\epsilon_h < \epsilon_l$.

---

9See, for example, Kongsamut et al. (2001), Ngai and Pissarides (2007), and Herrendorf et al. (2013).

10Sposi (2016) proceeds in a similar fashion.
Our calibration below will generate parameter values that are consistent with Assumption 3. The relative price effect, which is the first term on the right-hand side of Equation (9), then decreases the expenditure of services with high productivity growth relative to the expenditure of services with low productivity growth. Moreover, the income effect, which is the second term on the right-hand side, then increases the expenditure ratio of services with low productivity growth relative to services with high productivity growth if $C_t$ increases. The net effect is analytically ambiguous.

Assumption 3 is required for our model to replicate the patterns of structural transformation within the service sector as summarized by Figure 3. Until around 1970 the price of services with low productivity growth relative to services with high productivity growth increased along with the corresponding expenditure ratio. After 1970, the increase in the relative price of services with low productivity growth accelerated while the expenditure ratio remained roughly constant. Since we assume that services with low productivity growth are luxuries, the increases in consumption expenditure increase their share over the whole period. Since we assume that the two services are substitutes, the increases in the relative price of services with low productivity growth decreases their expenditure share over the whole period. Our model replicates the observed pattern by having the income effect dominate before 1970 and having the income and substitution effect offset each other after 1970. Alternative parameter constellations would not be able to generate the observed patterns. To see this, note first that the income effects do not change in 1970 but work in the same directions during the whole period. So the change in the relative expenditure share pattern must be coming from the acceleration in the relative prices. If the services were complements, then the expenditure share of services with low productivity growth would have increased by more after 1970 than before 1970, which is counterfactual. Given that the two services must be substitutes, services with low productivity growth must
be a luxury and services with high productivity growth must be a necessity. If the opposite was true, then the relative expenditure of service with high productivity growth would have increased during the whole period, which again is counterfactual.

It is important to realize that the usual forces behind the reallocation between goods and services are very different from the new forces behind the reallocation of production within the service sector. The usual forces imply that the relative price effect and the income effect both work in the same direction and increase the expenditure share of services relative to goods. In contrast, within the service sector the relative price effect and the income effect work in opposite directions: the increase in the relative price decreases the relative expenditure share of services with low productivity growth whereas the increases in total consumption increases them. If the elasticity of substitution is close to Cobb–Douglas, then the income effect will dominate; if the income effect is close to zero, then the substitution effect will dominate. In intermediate cases, both effects will interact with each other and it is impossible to say much more than that they work in opposite directions. Consider, for example, the effect on the relative expenditure share of an exogenous decline in the productivity of the services with low productivity growth. Since the two services are substitutes, the household will react by substituting away from services with low productivity growth so strongly that their expenditure share declines. This will increase the productivity of producing aggregate services, which in turn will increase aggregate productivity and total consumption. Since services with low productivity growth are luxuries, the resulting income effect will increase the relative expenditure of services with low productivity growth. The net effect of the two opposing effects is ambiguous analytically.

We emphasize that using a utility function with income effects that do not disappear in the long run is crucial for analyzing the effects of structural change within services. If, instead, we had used a standard Stone–Geary utility function without persistent income effects, then the substitution effect would have dominated in the long–run. In this case, the services with high productivity growth would have taken over the economy in the long run and there would have been no reason to worry about the future effects of Baumol disease. The situation is very different for our utility function where the outcome is not clear and a serious quantitative analysis becomes necessary. We turn to this task now.

5 Quantitative Analysis

In this section, we calibrate our model to match salient features of the postwar U.S. economy, including the behavior of the relative sectoral prices, the relative sectoral labor productivities, and the sectoral composition. We then use the calibrated model to predict by how much Baumol’s disease will affect productivity growth. As before, productivity is defined as the real value added per efficiency unit.
Before we turn to the details of the quantitative analysis, it is important to point out that it is challenging to connect our model to the data. The reason for this is that in the model GDP growth is given by the change in the non–homothetic CES utility index \( C_t \), whereas in the WORLD KLEMS data GDP growth is measured by the change in a Törnqvist index. Appendix A.2 shows that these two indexes are completely different objects when the utility is non–homothetic, implying that their growth factors cannot directly be compared to each other. Instead, one needs to construct the Törnqvist index in the same way in the model from quantities and prices of the model and as it was done in the data and compare the two Törnqvist indexes from the model and the data with each other. We will carefully follow this procedure when we calibrate the model and when we compare the productivity growth from the model with the data.

5.1 Calibration

We start with calibrating the sectoral taxes. Equation (5) implies that only relative taxes matter for the equilibrium. We therefore normalize the taxes in the goods sector to zero for all years \( t = 1947, \ldots, 2007 \): \( \tau_{gt} = 0 \). Defining nominal value added as \( VA_{jt} \equiv P_{jt} Y_{jt} \) and denoting observable variables by a tilde, Equation (5) can then be written as:

\[
1 + \tau_{jt} = \frac{\tilde{VA}_{jt}}{\tilde{VA}_{gt}} \frac{\tilde{H}_{gt}}{\tilde{H}_{jt}}, \quad j = h, l, \quad t = 1947, \ldots, 2007.
\]

This equation uniquely determines the values of the taxes in sectors \( h \) and \( l \), because the right–hand side is the observable nominal labor productivity per efficiency units in sector \( j \) relative to sector \( g \). The upper left panel of Figure 4 plots the resulting series for the distortions.

We continue with calibrating the sectoral TFPs. We normalize \( P_{g,1947} = P_{h,1947} = P_{l,1947} = A_{g,1947} = 1 \). We calibrate the TFPs in the goods sector by rewriting equation (1) as follows:

\[
\frac{A_{gt+1}}{A_{gt}} = \frac{\tilde{VA}_{gt+1}/(\tilde{P}_{gt+1} \tilde{H}_{gt+1})}{\tilde{VA}_{gt}/(\tilde{P}_{gt} \tilde{H}_{gt})}, \quad t = 1947, \ldots, 2007.
\]

This equation uniquely determines the growth factor of TFP in the goods sector. We calibrate the other sectoral TFPs by rewriting Equation (4) as:

\[
A_{jt} = \frac{\tilde{P}_{jt}}{P_{jt}} (1 + \tau_{jt}) A_{gt}, \quad j = h, l, \quad t = 1947, \ldots, 2007.
\]

We take \( \{\tilde{P}_{jt}\}_{j=g,h,l, t=1948, \ldots, 2007} \) from the data. We have already normalized \( \{P_{jt,1947}\}_{j=g,h,l} \) and have calibrated the taxes and the TFPs in the goods sector, implying that all variables on the right–hand side are given. The upper right panel of Figure 4 plots the implied sector TFPs.
Since only relative prices matter in the model, we are free to choose \( \{P_{gt}\}_{t=1948,...,2007} \) so as to match the growth factors of the goods prices in the data. The previous choices then imply that by construction we also match the growth of nominal labor productivity in the goods sector. Equation (4) and the previous choices then imply that we also match \( \{\tilde{P}_{jt}\}_{j=l, t=1948,...,2007} \) and the growth of nominal labor productivity in the other sectors by construction.

In sum, the parameter choices that we have made so far imply that our model will match the growth factors of nominal and real sectoral labor productivities and the growth factors of all prices. In addition, our model also matches the initial level differences between the real sectoral labor productivities.

This leaves ten parameters to be calibrated: the four relative weights \( \{\alpha_g, \alpha_s, \alpha_h, \alpha_l\} \), the two elasticities \( \{\sigma_s, \sigma_c\} \), and the four parameters governing the income effects \( \{\varepsilon_g, \varepsilon_s, \varepsilon_h, \varepsilon_l\} \). As we mentioned above already, \( \varepsilon_s \) and \( \varepsilon_g \) or \( \varepsilon_l \) and \( \varepsilon_h \) are not separately identified for the non-homothetic CES functional form that we are using, because the expenditure system (6a)–(7a) depends on the differences \( \varepsilon_s - \varepsilon_g \) and \( \varepsilon_l - \varepsilon_h \). We are therefore free to set \( \varepsilon_g \) and \( \varepsilon_h \) to whatever is convenient. We choose \( \varepsilon_g = 0.85 \) and \( \varepsilon_h = 0.58 \), because the resulting parameters will satisfy Assumption 1. We also impose that \( \alpha_s = 1 - \alpha_g \) and \( \alpha_l = 1 - \alpha_h \). This leaves the following six parameters to calibrate:

\[ \{\alpha_g, \alpha_h, \sigma_s, \sigma_c, \varepsilon_s, \varepsilon_l\} \]
To choose these six parameter values, we target the relative nominal value added of the different sectors from the data:

\[
\frac{\bar{VA}_t}{VA_{gt}}, \quad \frac{\bar{VA}_t}{VA_{ht}}, \quad t = 1947, \ldots, 2007.
\] (13)

We choose the values for the six parameters that minimize the sum of the squared deviations between the relative nominal value added that are implied by the expenditure system (6a)–(7a) and that are observed in the data. It is important in this context that when we solve for the equilibrium of the model we use the consumption indexes that are implied by the model, because the consumption indexes from the model differ from total consumption and service consumption in the data. The upper panels of Figure 5 show that this procedure matches well the trends of relative value added and employment shares in the data, which is what we care about in this context.

The calibration results are in Table 4. We find the expected parameter constellation: goods and services are complements \((\sigma_c < 1)\); goods are necessities and services are luxuries \((\epsilon_s - \epsilon_g > 0)\); services with high and low productivity growth are substitutes \((\sigma_s > 1)\); services with high productivity growth are necessities and services with low productivity growth are luxuries \((\epsilon_l - \epsilon_h > 0)\).

Before turning to the future effects of Baumol’s disease, two remarks about the calibration results are at order. First, the lower panel of Figure 4 shows that the calibrated parameters imply model sequences \(\{C_t, P_{st}\}_{t=1947, \ldots, 2007}\) that have upward trends. This justifies our assumption in the theory part that both \(C_t\) and \(P_{st}\) are increasing. Second, our model does a good job at generating aggregate labor productivity in the data. In this context two subtle measurement issues arise. First, to measure aggregate value added in the model we need to constructed it with the same Törnqvist index as it is done in the data.\(^{11}\) Second, we need to be careful in handling \(H_t\). In the data the quality–adjusted sectoral labor inputs \(\tilde{H}_t\) are indexes that do not add up to \(\tilde{H}_t\). This contradicts the feasibility constraint (3b), which imposes that \(\sum_{i \in \{g, h, l\}} H_{it} = H_t\). We deal with that by setting \(H_t = \sum_{i \in \{g, h, l\}} \tilde{H}_{it}\) in the model. While that way of proceeding assures that the adding up constraint will be satisfied in the model, it introduces an inconsistency when comparing aggregate productivity growth from the model with the data. The reason for this is that the former divides aggregate GDP by \(H_t\) whereas the latter divides GDP by \(\tilde{H}_t \neq H_t\). This matters for the quantitative results, because the difference between \(H_t\) and \(\tilde{H}_t\) is not negligible. We therefore adjust all productivity numbers from the model by multiplying them with the correction factor \((\sum_{i \in \{g, h, l\}} \tilde{H}_{it})/\tilde{H}_t\).

The lower panel of Figure 5 shows that, in fact, the resulting series for aggregate labor

\(^{11}\)In a companion paper, Duernecker et al. (2017), we demonstrate that this is essential for capturing the productivity growth slowdown. See also Whelan (2002) and Moro (2015).
productivity in the data and the model lie right on top of each other. The reason is that the model matches both nominal and real sectoral labor productivity and the relative prices by construction. How well the model matches aggregate labor productivity then depends only on how well it matches the last input into the Törnqvist index, namely relative sectoral efficiency units. The lower panel of Figure 5 shows that the model also does well in this respect. The fact that the model matches aggregate labor productivity during the postwar period well builds confidence that it is suitable for predicting how aggregate labor productivity will evolve in the future. We turn to this task now.

<table>
<thead>
<tr>
<th>Table 4: Calibrated Parameters</th>
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<tbody>
<tr>
<td>$\alpha_g$</td>
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<tr>
<td>0.48</td>
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</tbody>
</table>

Figure 5: Value Added, Employment, and Aggregate Productivity – Model and Data

5.2 Model simulation

To simulate our model forward, we assume that between 2015–2075, the variables $\{A_{gt}, A_{ht}, A_{lt}, H_t, P_{gt}\}$ and the correction factors $\{(\sum_{i \in \{g, h, l\}} \tilde{H}_i)/\tilde{H}_t\}$ grow at the same constant rates on average
as they did “in the past” and the tax rates \( \{ \tau_{ht}, \tau_{lt} \} \) equal the average of their “past values”. The key issue to settle is what we mean by “the past”. We consider three possibilities of what the past may be: 1987–2007; 1977–2007; 1967–2007. We add to these three possibilities a counterfactual that takes the values from 1987–2007 while imposing \( \% \Delta A_l = 0 \). This is meant to provide a natural lower bound of how low productivity might fall in the service sector with low productivity growth. Since past tax rates fluctuate quite a bit without showing a clear trend, we also provide some additional robustness analysis in the next subsection regarding the assumptions about the future tax rates (i.e. we also simulate the model for the minimum or maximum over all past values of the tax rates etc).

Table 5 shows the results. The top rows display the past growth rates of aggregate labor productivity. The bottom rows display the predicted future growth rates of aggregate labor productivity.\(^\text{12}\) During 2055–2075, the model predicts productivity growth rates of between 1.09\% and 1.26\%. In comparison, we have seen above in Section 3 that in the data productivity growth fell from 2.31\% during 1947–1967 and to 1.45\% during 1987–2007, of which 0.28 percentage points were due to structural change. So our predicted productivity growth slowdown of at most 0.14 percentage points is exactly half of the 0.28 percentage points that we saw between 1947–1967 and 1987–2007. Even if there was no productivity growth whatsoever in the services with low productivity growth (last row of the table), then aggregate productivity would only fall by 0.14 percentage points to 1.15\%. In other words, according to our model, the future effects of Baumol’s disease are considerably smaller than its past effects were.

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<tbody>
<tr>
<td></td>
<td>2.31 (2.33)</td>
<td>1.45 (1.48)</td>
<td>1.74 (1.75)</td>
</tr>
<tr>
<td>estimation period for exogenous variables</td>
<td>2015–35</td>
<td>Predicted</td>
<td>2015–75</td>
</tr>
<tr>
<td>1987–2007</td>
<td>1.40</td>
<td>1.26</td>
<td>1.33</td>
</tr>
<tr>
<td>1977–2007</td>
<td>1.23</td>
<td>1.09</td>
<td>1.16</td>
</tr>
<tr>
<td>1967–2007</td>
<td>1.34</td>
<td>1.22</td>
<td>1.28</td>
</tr>
<tr>
<td>1987–2007, % \Delta A_l = 0</td>
<td>1.29</td>
<td>1.15</td>
<td>1.22</td>
</tr>
</tbody>
</table>

It is important to realize that the model is essential for making these predictions. If instead we had just run a simple regression on past data and extrapolated the result into the future, we would have gotten different results. For example, during the period from 1987–2007, a simple linear regression of aggregate labor productivity growth on time gives a slope coefficient of 0.0293. This implies that a simple extrapolation by 20 years would predict a productivity

\(^{12}\)Background information on the other inputs is in Table 8 in Appendix C.
growth acceleration from 1.43% to 1.43% + (0.0293 \cdot 20)\% = 2.02\%. Alternatively, during the period from 1947–2007, a simple linear regression of aggregate labor productivity growth on time gives a slope coefficient of −0.0201. This implies that a simple extrapolation by 20 years would predict a slowdown from 1.43% to 1.43% − (0.0201 \cdot 20)\% = 1.03\%. Both predicted changes in aggregate productivity growth are very different from what the model predicts. This means that the effects of the reallocation of production among the three sectors that the model captures are important for our predictions.

We end this subsection with providing some intuition for why our model predicts that the future effects of Baumol’s disease on aggregate productivity growth are merely half of the past effects. To begin with, between 1947 and 2007 there was considerable reallocation to the service subsectors, which on average have smaller productivity growth than the goods sector. While that reallocation led to a productivity growth slowdown, the goods sector is rather small in 2007, implying that this force behind the productivity growth slowdown will be less important in the future when the center stage will be taken by reallocation to and within the service sectors. Since the data suggest that the two service subsectors are substitutes, the model predicts that the services with low productivity growth are not taking over the economy in the limit. This limits the future slowdown in productivity growth.

Our conclusion differs sharply from that of standard models of structural transformation. These models feature just one elasticity of substitution among sectors, which is typically set such that the different sector outputs are complements. They therefore imply that the sector with the slowest productivity growth takes over in the limit. If that sector has zero productivity growth, then a standard model of structural transformation would predict that in the limit productivity growth falls to zero. The last line of Table 5 shows that even if we impose that from now on onwards the productivity growth of that service sector is zero, aggregate future productivity growth in our model still comes out in excess of 1\%.\footnote{Strictly speaking, the previous statements are true only for structural change models that do not have capital. In contrast, structural change models with capital usually do not feature structural transformation within the investment sector, implying that the sector with the slowest productivity growth takes over the consumption sector, instead of the whole economy, in the limit. An exception to this statement is Herrendorf et al. (2017), who develop a recent model that captures structural change in the investment sector and the consumption sector.}

5.3 Robustness analysis

In this subsection, we explore how our predictions change if we adopt alternative assumptions about future taxes or take into account the possibility of underestimated quality improvements in services.

We start by conducting robustness analysis regarding the future values of the two tax processes. As the upper left panel of Figure 4 showed, the distortion series fluctuate widely without showing any clear trend. Above we therefore used average tax rates over the different past pe-
riods to predict future tax rates. This raises the question of how sensitive our results are to alternative specifications of the tax processes. Table 6 explores all possible mixes between the minimum and maximum values of $\tau_h$ and $\tau_l$ over the period 1987–2007. We can see that predicted aggregate productivity growth rates hardly change. We conclude from this that our results are not sensitive to the precise specification of the tax processes.

**Table 6: Future Productivity Growth under Different Assumptions about Future Taxes**
(baseline corresponds to line 1987–2007 of Table 5)

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<tr>
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<tbody>
<tr>
<td>baseline: average $\tau_h$, $\tau_l$</td>
<td>1.40</td>
<td>1.26</td>
<td>1.33</td>
<td></td>
</tr>
<tr>
<td>$\tau_h = \max{\tau_h}$, $\tau_l = \max{\tau_l}$</td>
<td>1.41</td>
<td>1.26</td>
<td>1.33</td>
<td></td>
</tr>
<tr>
<td>$\tau_h = \max{\tau_h}$, $\tau_l = \min{\tau_l}$</td>
<td>1.38</td>
<td>1.24</td>
<td>1.31</td>
<td></td>
</tr>
<tr>
<td>$\tau_h = \min{\tau_h}$, $\tau_l = \max{\tau_l}$</td>
<td>1.41</td>
<td>1.27</td>
<td>1.34</td>
<td></td>
</tr>
<tr>
<td>$\tau_h = \min{\tau_h}$, $\tau_l = \min{\tau_l}$</td>
<td>1.38</td>
<td>1.25</td>
<td>1.31</td>
<td></td>
</tr>
</tbody>
</table>

In Section 3 above, we have already touched on the possibility that the low productivity growth of many service industries may in part come from the fact that quality improvements in services are not properly measured. If this is the case, then under−estimated quality improvements translate into an overestimated slowdown of aggregate productivity growth. So far, we have taken the numbers from WORLD KLEMS at face value and have pretended that there are no issues with under−estimated quality improvements. We close this section by looking more seriously at this possibility. The goal is to substantiate the claim we made in Section 3 above that our estimates of the effects of Baumol’s disease provide an upper bound, that is, if there are unmeasured quality improvements in services, then the future slowdown in productivity growth will be smaller than our estimates.

To entertain various possibilities of by how much the price increases of services with low productivity growth are overestimated, consider the counterfactual price increase

$$\Delta\tilde{P}_l = \omega \Delta P_l + (1 - \omega)\Delta P_h,$$

where $\omega \in [0, 1]$. If $\omega = 1$, then $\Delta\tilde{P}_l = \Delta P_l$ and there is no underestimation of quality improvements. If $\omega = 0$, then $\Delta\tilde{P}_l = \Delta P_h$ and there is such a severe underestimation of quality improvements in the service sector with low productivity growth that the actual price increases of both services subsectors are the same. We vary $\omega$ between these extremes, recalibrate our model after replacing $P_l$ by $\tilde{P}_l$, take the period 1987–2007 as the past which we use to obtain the estimates of future exogenous processes, and redo the projection exercise for the next 60 years.

Table 7 reports the results. Recall that $\omega = 1$ corresponds to the previous benchmark
case and a lower value of $\omega$ corresponds to a more severe underestimation of quality of the value added produced in the service sector with low productivity growth. As $\omega$ increases, the predicted future slowdown in productivity growth decreases. This confirms that if quality improvements are underestimated in the services with low productivity growth, then the predicted future slowdown in productivity growth that was reported in Table 5 above provides an upper bound on how much productivity growth will fall.

<table>
<thead>
<tr>
<th>$\omega$</th>
<th>Data 1987–2007</th>
<th>Model Predictions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>1.45</td>
<td>1.40 1.26 1.33</td>
</tr>
<tr>
<td>0.75</td>
<td>1.73</td>
<td>1.71 1.60 1.65</td>
</tr>
<tr>
<td>0.50</td>
<td>2.01</td>
<td>2.01 1.94 1.98</td>
</tr>
<tr>
<td>0.25</td>
<td>2.29</td>
<td>2.31 2.09 2.30</td>
</tr>
</tbody>
</table>

6 Conclusion

We have demonstrated that Baumol’s disease considerably slowed down productivity growth in the post World War II period. We have built a model that accounts for this and that implies that in the future Baumol’s disease will be much less of a drag on productivity growth than it has been in the past. The key novel feature that has been crucial for reaching this conclusion is that we have disaggregated services into the two subsectors that have high and low productivity growth. The data suggest that these two subcategories of services are substitutes. This implies that the services with low productivity growth will not take over the economy in the limit, which is in sharp contrast to what existing models of structural transformation imply.

We have taken the sectoral growth rates as given and we have explored which consequences changes in the sectoral composition have for future productivity growth. A first interesting question for future work is why different sectors show different productivity growth. A second interesting question for future work is to study whether the slow growing sectors will continue to grow slowly even when they comprise sizeable shares of the economy. We have made some initial progress on these questions in Herrendorf and Valentinyi (2015).

We have modeled structural change as resulting from the interaction between technological progress and the features of preferences. While this way of proceeding has been shown to work well in many contexts, it abstracts from important features of reality. One such feature is structural change within the investment sector: in the data, the share of investment produced in the goods (service) sector is decreasing (increasing). Recent work by Garcia-Santana et
al. (2016) and Herrendorf et al. (2017) started to get a handle on structural change within the investment sector. Another feature from which we have abstracted is changes in the input–output structure of the economy, which Oulton (2001) argued may be important in the context of Baumol’s disease. Our way of proceeding implies that these changes are captured by the income and substitution effects that originate from the features of household preferences which are formulated over value added. While the functional form that we have used is general enough to do a good job at capturing past structural change in the postwar U.S., it may not work as well anymore if in the future the input–output structure change considerably. Recent work by Sposi (2016) has started to study the role that input–output linkages play in the context of structural transformation. We leave it to future research to think seriously about the role of structural change in the investment sector or the input–output structure in the context of Baumol’s disease.

References


—— and ——, “Relative Prices and Sectoral Productivity,” Manuscript, University of Toronto 2016.


Appendix A: Theory behind the Data Work

A.1 Calculations behind Tables 1–2

A.1.1 Calculating real value added

The WORLD KLEMS 2017 March Release consists of nominal and real gross outputs, intermediate inputs, capital inputs (capital services), and labor inputs (efficiency units of labour) for 65 industries. Nominal capital and labor inputs are defined by nominal capital and labor compensation. The data set uses Törnqvist indexes to calculate real quantities. This implies that industry quantities are not additive and one has to be careful when manipulating real quantities. For example, real value added does not equal the difference between real gross outputs and real intermediate inputs. The objects that remain additive when Törnqvist indexes are used are nominal outputs and inputs (outputs and inputs in current dollars).

Here, we describe how the growth rates of real value added are constructed. The first step is to define the growth rate of a generic variable $X$ between periods $t$ and $t + 1$ as:

$$\Delta \log(X_t) \equiv \log(X_{t+1}) - \log(X_t).$$

The growth rates of real gross output, real value added and real intermediate inputs in industry $i$ are linked by the following identity:

$$\Delta \log(GO_{it}) = [1 - V(P_{II_{it}}^I)]\Delta \log(Y_{it}) + V(P_{II_{it}}^I)\Delta \log(II_{it}),$$

where $GO_{it}$, $II_{it}$, $Y_{it}$, $P_{GO_{it}}^I$, $P_{II_{it}}^I$ and $P_{it}$ denote real gross output, real intermediate inputs, real value added, the price of real gross outputs, the price of real intermediate inputs and the price of real value added in industry $i$. Moreover, $V(P_{II_{it}}^I)$ denotes the averages over periods $t$ and $t + 1$ of the shares of industry $i$’s nominal intermediate inputs in the industry’s nominal gross output:

$$V(P_{II_{it}}^I) = \frac{1}{2}\left(\frac{P_{II_{it}}^I}{P_{GO_{it}}^I} + \frac{P_{II_{it+1}}^I}{P_{GO_{it+1}}^I}\right).$$

Note that these shares are meaningful concepts because they are constructed in terms of nominal variables that are additive. We can calculate $\Delta \log(Y_{it})$ by solving equation (15) for $\Delta \log(Y_{it})$. 
and substituting in $GO_t, II_t, P^G_t GO_t, P^{II}_t II_t$ from WORLD KLEMS:

$$\Delta \log(Y_t) = \frac{\Delta \log(GO_t) - V(P^{II}_t II_t) \Delta \log(II_t)}{1 - V(P^{II}_t II_t)}. \quad (16)$$

**A.1.2 Calculating aggregate productivity growth**

The aggregate growth rates of real value added, $Y_t$, and efficiency units of labor, $H_t$, are defined as the weighted averages of the $n$ corresponding industry growth rates:

$$\Delta \log(Y_t) \equiv \sum_{i=1}^{n} S(P_i Y_i) \Delta \log(Y_i), \quad (17a)$$

$$\Delta \log(H_t) \equiv \sum_{i=1}^{n} S(W_i H_i) \Delta \log(H_i), \quad (17b)$$

where $H_i$ denotes efficiency units of labor in industry $i$ and $S(P_i Y_i)$ and $S(W_i H_i)$ denote the averages over periods $t$ and $t+1$ of the shares of industry $i$'s nominal value added and nominal labor compensation in the corresponding totals:

$$S(P_i Y_i) \equiv \frac{1}{2} \left( \frac{P_i Y_i}{\sum_{i=1}^{n} P_i Y_i} + \frac{P_{i+1} Y_{i+1}}{\sum_{i=1}^{n} P_{i+1} Y_{i+1}} \right), \quad (18)$$

$$S(W_i H_i) \equiv \frac{1}{2} \left( \frac{W_i H_i}{\sum_{i=1}^{n} W_i H_i} + \frac{W_{i+1} H_{i+1}}{\sum_{i=1}^{n} W_{i+1} H_{i+1}} \right). \quad (19)$$

(Real labor) productivity in the aggregate and for industry $i$ is defined as the corresponding real value added per efficiency unit:

$$LP_t \equiv \frac{Y_t}{H_t} \quad \text{and} \quad LP_i \equiv \frac{Y_i}{H_i}. \quad (20)$$

Applying the definitions in (17)–(20), the growth rate of aggregate productivity is given by:

$$\Delta \log(LP_t) \equiv \Delta \log(Y_t) - \Delta \log(H_t)$$

$$= \sum_{i=1}^{n} S(P_i Y_i) \Delta \log(Y_i) - \sum_{i=1}^{n} S(W_i H_i) \Delta \log(H_i)$$

$$= \sum_{i=1}^{n} S(P_i Y_i) \Delta \log(LP_i) + \sum_{i=1}^{n} \left[ S(P_i Y_i) - S(W_i H_i) \right] \Delta \log(H_i). \quad (21)$$

The growth rate of aggregate productivity has two main components. The first one is the sum of the growth rates of industry productivity weighted by the shares of nominal industry value added. This term captures the effect of productivity growth in the different industries. The second component is the sum of the changes in the industry efficiency units weighted by the
relative nominal industry labor productivity, \( S(P_i Y_i) - S(W_i H_i) \). This term captures the effect of reallocating efficiency units among industries with different levels of productivity. If \( S(P_i Y_i) - S(W_i H_i) > 0 \), for example, then industry \( i \) is relatively more productive than the average industry and reallocating efficiency units to industry \( i \) has a positive effect on productivity. This is sometimes called the “Denison effect”.

### A.1.3 Calculating counterfactual aggregate productivity growth

To calculate the effect of Baumol’s disease on aggregate productivity growth, we define the aggregate labor productivity growth for period \( t \) with fixed industry weights of period \( T \). For example, \( T \) is often the initial period 1947, which freezes the sectoral composition to that before structural change takes place. Using expression (21), we define:

\[
\Delta \log(LP_t(T)) \equiv \sum_{i=1}^{n} S(P_i Y_i) \Delta \log(LP_i) + \sum_{i=1}^{n} \left[ S(P_i Y_i) - S(W_i H_i) \right] \Delta \log(H_i). \tag{22}
\]

Note that given our notation, \( S(P_i Y_i) \) fixes the value added share to the years \( T \) and \( T + 1 \), because it is defined as a weighted average over two adjacent years; compare (18). The difference between the actual and the counterfactual productivity growth rates can then be expressed as follows:

\[
\Delta \log(LP_t) - \Delta \log(LP_t(T)) = \sum_{i=1}^{n} \left[ S(P_i Y_i) - S(P_i Y_i) \right] \Delta \log(LP_i)
\]

\[
+ \sum_{i=1}^{n} \left[ \left( S(P_i Y_i) - S(W_i H_i) \right) - \left( S(P_i Y_i) - S(W_i H_i) \right) \right] \Delta \log(H_i). \tag{23}
\]

The first term after the equality sign is about the effects of the growth rates of industry productivity with the industry compositions of time \( t \) versus \( T \) (“Baumol effect”). The second term after the equality sign is about the effects of the levels of the relative industry productivities with the relative productivities of time \( t \) versus \( T \). Note that the second effect is the difference between the Denison effects for the relative productivities of times \( t \) and \( T \). We do not distinguish between the Baumol effect and the difference between the Denison effects, because structural change gives rise to both of them. Somewhat liberally we call the sum of them the effect of Baumol’s disease.
A.1.4 Constructing Tables 1–2

We start with the upper part of Table 1. The second column of the table shows the average aggregate productivity growth rate with the actual industry weights over the period \([t_1, t_2]\):

\[
\bar{LP}([t_1, t_2]; t) \equiv \frac{1}{t_2 - t_1} \sum_{t=t_1}^{t_2-1} \Delta \log(LP_t).
\]

The third column is the average aggregate productivity growth rate calculated with the fixed industry weights from \(T = 1947\):

\[
\bar{LP}([t_1, t_2]; 1947) = \frac{1}{t_2 - t_1} \sum_{t=t_1}^{t_2-1} \Delta \log(LP_t(1947)).
\]

The first two numbers in the row starting with “difference” are given by:

\[
\begin{align*}
\bar{LP}([1987, 2007]; t) - \bar{LP}([1947, 1967]; t) \quad \text{and} \\
\bar{LP}([1987, 2007]; 1947) - \bar{LP}([1947, 1967]; 1947).
\end{align*}
\]

The last number in that row is given by:

\[
\left[\bar{LP}([1987, 2007]; t) - \bar{LP}([1947, 1967]; t)\right] - \\
\left[\bar{LP}([1987, 2007]; 1947) - \bar{LP}([1947, 1967]; 1947)\right].
\]

This is what we call the effect of Baumol disease on productivity growth.

We continue with the lower part of Table 1. The number in the fourth column is calculated with the fixed industry weights from \(T = 2006\).

We continue with Table 2. Note that the growth rate of any aggregate of a subset of industries is defined analogously to (17). For example, we obtain the growth rate of real value added of the goods sector by applying the formula (17) to the industries in agriculture, mining, manufacturing, construction and utilities. The numbers in the rows starting with “1947/8” and “2006/7” are the aggregate productivity growth rates for fixed industry weights for \(T = 1947\) and \(T = 2006\):

\[
\bar{LP}([1947, 2007]; T) = \frac{1}{60} \sum_{t=1947}^{2006} \Delta \log(LP_t(T)),
\]

The numbers in the row starting with “difference” are given by:

\[
\bar{LP}([1947, 2007]; 1947) - \bar{LP}([1947, 2007]; 2006).
\]

33
A.2 Difference between the price indices in the data and the model

In the data, \( P_t(D) \) is calculated with a Törnqvist index, implying that:

\[
\Delta \log \left( P_t(D) \right) = S \left( Y_{gt}(D) \right) \Delta \log \left( P_{gt}(D) \right) + S \left( Y_{st}(D) \right) \Delta \log \left( P_{st}(D) \right),
\]

where

\[
\Delta \log \left( P_t(D) \right) \equiv \log \left( P_{t+1}(D) \right) - \log \left( P_t(D) \right),
\]

\[
S \left( Y_t(D) \right) \equiv \frac{1}{2} \left( \frac{P_{it} Y_{it}}{P_{it} Y_{t+1}} + \frac{P_{it+1} Y_{it+1}}{P_{t+1} Y_{t+1}} \right),
\]

\[
\Delta \log \left( P_{it}(D) \right) \equiv \log \left( P_{it+1}(D) \right) - \log \left( P_{it}(D) \right).
\]

In the model the aggregate price index satisfies:

\[
P_t = \left( \alpha_s C_t^{\epsilon_t-1} P_{gt}^{1-\sigma_t} + \alpha_s C_t^{\epsilon_t-1} P_{st}^{1-\sigma_t} \right)^{\frac{1}{1-\sigma_t}}.
\]

Linearising around \( C_t, P_{gt}, P_{st} \) leads to:

\[
P_{t+1} - P_t \\
\approx \frac{1}{1 - \sigma_t} \left( \alpha_s C_t^{\epsilon_t-1} P_{gt}^{1-\sigma_t} + \alpha_s C_t^{\epsilon_t-1} P_{st}^{1-\sigma_t} \right)^{\frac{1}{1-\sigma_t} - 1} \left( \epsilon_t - 1 \right) \alpha_s C_t^{\epsilon_t-1} P_{gt}^{1-\sigma_t} \frac{C_{t+1} - C_t}{C_t} \\
+ \frac{1}{1 - \sigma_t} \left( \alpha_s C_t^{\epsilon_t-1} P_{gt}^{1-\sigma_t} + \alpha_s C_t^{\epsilon_t-1} P_{st}^{1-\sigma_t} \right)^{\frac{1}{1-\sigma_t} - 1} \left( \epsilon_t - 1 \right) \alpha_s C_t^{\epsilon_t-1} P_{gt}^{1-\sigma_t} \frac{P_{gt+1} - P_{gt}}{P_{gt}} \\
+ \frac{1}{1 - \sigma_t} \left( \alpha_s C_t^{\epsilon_t-1} P_{gt}^{1-\sigma_t} + \alpha_s C_t^{\epsilon_t-1} P_{st}^{1-\sigma_t} \right)^{\frac{1}{1-\sigma_t} - 1} \left( \epsilon_t - 1 \right) \alpha_s C_t^{\epsilon_t-1} P_{gt}^{1-\sigma_t} \frac{C_{t+1} - C_t}{C_t} \\
+ \frac{1}{1 - \sigma_t} \left( \alpha_s C_t^{\epsilon_t-1} P_{st}^{1-\sigma_t} + \alpha_s C_t^{\epsilon_t-1} P_{st}^{1-\sigma_t} \right)^{\frac{1}{1-\sigma_t} - 1} \left( \epsilon_t - 1 \right) \alpha_s C_t^{\epsilon_t-1} P_{st}^{1-\sigma_t} \frac{P_{st+1} - P_{st}}{P_{st}},
\]

which can be rewritten as

\[
\frac{P_{t+1} - P_t}{P_t} \approx \frac{\epsilon_t - 1}{1 - \sigma_t} \chi_{gt} C_{t+1} - C_t \\
+ \chi_{gt} \frac{P_{gt+1} - P_{gt}}{P_{gt}} + \frac{\epsilon_t - 1}{1 - \sigma_t} \chi_{st} C_{t+1} - C_t \\
+ \chi_{st} \frac{P_{st+1} - P_{st}}{P_{st}},
\]

where

\[
\chi_{it} \equiv \frac{P_{it} Y_{it}}{P_{it} Y_t} = \frac{\alpha_s C_t^{\epsilon_t-1} P_{it}^{1-\sigma_t}}{\alpha_s C_t^{\epsilon_t-1} P_{it}^{1-\sigma_t} + \alpha_s C_t^{\epsilon_t-1} P_{it}^{1-\sigma_t}}, \quad i = \{ g, s \}.
\]
Approximating \((X_{t+1} - X_t)/X_t\) with log differences, we get:

\[
\Delta \log(P_t) \approx \chi_{gt}\Delta \log(P_{gt}) + \chi_{st}\Delta \log(P_{st}) + \left(\frac{\varepsilon_g - 1}{1 - \sigma_c} \chi_{gt} + \frac{\varepsilon_s - 1}{1 - \sigma_c} \chi_{st}\right) \Delta \log(C_t).
\]  \hspace{1cm} (25)

We can also evaluate (7b) for \(P_t\) and linearise it around \(C_{t+1}, P_{gt+1}, P_{st+1}\):

\[
\frac{P_t - P_{t+1}}{P_{t+1}} \approx \frac{\varepsilon_g - 1}{1 - \sigma_c} \chi_{gt+1} \frac{C_t - C_{t+1}}{C_{t+1}} + \chi_{gt+1} \frac{P_{gt} - P_{gt+1}}{P_{gt+1}} + \frac{\varepsilon_s - 1}{1 - \sigma_c} \chi_{st+1} \frac{C_t - C_{t+1}}{C_{t+1}} + \chi_{st+1} \frac{P_{st} - P_{st+1}}{P_{st+1}}.
\]

Approximating again with log differences gives:

\[
\Delta \log(P_t) \approx \chi_{gt+1}\Delta \log(P_{gt}) + \chi_{st+1}\Delta \log(P_{st}) + \left(\frac{\varepsilon_g - 1}{1 - \sigma_c} \chi_{gt+1} + \frac{\varepsilon_s - 1}{1 - \sigma_c} \chi_{st+1}\right) \Delta \log(C_t).
\]  \hspace{1cm} (26)

Combining (25) and (26) leads to

\[
\Delta \log(P_t) \approx S(Y_{gt})\Delta \log(P_{gt}) + S(Y_{st})\Delta \log(P_{st}) + \left(\frac{\varepsilon_g - 1}{1 - \sigma_c} S(Y_{gt}) + \frac{\varepsilon_s - 1}{1 - \sigma_c} S(Y_{st})\right) \Delta \log(C_t),
\]  \hspace{1cm} (27)

where

\[S(Y_{it}) \equiv \frac{1}{2}(\chi_{it} + \chi_{it+1})\]

Comparing (24) and (27), it is clear that if \(\varepsilon_i \neq 1\), then

\[
\Delta \log(P_t(D)) \neq \Delta \log(P_t).
\]

**Appendix B: Derivations and Proofs**

**Derivation of equilibrium conditions**

The first–order condition to the inner and outer parts of the household’s problem are:

\[
P_{jt} = \mu_j \alpha_j^\sigma C_{jt}^{\sigma_j} C_{jt}^{\frac{1}{\sigma_j}}, \quad j = h, l.
\]  \hspace{1cm} (28a)

\[
P_{it} = \lambda_i \alpha_i^\sigma C_{it}^{\sigma_i} C_{it}^{\frac{1}{\sigma_i}}, \quad i = g, s.
\]  \hspace{1cm} (28b)

To derive (6b), multiply both sides of (28a) with \(C_{jt}\) and adding up the resulting equations,
we get

\[ P_{ht}C_{ht} + P_{lt}C_{lt} = \mu \left( \frac{1}{\alpha_h} C_t^{1-\sigma_h} C_{ht}^{\sigma_h} + \frac{1}{\alpha_l} C_t^{1-\sigma_l} C_{lt}^{\sigma_l} \right) C_{st}^{\sigma_s} = \mu C_{st}^{\sigma_s} = \mu C_{st}. \]  

(29)

which implies that

\[ P_{st} = \frac{P_{ht}C_{ht} + P_{lt}C_{lt}}{C_{st}} = \mu. \]  

(30)

Rewriting (28a) again

\[ P_{jt}^{1-\sigma_j} = P_{st}^{1-\sigma_j} \frac{1-\sigma_j}{\alpha_j} \frac{1-\sigma_j}{\sigma_j} C_{jt}^{\sigma_j} C_{st}^{\sigma_j} \]  

which implies

\[ \alpha_j C_{jt}^{\sigma_j} P_{jt}^{1-\sigma_j} = P_{st}^{1-\sigma_j} \frac{1-\sigma_j}{\alpha_j} \frac{1-\sigma_j}{\sigma_j} C_{jt}^{\sigma_j} C_{st}^{\sigma_j}. \]

Adding this up over \( j = h, l \) yields

\[ \alpha_h C_{ht}^{\sigma_h-1} P_{ht}^{1-\sigma_h} + \alpha_l C_{lt}^{\sigma_l-1} P_{lt}^{1-\sigma_l} = P_{st}^{1-\sigma_j} \left( \frac{1}{\alpha_h} C_t^{1-\sigma_h} C_{ht}^{\sigma_h} + \frac{1}{\alpha_l} C_t^{1-\sigma_l} C_{lt}^{\sigma_l} \right) C_{st}^{\sigma_s} \]

\[ = P_{st}^{1-\sigma_j} C_{st}^{\sigma_s} = P_{st}^{1-\sigma_j}, \]

implying that the price index is given as

\[ P_{st} = \left( \alpha_h C_{ht}^{\sigma_h-1} P_{ht}^{1-\sigma_h} + \alpha_l C_{lt}^{\sigma_l-1} P_{lt}^{1-\sigma_l} \right)^{\frac{1}{1-\sigma_j}}. \]

This is (6b). Similar steps give (7b). QED
## Appendix C: Inputs for the Predictions

### Table 8: Inputs for Table 5

<table>
<thead>
<tr>
<th>Estimation period for endogenous variables</th>
<th>$%\Delta A_g$</th>
<th>$%\Delta A_h$</th>
<th>$%\Delta A_l$</th>
<th>$%\Delta P_{gl}$</th>
<th>$%\Delta H_l$</th>
<th>$\tau_h$</th>
<th>$\tau_l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1987–2007</td>
<td>2.25</td>
<td>3.04</td>
<td>0.20</td>
<td>1.79</td>
<td>0.30</td>
<td>0.07</td>
<td>0.70</td>
</tr>
<tr>
<td>1977–2007</td>
<td>2.04</td>
<td>2.67</td>
<td>0.15</td>
<td>2.72</td>
<td>0.51</td>
<td>0.04</td>
<td>0.66</td>
</tr>
<tr>
<td>1967–2007</td>
<td>2.03</td>
<td>2.75</td>
<td>0.31</td>
<td>3.52</td>
<td>0.40</td>
<td>0.02</td>
<td>0.67</td>
</tr>
</tbody>
</table>

## Appendix D
(productivity equals real value added per efficiency unit)

<table>
<thead>
<tr>
<th>Service industries with high productivity growth</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Pipeline Transportation</td>
<td>6.23%</td>
</tr>
<tr>
<td>Air Transportation</td>
<td>5.22%</td>
</tr>
<tr>
<td>Broadcasting And Telecommunications</td>
<td>4.47%</td>
</tr>
<tr>
<td>Wholesale Trade</td>
<td>3.44%</td>
</tr>
<tr>
<td>Securities, Commodity Contracts, And Investments</td>
<td>3.30%</td>
</tr>
<tr>
<td>Waste Management And Remediation Services</td>
<td>3.29%</td>
</tr>
<tr>
<td>Social Assistance</td>
<td>2.87%</td>
</tr>
<tr>
<td>Railroad Transportation</td>
<td>2.75%</td>
</tr>
<tr>
<td>Publishing Industries (Includes Software)</td>
<td>2.74%</td>
</tr>
<tr>
<td>Water Transportation</td>
<td>2.71%</td>
</tr>
<tr>
<td>Administrative And Support Services</td>
<td>2.58%</td>
</tr>
<tr>
<td>Rental And Leasing Services And Lessors Of Intangible Assets</td>
<td>2.49%</td>
</tr>
<tr>
<td>Truck Transportation</td>
<td>2.45%</td>
</tr>
<tr>
<td>Retail Trade</td>
<td>2.29%</td>
</tr>
<tr>
<td>Insurance Carriers And Related Activities</td>
<td>1.78%</td>
</tr>
<tr>
<td>Performing Arts, Spectator Sports, Museums, And Related Activities</td>
<td>1.62%</td>
</tr>
<tr>
<td>Warehousing And Storage</td>
<td>1.45%</td>
</tr>
<tr>
<td>Motion Picture And Sound Recording Industries</td>
<td>1.40%</td>
</tr>
</tbody>
</table>

| Service sector average                                                     | 1.40% |

<table>
<thead>
<tr>
<th>Service industries with low productivity growth</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Management Of Companies And Enterprises</td>
<td>1.39%</td>
</tr>
<tr>
<td>Miscellaneous Professional, Scientific, And Technical Services</td>
<td>1.38%</td>
</tr>
<tr>
<td>Accommodation</td>
<td>1.18%</td>
</tr>
<tr>
<td>Federal General Government</td>
<td>1.17%</td>
</tr>
<tr>
<td>Federal Reserve Banks, Credit Intermediation, And Related Activities</td>
<td>1.08%</td>
</tr>
<tr>
<td>Educational Services</td>
<td>0.89%</td>
</tr>
<tr>
<td>Real Estate</td>
<td>0.79%</td>
</tr>
<tr>
<td>Legal Services</td>
<td>0.74%</td>
</tr>
<tr>
<td>Federal Government Enterprises</td>
<td>0.73%</td>
</tr>
<tr>
<td>Ambulatory Health Care Services</td>
<td>0.68%</td>
</tr>
<tr>
<td>Computer Systems Design And Related Services</td>
<td>0.39%</td>
</tr>
<tr>
<td>Hospitals, Nursing And Residential Care Facilities</td>
<td>0.16%</td>
</tr>
<tr>
<td>Other Transportation And Support Activities</td>
<td>−0.04%</td>
</tr>
<tr>
<td>State And Local Government Enterprises</td>
<td>−0.13%</td>
</tr>
<tr>
<td>Amusements, Gambling, And Recreation Industries</td>
<td>−0.23%</td>
</tr>
<tr>
<td>Information And Data Processing Services</td>
<td>−0.24%</td>
</tr>
<tr>
<td>Food Services And Drinking Places</td>
<td>−0.41%</td>
</tr>
<tr>
<td>Transit And Ground Passenger Transportation</td>
<td>−0.41%</td>
</tr>
<tr>
<td>State And Local General Government</td>
<td>−0.50%</td>
</tr>
<tr>
<td>Other Services, Except Government</td>
<td>−0.63%</td>
</tr>
<tr>
<td>Funds, Trusts, And Other Financial Vehicles</td>
<td>−0.83%</td>
</tr>
</tbody>
</table>