On the Allocation of Time – A Quantitative Analysis of the Roles of Taxes and Productivities*

Georg Duernecker† (University of Mannheim, CEPR, and IZA)
Berthold Herrendorf‡ (Arizona State University)

July 22, 2017

Abstract

Basic theory suggests that increases in labor–income taxes induce people to substitute household production for market work. Time–use surveys for 12 OECD countries during 1970–2010, however, show that instead people substituted leisure for market work. To understand why this happened, we carefully measure the labor productivity of household production and find that it grew strongly in many countries of our sample. Employing a calibrated model of household production, we show that strong growth in the labor productivity of household production implies that leisure absorbs the reductions in market work after labor–income tax increases.

Keywords: household production; income tax; labor productivity; leisure.

JEL classification: E1; J4.

*The previous version of this paper was entitled “On the Allocation of Time – A Quantitative Analysis for France and the U.S.” For helpful comments and suggestions, we would like to thank Alexander Bick, Lei Fang, Charles Gottlieb, John Hassler, Cara McDaniel, Franck Portier, Valerie Ramey, Richard Rogerson, Todd Schoellman, Ákos Valentinyi, Gustavo Ventura and the audiences at ASU, Cardiff Business School, the Canadian Macro Study Group, the Central Banks of Chile and Finland, the CEPR Summer Symposium in International Macroeconomics, the EUI Alumni Meeting, the Federal Reserve Banks of Atlanta and St. Louis, the IIIES in Stockholm, the Ifo Conference on Macroeconomics and Survey Data, the Meetings of the Society of Economics Dynamics, the NBER Summer Institute (Macro Perspectives), the Nordic Summer Symposium in Macroeconomics, the North American Meetings of the Econometric Society, Rice and Sabanci Universities, the Stockholm School of Economics, Stony Brook, and the Universities of Adelaide, Frankfurt, Hamburg, Köln, Konstanz, Mannheim, München, and St. Gallen. Herrendorf thanks the Spanish Ministry of Education for research support (Grant ECO2012-31358) and the University of Mannheim for its hospitality during a sabbatical in 2012 when this project originated. Duernecker thanks the Institute for International Economic Studies for its hospitality during 2014. All errors are our own.

†Address: Department of Economics, University of Mannheim, 68131 Mannheim, Germany. E-mail: duernecker@uni-mannheim.de
‡Address: Department of Economics, W.P. Carey School of Business, Arizona State University, Tempe, AZ 85287–9801, U.S.A. E-mail: berthold.herrendorf@asu.edu
1 Introduction

Starting with the seminal paper of Becker (1965), many scholars have studied what determines how people allocate their time among different activities. A prominent example is Prescott (2004), who argued that differences in labor–income taxes are an important reason for why Americans work so much more in the market than Continental Europeans. Prescott’s contribution stimulated a growing literature in quantitative macroeconomics that confirmed his basic insight for various developed countries.\(^1\) A consensus view is emerging in this literature that hours differ as basic theory predicts: in the developed countries which increased labor–income taxes, people reduced their hours worked in the market and increased their hours worked in the household. To the best of our knowledge, however, there is little evidence corroborating this view.\(^2\) Instead, most of the existing evidence on time allocation comes from the work of Aguiar and Hurst (2007b), Ramey (2009), and Ramey and Francis (2009) about the U.S., whereas outside of the U.S. the evidence is spotty and often based on individual cross sections.

In this paper we shed light on how people allocate their time in a sample of developed countries during the period 1970–2010. Our sample consists of the U.S. and 11 OECD countries for which we have two time–use surveys that are at least 20 years apart and are sufficiently detailed to be able to conduct our empirical analysis. We find that changes in market hours are mostly offset by changes in leisure while hours devoted to household production hardly changed. This is at odds with the prediction of basic theory like that in Rogerson (2008) which implies that household hours increase when labor–income taxes increase. The discrepancy between the evidence and basic theory is important for assessing the welfare implications of tax increases. Given the standard notion that household hours lead to dis–utility, or at least to less utility than leisure, the evidence suggests that basic theory exaggerates the welfare costs that result from increases in labor–income taxes.

In order to understand the reasons for the discrepancy between the evidence and basic theory, we view the time–allocation decision through the lens of a simple model with household production. The model has three key features: a subsistence consumption level implies that the income elasticity of consumption is smaller than one and the income elasticity of leisure is larger than one; consumption is either produced in the market or the household; hours worked are the only input into market and household production. We calibrate our model to match key features of the time allocation in the U.S., taking as given the paths of U.S. labor–income taxes and labor productivities of market and household production. Except for the labor productivity


\(^2\)In independent work that came to our attention after the first draft of this paper was completed, Fang and McDaniel (2017) documented similar facts for developed countries. In contrast to us, however, they did not attempt to explain them.
of household production, we use off-the-shelf numbers for these statistics.

Obtaining estimates of the labor productivity of household production is challenging because the value added produced in the household is not traded in the market. The literature, therefore, calibrates the labor productivity of household production to capture key features of the U.S. time allocation and then makes “reasonable” assumption for the other countries. One contribution of our paper is to measure the labor productivity of household production for our 12 sample countries in a consistent way that follows the income approach that the BEA uses for its satellite accounts. According to this approach, the value added of household production equals the sum of hours worked in the household and capital used in the household, each multiplied by an appropriate rental price. To obtain real labor productivity of household production, we divide value added by an appropriate market price of close substitutes to household value added and by hours worked in the household.\(^3\)

Two insights emerge from our measurement of the real labor productivity of household production. First, in the U.S. it has almost stagnated during 1970–2005. It is reassuring that most calibration exercises reach similar conclusions; see for example McDaniel (2011). Second, in most other countries, the real labor productivity of household production started at a lower level than in the U.S. from which it has grown to catch up with or even overtake the U.S. level. This is different from what typical calibration exercises assume and crucial for our findings.

For each of our 11 sample countries other than the U.S., we feed into the calibrated model its labor productivity of household production along with its labor–income tax and its labor–productivity of market production. We find that our model generates most of the changes in the time allocation in these countries. In particular, for all 11 countries it implies the correct direction of change in household hours. Our model also captures that tax increases led to strong increases in leisure in many of these countries. The key reason why our model replicates the changes in the allocation of time is that the labor productivity of household production grew strongly in most countries other than the U.S. We establish that if instead we had used the slower productivity growth of household production that the literature assumes, then our model would have performed considerably worse.

Our findings have two important implications. First, increases in labor–income taxes increase hours of household production only if the labor productivity of household production grows little. Although it is typically assumed in the literature that this is the case, our evidence suggests otherwise. Second, although there are potential issues with measuring the value added of household production according to the income approach, the resulting numbers reconcile the implications of a standard model of household production with the evidence for 11 OECD countries. We think that this is a remarkable success of the income method of measuring household labor–productivity.

\(^3\)Kendrick (1979), Juster and Stafford (1991), and Landefeld et al. (2009) offer detailed justifications for using the income approach to impute the value added of household production.
The rest of the paper is organized as follows. In the next section, we document evidence about hours worked and taxes in our sample. In the subsequent section, we lay out our environment and characterize the equilibrium. Section 4 connects our model with the U.S. data. Section 5 contains our results and Section 6 concludes. An appendix contains further documentation of our data work and the technical details of some derivations.

2 Evidence on the Allocation of Time and Taxes

In this section, we document for 12 OECD countries the patterns of the allocation of time among market hours, household hours, and leisure along with labor–income taxes. While for the U.S. the facts about the time allocation are well known from the work of Aguiar and Hurst (2007b), Ramey (2009), and Ramey and Francis (2009), most of the evidence outside of the U.S. is spotty and comes mostly from single cross sections or from a few cross sections that are fairly close together in time; see for example Freeman and Schettkat (2001), Juster and Stafford (1991), Burda et al. (2008), and Ragan (2013). A notable exception is Gimenez-Nadal and Sevilla-Sanz (2012), who documented facts about the time allocation in seven OECD countries during the 1970–2000. Unfortunately, however, some of their choices make their work hard to interpret. For example, they included school/classes and study time in market work and found that market hours increased in France during 1970–1990. This is the opposite of what the other existing evidence suggests.

2.1 Stylized facts

We focus on the working–age population 15–64 years old so as to ensure the comparability of our results with the literature on cross–country labor supply. Market work is defined as paid work plus travel to and from work. We use the concept of household production of Ramey (2009), which includes gardening, shopping, and childcare. Leisure is defined as the remaining disposable time where disposable time is defined as total hours available minus the hours allocated to the sleep, personal care, and education. Table 1 lists the activities included in market work, household production, and leisure. Note that this definition implies that time spent on education is not included in our facts. To establish that the details of the definition do not matter for our results, we conduct some robustness analysis below.

The data sources for hours are the time–use surveys that have recently been standardized by the Multinational Time Use Studies (MTUS henceforth); see Gershuny and Fisher (2013). We combine these data with additional sources to document the patterns of the time allocation and of household labor productivity for 12 OECD countries, both in terms of levels in and changes between an initial year and a final year. The data required for our analysis are market hours, household hours, leisure, effective taxes, and labor productivity both in the market and
### Table 1: Definitions of Aggregate Time–Use Categories

<table>
<thead>
<tr>
<th>Category</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disposable Time</td>
<td>Total available time minus time spent on School, classes; Dress/personal care; Consume personal services; Sleep.</td>
</tr>
<tr>
<td>Market Work</td>
<td>Share of Disposable time spent on Paid work; Paid work at home; Paid work, second job; Travel to/from work.</td>
</tr>
<tr>
<td>Household Production</td>
<td>Share of Disposable time spent on Cook, wash up; Housework; Odd jobs; Gardening; Shopping; Childcare.</td>
</tr>
<tr>
<td>Leisure</td>
<td>Share of Disposable time spent on Domestic travel; Meals and snacks; Free time travel; Excursions; Active sports participation; Passive sports participation; Walking; Religious activities; Civic activities; Cinema or theatre; Dances or parties; Social clubs; Pubs; Restaurants; Visit friends at their households; Listen to radio; Watch television or video; Listen to records, tapes, cds; Study, household work; Read books; Read papers, magazines; Relax; Conversation; Entertain friends at home; Knit, sew; Other leisure.</td>
</tr>
</tbody>
</table>

### Table 2: Sample Countries and Years

<table>
<thead>
<tr>
<th>Countries</th>
<th>Initial and final Years</th>
<th>Closest Years with Time–use Surveys</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finland</td>
<td>1979, 2009</td>
<td>1979, 2009</td>
</tr>
<tr>
<td>Italy</td>
<td>1979, 2002</td>
<td>1979, 2002</td>
</tr>
</tbody>
</table>
the household. Table 2 lists the countries, the initial and the final years, and the closest years with time–use surveys. We start in 1970 because it is the first year for which we have data on household labor productivity and household hours in a large set of OECD countries. We stop with 2010 because it is the last year for which we have time–use surveys for most of these countries. For the U.S., we focus on the subperiod 1970–2005, because severe recessions started in 1974 and 2008. Since we will calibrate our model to the U.S., focusing on 1970–2005 will avoid that we muddle secular trends with the effects of these recessions. For the other countries in our sample, we pick the initial and last years that satisfy three criteria: they fall into the period 1970–2010; they are as far apart from each other as possible; they are within five years of actual time–use surveys.\footnote{The working–paper version of this paper, Duernecker and Herrendorf (2015), explains how we do the interpolation to near–by years. If outside of the U.S. our choices of initial and final year coincide with recession years, then this makes it more challenging for our model to capture the changes in the patterns of time allocation. Unfortunately, there is little we can do about this because for most countries, time–use surveys are available only for isolated years, instead of annually. Hence, we cannot resort to any of the usual smoothing tricks like averaging hours over a few years.}

Figure 1: Changes in Effective Labor–Income Taxes and Time Allocation
We define effective (labor–income) taxes as

\[
\tau \equiv \frac{\tau_c + \tau_h}{1 + \tau_c}
\]

where \(\tau_c\) denotes consumption taxes and \(\tau_h\) denotes labor income taxes. Prescott (2004) showed that this concept of effective tax rates captures the relevant distortion for the allocation of time between market work and non–market time. We use the tax rates that McDaniel provided on her webpage.\(^5\)

Figure 1 plots the changes in hours against the changes in the effective labor–income taxes for the country–year pairs in our sample. The x axes depict percentage–point changes of effective labor–income taxes. The y axes depict percentage–point changes of market hours, household hours, and leisure, which in turn are reported as shares of the disposable time of an average working–age person. There are three clear patterns: changes in market hours are negatively related to effective taxes; changes in leisure hours are positively related to effective taxes; changes in household hours are largely unrelated to effective taxes. Moreover, among the 11 countries which increased their effective taxes, eight experienced decreases in household hours. The lack of a positive relation between effective taxes and household hours is particularly striking in France, Germany, and Japan: although each of these countries increased its effective taxes by more than 10 percentage point, household hours decreased.\(^6\)

We emphasize that the stylized facts about taxes and non–market hours are the opposite of what standard theory predicts. For example, the seminal paper of Rogerson (2008) would have household hours increase when taxes increase. Instead, leisure absorbed the hours that the decline in market hours freed up. This is the surprising fact that the rest of this paper will shed light on.

2.2 Robustness

This subsection conducts several robustness checks of our estimates and compares them with those in the literature. Readers who are interested in the big picture may skip this subsection without losing the flow of the argument.

In presenting the stylized facts above, we have abstracted from unemployment. One may wonder whether differences in unemployment rates across countries are behind our finding that market hours in most countries of Continental Europe have decreased whereas market hours in the U.S. have been almost constant. One way to address this question is to conduct the counterfactual exercise suggested by Rogerson (2006). He asked by how much market hours

---


\(^6\)The largest decrease in market hours and the largest increase in leisure happened in France which also experienced one of the largest tax increases. This was the reason why the working–paper version of this paper, Duernecker and Herrendorf (2015) focused on France and the U.S.
worked in given country would increase if the country had the same unemployment rate as the U.S. For most countries in Continental Europe that requires to reduce the unemployment rate and put additional individuals to work. He assumed that these individuals work the average market hours in that country. We find that in no country this affects the direction of change in market hours. Moreover, on average over all 11 sample countries other than the U.S., this affects the change in market hours by just 16%. There is some heterogeneity within Continental Europe, with France seeing the smallest difference and Austria seeing the largest difference. Overall, however, changes in unemployment cannot be the main driver of our findings. The results are available upon request.

Given that we adopt the concept of household work of Ramey (2009), it is natural to compare our estimates of household work with hers. Several subtle differences between our and her work make this comparison less straightforward than one might think. To begin with, the age groups are different: Ramey considers individuals who are at least 21 years old. In contrast, we focus on the working–age population 15–64 years old because we want to compare our results with those in the literature on cross–country labor supply. Second, Ramey uses time–use information provided by AHTUS which has the most detailed time–use categories. In contrast, we use MTUS because its data are harmonized for all OECD countries. Third, in order to ensure comparability with the data from the first half of the 20th century, Ramey excluded a number of activities from household production. In contrast, we include these categories because they are arguably part of household production. Using additional information that MTUS offers for the U.S., we are able to replicate the procedure of Ramey (2009) very closely. Appendix A shows that we then get virtually the same numbers for the U.S. as she did.

Table 3: U.S. Hours under different Definitions of Household Production (hours per working–age population in shares of available time)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Market</td>
<td>32.6</td>
<td>30.2</td>
<td>32.6</td>
<td>29.4</td>
<td>32.6</td>
<td>30.2</td>
<td>32.1</td>
<td>29.5</td>
</tr>
<tr>
<td>Home</td>
<td>19.9</td>
<td>20.4</td>
<td>19.9</td>
<td>20.1</td>
<td>17.2</td>
<td>16.2</td>
<td>19.6</td>
<td>19.9</td>
</tr>
<tr>
<td>Leisure</td>
<td>47.5</td>
<td>49.4</td>
<td>47.5</td>
<td>50.5</td>
<td>51.2</td>
<td>53.4</td>
<td>48.3</td>
<td>50.6</td>
</tr>
</tbody>
</table>

Sources: MTUS.

The literature has debated some of the choices Ramey (2009) made. To begin with, Aguiar and Hurst (2007b) argued that it is instructive to separate the effects of changes in individual behavior from those of changes in the demographic composition of the population, and so they focused on what happens to the time allocation if one keeps the demographic composition unchanged. They also argued that it is more appropriate to include child care in leisure instead
of in household production. Rogerson (2008) chose to include education as part of disposable
time and classified it as leisure whereas we excluded it from disposable time. It turns out that the
difference between the choices of these authors and of Ramey are inconsequential for the basic
patterns we find. Table 3 establishes that for the U.S. the qualitative patterns of our baseline
reported in panel (i) are similar to those in panel (ii), which keeps demographics unchanged,\textsuperscript{7}
to those in panel (iii), which includes childcare in leisure, and to those in panel (iv), which
includes education in leisure. Note that the levels change somewhat, which is entirely expected
though.

Another natural comparison is between our estimates of market hours and those of the
OECD and the GGDC. We choose not to use their off–the–shelf numbers because we prefer to
base our estimates of market hours, household hours, and leisure all on the same data source. In
addition, the market hours from the standard data sources are in fact not meant to be comparable
across countries.\textsuperscript{8} In any case, the long–run trends of our series for hours worked in the market
are reassuringly similar to those reported by the OECD and GGDC. The main difference is that
our levels are higher than those of the OECD and the GGDC, which reflects that our measure of
market hours includes commuting time. The results can be found in the working–paper version,
Duerneccker and Herrendorf (2015).

3 Model

In this section, we build a simple model with endogenous labor supply and household produc-
tion. Benhabib et al. (1991) and Greenwood and Hercowitz (1991) were the first papers that
developed versions of the growth model with household production. Our model shares two key
features with that of Rogerson (2008): there is no capital and there is a stand–in household.
The stand–in household captures what happens at the aggregate level, but abstracts from what
happens at the disaggregate level at the intensive and extensive margins, for men and women, in
different age groups etc. Nonetheless, our model will do a good job of accounting for aggregate
hours per capita. Our model differs from that of Rogerson (2008) in that we focus our attention
on market and household production, and so we do not disaggregate market production into the
production of goods and services.

\textsuperscript{7}Since the closest date to 1970 at which a time–use survey was taken is 1965, we fix the 1965 demographics.
\textsuperscript{8}The OECD explicitly warns the reader on its webpage that “... the data are intended for comparisons of
trends over time; they are unsuitable for comparisons of the level of average annual hours of work for a given
year, because of differences in their sources”. This statement appears when one clicks the info button next to the
into the similarities and differences of the market hours from the OECD, concluding that they are not comparable
across countries.
3.1 Environment

The stand–in household is endowed with one unit of time and derives utility from market–produced and household–produced consumption, \( C_m \) and \( C_h \), and leisure, \( L \). The period–utility function is given by:

\[
U(C, L) = \alpha_u \log(C - \bar{C}) + (1 - \alpha_u) \log(L)
\]  

(2)

\[
C = \left[ \alpha_c C_m^{\sigma - 1} + (1 - \alpha_c) C_h^{\sigma - 1} \right]^{\frac{1}{\sigma - 1}}
\]  

(3)

where \( \alpha_u, \alpha_c \in (0, 1) \) are relative weights, \( C \) is a composite consumption good, \( \bar{C} > 0 \) is a subsistence level of consumption, \( L \) is leisure, and \( \sigma \in [0, \infty) \) is the elasticity of substitution between market–produced and household–produced consumption (with \( \sigma = 1 \) being the Cobb–Douglas case).

Except for the subsistence term \( \bar{C} \), the functional form of the utility function (2) is the same as that in Prescott (2004), which is commonly used in the business cycle literature. It is important to realize though that in this literature \( L \) captures non–market time whereas here it captures leisure, that is, the non–market time that is not used for household production. The new term \( \bar{C} \) implies that preferences are non–homothetic in that the income elasticity of leisure is larger than one. This feature will be essential for matching the fact that leisure has increased in many countries. In related work, Restuccia and Vandenbroucke (2013) and Restuccia and Vandenbroucke (2014) found that subsistence consumption is important for accounting for the long run trends of market work, leisure and educational attainment.

The technologies for producing market and household value added are represented by production functions that are linear in hours:

\[
Y_i = A_i H_i
\]  

(4)

where the index \( i \in \{m, h\} \) stands for market and household, \( Y_i \) denotes value added, \( A_i \) total factor productivity, and \( H_i \) hours.

The feasibility constraints require that consumption does not exceed production and the sum of the hours allocated to market production, household production, and leisure does not exceed the time endowment of one:

\[
Y_i \geq C_i
\]

\[
1 \geq H_m + H_h + L
\]

\[9\]To economize on notation, we will drop the time indexes when this is unlikely to cause confusion.
3.2 Equilibrium

We choose market consumption as the numeraire and denote by $w$ the rental price for labor in terms of market consumption and by $\tau_c$ and $\tau_h$ the tax rates on consumption and on labor income. As in Prescott (2004), the tax revenues are rebated, instead of consumed by the government, which implies that taxes do not have an income effect.\footnote{There a literature on how Scandinavian countries like Sweden increased market hours worked by subsidizing services like child care; see for example Rogerson (2007), Olovsson (2009), and Ragan (2013). Or model will not capture this because we assume that tax revenues are lump–sum rebated.} Dividing the budget constraint by the consumption taxes gives:

$$C_m = (1 - \tau)wH_m + T$$

(5)

where $\tau$ is the effective tax rate that we defined in equation (1) above and $T$ denotes the lump–sum transfer.

In each period, the household problem is:

$$\max_{(C, C_m, C_h, L, H_m, H_h)} \left[ \alpha_u \log(C) + (1 - \alpha_u) \log(L) \right] \quad \text{s.t.} \quad C = \left[ \alpha_c C_m^{\sigma_c} + (1 - \alpha_c)C_h^{\sigma_h} \right]^{\frac{1}{\sigma}},$$

$$C_m = (1 - \tau)wH_m + T, \quad C_h = A_hH_h, \quad 1 = H_m + H_h + L$$

(6)

Appendix B derives the first–order conditions for an interior solution to this problem.

Definition. A competitive equilibrium is an effective tax rate, $\tau$, a lump sum transfer, $T$, a rental price for labor $w$, and allocations $(C, C_m, C_h, L, H_m, H_h)$ such that

- taking $(\tau, T, w)$ as given, $(C, C_m, C_h, L, H_m, H_h)$ solve the household problem
- taking $w$ as given, $H_m$ maximizes profits
- markets clear.

The first–order conditions to the household problem imply two consolidated conditions that have obvious intuitive interpretations. First, the optimal allocation of time between household work and market work equalizes the marginal utilities of household work and market work:

$$(1 - \alpha_c)C_h^{\frac{1}{\sigma_c}}A_h = \alpha_c C_m^{\frac{1}{\sigma_c}}(1 - \tau)w$$

Note that the marginal utility of market work is distorted by the income tax $\tau$. Using the production functions (4), we can rewrite the last equation as:

$$\frac{H_m}{H_h} = \left[ \frac{\alpha_c}{1 - \alpha_c} \frac{A_m}{A_h} (1 - \tau) \right]^{\sigma} \left[ \frac{A_m}{A_h} \right]^{\sigma - 1}$$

(7)
Since $\sigma \in [0, \infty)$, an increase in the effective labor income tax decreases market hours relative to household hours. This makes intuitive sense because the value added of household production is not taxed. The effect on market hours relative to household hours of an increase in the TFP of market production relative to household production depends on the value of $\sigma$. If market and household–produced consumption are substitutes ($\sigma > 1$), then the ratio increases. If market and household–produced consumption are complements ($\sigma < 1$), then the ratio decreases.\footnote{This is similar to Ngai and Pissarides (2007), who showed that if sector outputs are complements in the utility function, then uneven technological progress leads to the reallocation of labor to the sector with the slower productivity growth. See Herrendorf et al. (2014) for a generalization to non–homothetic CES utility functions.}

Second, the optimal allocation of time between leisure and market work equalizes the marginal utilities of leisure and market work:

$$(1 - \alpha_u)L^{-1} = \alpha_u \alpha_c (C - \bar{C})^{-1} C^{\frac{1}{\sigma}} C_m^{-\frac{1}{\sigma}} (1 - \tau)w$$

Again, the marginal utility of market work is distorted by the income tax $\tau$. In Appendix B we show that the last equation can be rewritten to:

$$L = \frac{1 - \alpha_u}{\alpha_u(1 - \tau) + (1 - \alpha_u)} \left(1 - \tau H_h - \frac{\bar{C}}{w/P} \right) \quad (8)$$

where $P$ is the price level associated with the CES aggregator,

$$P \equiv \left[\alpha_c^{\sigma} + (1 - \alpha_c)^{\sigma} p_h^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad (9)$$

and $p_h = (1 - \tau)w/A_h$ denotes the shadow price of household production.\footnote{With a linear production function, $p_h = (1 - \tau)w/A_h$ is the shadow price of household production because one unit of household production requires $1/A_h$ units of time, which has a market price of $(1 - \tau)w$.} This equation brings out how real wages and taxes affect the optimal leisure choice of the stand–in household. An increase in $w/P$ implies that $L$ increases, and hence hours worked $1 - L$ decrease. Note that leisure $L$ would not respond to changes in $w/P$ if $\bar{C} = 0$, because then preferences would be homothetic and the income and substitution effects would cancel. An increase in $\tau$ has three effects: it decreases the denominator of (8); it decreases the second term in parenthesis; it decreases $P$ and therefore decreases the third term in parenthesis. The first effect of an increase in $\tau$ captures the usual substitution from market work to leisure when the income tax increases; the second effect captures that the income tax only affects market production, implying that additional hours are substituted from market hours to household hours and the usual effect is smaller; the third effect captures that a tax increase implies an increase in the real wage (because it decreases the price index), which reduces the bite of the subsistence term.
4 Connecting the Model with the Data

4.1 Model Solution

Since there is no capital, the model is static and can be solved period by period. There are seven endogenous variables in each period: \( w, H_m, H_h, L, C, C_m, C_h \). To determine them, we need seven equations. The first three equation are (3) and the two first–order conditions (7) and (8) from above. In addition, there is one first–order condition from the problem of the stand–in firm: \( w = A_m \). Lastly, there are two technology constraints and one adding–up constraints:

\[
C_i = A_i H_i \quad i \in \{m, h\}
\]

\[
1 = H_m + H_h + L
\]

4.2 Labor Productivity

We calculate the labor productivity of market production by dividing GDP in 2005 PPP–adjusted international dollars by hours worked in the market. We obtain GDP from the Penn World Table and hours worked from the Total Economy Database.

We estimate the labor productivity of household production and then use the estimates as calibration targets. In contrast, the literature calibrates the labor productivity of household production for the U.S. and then makes ad hoc assumptions for other countries; see for example Rogerson (2008) and McDaniel (2011). Estimating the labor productivity of household production is challenging because its value added is not traded in the market and is not part of NIPA.\(^{13}\) As Bridgman (2016), we proceed in two steps. First, we impute the value added of household production following the income method of calculating GDP, which was developed by Kendrick (1979), Juster and Stafford (1991), and Landefeld et al. (2009). Second, we divide the imputed value added by the hours of household production from time–use surveys.

Before describing the details of the imputation method, we emphasize that we apply is consistently in all 12 countries of our sample. Although the method was not at all tailored to the present application, we will show below that the resulting estimates resolve most of the discrepancy between data and model implications for household hours that the assumptions of the literature imply. We will also show that the standard assumptions about household labor productivity fail in this regard.

According to the income method, the value added of household production in current prices is defined as the sum of all inputs evaluated at their rental prices:

\[
Y_h = \frac{w_h H_h + \sum_j (r_{hj} + \delta_{hj}) K_{hj}}{p_h}
\]

\(^{13}\)An exception to this statement is the measured value added for owner–occupied housing.
where \( w_h \) is the rental price of household hours \( H_h, r_{hj} \) and \( \delta_{hj} \) are the rental price and the depreciation rate of the stock of household capital of subcategory \( K_{hj} \), and \( p_h \) is the price of household value added. The inputs \( H_h \) and \( K_{hj} \) are observable. The former comes from time use surveys and the latter comes from NIPA information on the investment expenditure on consumer durables that are related to household production. We calculate their stocks by using the perpetual inventory with U.S. depreciation rates. The depreciation rates can then be calculated by dividing each category’s capital consumption from NIPA by that category’s stock of durables.

The challenge in implementing the income method is that the prices are not observable because the value added and the inputs of household production are not traded in the market. The method therefore replaces them with observed market prices \( r_{mj}, w_m, \) and \( p_m \) of close substitutes:

\[
Y_h = \frac{w_m H_h + \sum_j (r_{mj} + \delta_{hj}) K_{hj}}{p_m}
\]

For \( w_m \), we use the observed market wages of workers in the private–households sub-sector (“private–household workers”), which are hired by private individuals to work in their household as cooks, gardeners, nannies etc. Bridgman et al. (2017) provide evidence that suggests that the time of private–household workers is a reasonably close substitute for the time that households allocate to household production. For \( r_m \), we use the observed real interest rate of 10–year government bonds and add the depreciation rate to it. 10–years government bonds have a similar horizon as many household durables and their rates of returns are widely available. Bridgman (2016) uses instead the aggregate rate of return on market capital as implied by NIPA and finds that imputed household labor productivity for the U.S. hardly change. This leaves the price of household value added. We use the OECD price indexes for expenditure on market services that are close substitutes to household consumption. In contrast, Bridgman (2016) uses the price index for the value added produced by household workers. Unfortunately, this index is not available for all countries in our sample. Fortunately, for the U.S., the results for both price indexes are similar.

Table 4 shows the resulting estimates of household labor productivity. The first notable feature of the table is that the labor productivity of household production has hardly grown in the U.S. since 1970; in accumulated terms it increased by just 5 percentage points between 1970 and 2005. This is similar to what the imputation of Bridgman (2016) gave and what some calibrations of U.S. household productivity deliver; see for example McDaniel (2011).
Table 4: Estimates for Labor Productivities and Effective Labor–Income Taxes
(all labor productivities are relative to the U.S. labor productivity in the market in 1970, first columns are for the initial and second columns are for the final years in sample)

<table>
<thead>
<tr>
<th>Country</th>
<th>$A_m$</th>
<th>$A_h$</th>
<th>$\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>0.95</td>
<td>1.64</td>
<td>0.41</td>
</tr>
<tr>
<td>Canada</td>
<td>1.01</td>
<td>1.52</td>
<td>0.27</td>
</tr>
<tr>
<td>Finland</td>
<td>0.71</td>
<td>1.43</td>
<td>0.21</td>
</tr>
<tr>
<td>France</td>
<td>0.70</td>
<td>1.82</td>
<td>0.22</td>
</tr>
<tr>
<td>Germany</td>
<td>0.64</td>
<td>1.60</td>
<td>0.22</td>
</tr>
<tr>
<td>Italy</td>
<td>0.96</td>
<td>1.39</td>
<td>0.34</td>
</tr>
<tr>
<td>Japan</td>
<td>0.54</td>
<td>1.24</td>
<td>0.17</td>
</tr>
<tr>
<td>Netherlands</td>
<td>1.30</td>
<td>1.71</td>
<td>0.35</td>
</tr>
<tr>
<td>Norway</td>
<td>1.24</td>
<td>2.29</td>
<td>0.38</td>
</tr>
<tr>
<td>Sweden</td>
<td>1.13</td>
<td>1.59</td>
<td>0.50</td>
</tr>
<tr>
<td>U.K.</td>
<td>0.54</td>
<td>1.52</td>
<td>0.20</td>
</tr>
<tr>
<td>U.S.</td>
<td>1.00</td>
<td>1.83</td>
<td>0.49</td>
</tr>
</tbody>
</table>

The second notable feature of the table is that in all but two of the other countries the labor productivity of household production grew much more strongly than in the U.S. Particularly noteworthy cases are France, Germany and the U.K. where the labor productivity of household production started from much lower levels than in the U.S. and then more than doubled. This is very different from what existing calibrations of household productivity deliver.

Our estimates raise the question as to why the labor productivity of household production stagnated in the U.S. but grew in most of the other countries in our sample. We offer two channels through which the labor productivity of household production can increase strongly. The first one works through the adoption of new household appliances and the second one works through increases in the wages of household workers.

The adoption patterns of new household durables affect household labor value added directly and indirectly. The direct effect comes from the increase in the input household capital and the indirect effect comes from decrease in the input household hours (“labor–saving appliances”). In addition, the productivity of each hour of household work increases because household labor becomes more productive when working with the new appliances. The work of Greenwood et al. (2005) suggests that in the U.S. most of the adoption of major labor–saving household capital goods like refrigerator, vacuum cleaner, and washer happened before our period of investigation took place for the 1970s. Bridgman (2016) argued that this is part of the reason why he finds that U.S. household labor productivity has grown strongly before the 1970s and only weakly afterwards. In Continental Europe, the adoption of these new appliances happened considerably later. Figure 2 plots the adoption rates in Germany taken from the German Statistical Yearbooks. Much of the adoption of household durables took place well after household appliances after World War II as documented by Greenwood et al. (2005).
1970, implying that our period of investigation was a period of significant increase in household capital in Germany. We conjecture that similar patterns apply to other Continental European countries like France, but we have not been able to come up with sufficient data to back up that conjecture.

**Figure 2: Adoption of Household Appliances in Germany**

The wages of private–household workers are a second important determinant of the labor productivity of household production. Table 5 shows for each country how the wages of household workers (in constant international prices) have changed between the initial and the final year. We can see that in many countries, the wages of household workers have gone up considerably whereas in the U.S. they have all but stagnated. If the wages of household workers grow considerably, as they did in Germany, then the marginal productivity of household workers will increase considerably. In contrast, if the wages of household workers stagnate, as they did for example in the U.S., then the marginal productivity of household workers will remain roughly unchanged. Assuming that labor hired in the market and labor provided by the households themselves are close substitutes, each hour of household production gets evaluated by an increasing wage in Germany but a stagnating wage in the U.S. In economic terms, this captures that American households let household workers do chores for them that German households do themselves. A concrete example may help: Ever since one of us owned a house in the U.S., he has hired a regular gardening service to take care of his yard. Compared to his salary, the costs are minor. His father and his brother both own houses back in Germany, but none of them has ever hired a regular gardening service. That would just be too expensive compared to their salaries. As a result, his aging father still does most the yard work himself that his American son outsources to a gardener.

This argument raises the question as to why the wages of household workers behaved differently in the two countries. We conjecture that the key reason is that the U.S. has seen a large
influx of low skilled labor from the South which kept the wages for household workers low. In contrast, despite the recent spike in immigration, Germany had much less immigration than the U.S. In addition, Germany had higher minimum wages and tighter labor market regulations that both tend to keep the wages for household workers up.

Table 5: Changes in Wages of Private–Household Workers and in Imputed Labor Productivities of Household Production (wages and productivities in constant international prices, changes in percent)

<table>
<thead>
<tr>
<th>Country</th>
<th>Wages</th>
<th>Productivities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>-0.11</td>
<td>-0.12</td>
</tr>
<tr>
<td>Canada</td>
<td>1.92</td>
<td>1.05</td>
</tr>
<tr>
<td>Finland</td>
<td>2.32</td>
<td>2.16</td>
</tr>
<tr>
<td>France</td>
<td>2.87</td>
<td>2.57</td>
</tr>
<tr>
<td>Germany</td>
<td>1.78</td>
<td>3.11</td>
</tr>
<tr>
<td>Italy</td>
<td>-2.14</td>
<td>-1.17</td>
</tr>
<tr>
<td>Japan</td>
<td>1.55</td>
<td>1.98</td>
</tr>
<tr>
<td>Netherlands</td>
<td>1.76</td>
<td>2.97</td>
</tr>
<tr>
<td>Norway</td>
<td>2.19</td>
<td>2.21</td>
</tr>
<tr>
<td>Sweden</td>
<td>2.39</td>
<td>1.19</td>
</tr>
<tr>
<td>U.K.</td>
<td>2.55</td>
<td>2.54</td>
</tr>
<tr>
<td>U.S.</td>
<td>0.01</td>
<td>0.30</td>
</tr>
</tbody>
</table>

4.3 Calibration

We will restrict the parameters of our model so that it matches key features of the U.S. economy in 1970 and in 2005. There are two crucial differences with the calibration procedure of Rogerson (2008). Rogerson considered a longer time period 1950–2003, which is not feasible here because before 1970 we do not have all data required for our analysis. Rogerson chose an off–the–shelf value for the elasticity of substitution between market and household consumption and calibrated the growth of the labor productivity of household production. In contrast, we use our measured growth of the labor productivity of household production and calibrate the elasticity of substitution.

We calculate the effective tax rates for 1970 and for 2005 from the information provided on McDaniel’s webpage.\(^{17}\) We normalize the initial market TFP in the U.S. to one and choose the remaining three TFPs to match the relative labor productivities:

\[
A_i(t) = \frac{Y_i(t)/H_i(t)}{Y_m(1970)/H_m(1970)}, \quad t \in \{1970, 2005\}, \quad i \in \{m, h\}
\]

We calculate the market labor productivities by dividing GDP by hours worked in the market, where GDP is from the Penn World Table and hours worked are from the Total Economy.

Database. We take the household labor–productivities from our estimates. All value–added
estimates are in 2005 PPP–adjusted international prices.

This leaves four parameters to calibrate: $\alpha_u, \alpha_c, \bar{C}, \sigma$. We choose them jointly to match mar-
et and household hours in 1970 and in 2005 in the U.S. The first columns of Table 6 shows that
our model hits the targets and the last column reports the resulting parameter values. Two fea-
tures of our calibration results warrant further discussion: the subsistence level of consumption
is positive; market and household–produced consumption are complements.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_m$</td>
<td>0.33</td>
<td>0.30</td>
<td>0.33</td>
<td>0.30</td>
<td>$\alpha_u = 0.52$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_h$</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>$\alpha_c = 0.74$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_m$</td>
<td>1.00</td>
<td>1.83</td>
<td>1.00</td>
<td>1.83</td>
<td>$\bar{C} = 0.04$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_h$</td>
<td>0.49</td>
<td>0.54</td>
<td>0.49</td>
<td>0.54</td>
<td>$\sigma = 0.82$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The importance of the subsistence term can be best gauged by expressing it as a share of
total consumption: in the U.S., $\bar{C}$ amounted to 19% of total consumption in 1970 and 13% in
2005. Compared to other studies, these numbers are sizeable. Given that balanced growth is
often viewed as the natural benchmark when modeling developed economies, having a sizeable
subsistence term on total consumption needs justification. A first justification comes from
looking at the changes in the ratio of total hours worked over leisure between the initial and the
final year we consider. Under strict balanced growth, this ratio would not change, of course.
Table 7 reports the changes in the ratio of total hours worked over leisure between the initial and
the final years we consider. The ratio fell in all but three countries of our sample. Moreover,
it fell in both the U.K. and the U.S., although taxes hardly changed. Particularly the fall in
the U.K. is far too big to be explained away with business cycle fluctuations. We think that is
suggestive of a deviation from the balanced–growth assumption, which our non–homotheticity
term captures.

A different way of justifying the presence of $\bar{C}$ is to set it equal to zero and try to calibrate
the model for the U.S. without it. It turns out that it is then impossible to exactly match the
observed time allocation in the U.S. during 1970–2005. So that is a non–starter. Below we
will instead analyze the alternative specification of McDaniel (2011) who put the subsistence
term on $C_m$ instead of on $C$. While this specification can be calibrated to capture the U.S. time
allocation, it implies counterfactually high household hours in most of the other 11 countries.

Our calibration also implies that the value added of market production and the value added
of household production are complements in the utility function, with an elasticity of substitu-
tion equal to 0.8. In contrast, the prevailing view in the literature is that they are substitutes
Table 7: Changes in the Ratios of Total Hours Worked to Leisure (in percent)

<table>
<thead>
<tr>
<th>Country</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>-11</td>
</tr>
<tr>
<td>Canada</td>
<td>2</td>
</tr>
<tr>
<td>Finland</td>
<td>-26</td>
</tr>
<tr>
<td>France</td>
<td>-43</td>
</tr>
<tr>
<td>Germany</td>
<td>-37</td>
</tr>
<tr>
<td>Italy</td>
<td>-11</td>
</tr>
<tr>
<td>Japan</td>
<td>-18</td>
</tr>
<tr>
<td>Netherl.</td>
<td>10</td>
</tr>
<tr>
<td>Norway</td>
<td>1</td>
</tr>
<tr>
<td>Sweden</td>
<td>-1</td>
</tr>
<tr>
<td>U.K.</td>
<td>-28</td>
</tr>
<tr>
<td>U.S.</td>
<td>-7</td>
</tr>
</tbody>
</table>

with an elasticity of substitution of around two. In what follows, we shall argue that although the elasticity is a structural parameter within a given environment, it can plausibly vary across both different environments and different time periods. There are two particular reasons why one should expect our elasticity to be considerably lower than the usual value of 2. First, our elasticity is between household value added and the broad category market value added, instead of a narrower subcategory of market value added. Second, our elasticity pertains to a shorter, more recent subperiod than the usual elasticities in the literature. To be concrete, we illustrate the implications of the previous points by comparing the elasticity in our setting with that in Rogerson’s (2008) setting. Our calibration implies a value of 0.8 whereas he chose a value of 1.8.

The first reason for why our elasticity comes out relatively low compared to that of Rogerson that his elasticity governs the substitutability between household value added and market services whereas our elasticity governs the substitutability between household value added and market value added. Market value added also contains goods that tend to be less substitutable with household value added than services. To investigate how much this difference matters for the value of the elasticity, we calibrate both our model and Rogerson’s model to the same data from his period of investigation 1950–2003. The targets are Rogerson’s U.S. data for market hours in goods and service production, for household hours, and for market labor productivities. Since Rogerson did not have data on the growth of household productivity, we consider five scenarios, ranging from the factor decrease of 0.87 in household value added that his calibration implies to a factor increase of 1.7 that is close to what the measurement of Bridgman (2016) implies. For each of these five scenarios, we calibrate both models to the same data, with the one difference that for his model we target separate information on market goods and market services. Table 8 shows that the elasticities of our model come out way below the elasticities of his model.18

Given that all elasticities of Table 8 are larger than one, the question remains how plausible it is that the calibration of our model to 1970–2005 yields an elasticity below one. After all,

---

18Similar statements apply to micro estimates of the elasticity of substitution. Aguiar and Hurst (2007a), Rupert et al. (1995), and Fang and Zhu (2017) all estimated elasticities of substitution around two. These estimations share at least one of two features that imply a lot of substitutability: they consider the substitution between household value added and a narrow subset of market value added (like food consumption at home or in the market); they focus on the subset of individuals who are working and so have positive market hours, household hours, and leisure.
Table 8: Calibrating the Elasticities of Rogerson’s and our Model to Rogerson’s Data

<table>
<thead>
<tr>
<th>( A_h(2003) )</th>
<th>Elasticity of ( A_h(1950) ) Rogerson’s Model</th>
<th>our Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>1.8</td>
<td>1.3</td>
</tr>
<tr>
<td>1.1</td>
<td>2.2</td>
<td>1.4</td>
</tr>
<tr>
<td>1.3</td>
<td>2.7</td>
<td>1.5</td>
</tr>
<tr>
<td>1.5</td>
<td>3.7</td>
<td>1.7</td>
</tr>
<tr>
<td>1.7</td>
<td>6.7</td>
<td>1.9</td>
</tr>
</tbody>
</table>

the only difference is that, instead of 1950–2003, we calibrate to the more recent period 1970–2005. In order to show that the calibration period matters as well for the resulting value of the elasticity, we also calibrate Rogerson’s model to the more recent period 1970–2005. To implement this, we supplement our data for market and household production with disaggregate data from WORLD KLEMS about the hours and the labor productivities for market–goods and market–services production. This calibration of Rogerson’s model results in an elasticity of substitution between market–services and household value added of 1.3. This is well below the values that calibrating his model to 1950–2003 imply.

We think that a plausible reason for the differences in the calibration results between the two periods is that the composition of the aggregate category services changed between 1950–1970 and 1970–2005. As has been pointed out by the work of Buera and Kaboski (2012a,b), and more recently by Dueeneker et al. (2017), most of the growth in services in recent decades has come from growth in modern services such as business services, financial services, legal service, medical services etc. Producing these services requires high–skilled labor input. Hence, it is difficult or impossible to produce them at home with low–skilled labor input, implying that they are not closely substitutable to household production. The rise of the expenditure share of not substitutable services implies that market and household value added become less substitutable over time.

5 Results

Having calibrated our model to the U.S. economy in 1970 and 2005, we now explore to what extent it can account for the allocation of time between market work, household work, and leisure in the other 11 OECD countries of our sample. To this end, we will keep the calibrated parameters unchanged and feed into the model the following country–specific numbers for the initial and final sample years: the effective labor income taxes; the labor productivities in the market; the labor productivities in the household. For the first two sets of numbers, we follow the literature and use standard data sources: we take the effective taxes from McDaniel’s
webpage and we calculate the market labor productivities as for the U.S. by dividing GDP in 2005 PPP–adjusted international dollars by hours worked in the market, where GDP is from the Penn World Table and hours worked are from the Total Economy Database. For the third set of statistics, we use our own estimates of household labor productivities.

5.1 Implied changes in time allocation

Table 9: Changes in Hours — Data and Model with different Labor Productivities of Household Production (in percentage points)

<table>
<thead>
<tr>
<th></th>
<th>Data $\Delta H_m$</th>
<th>$\Delta H_h$</th>
<th>$\Delta L$</th>
<th>Our $A_h$ $\Delta H_m$</th>
<th>$\Delta H_h$</th>
<th>$\Delta L$</th>
<th>Rogerson’s $A_h$ $\Delta H_m$</th>
<th>$\Delta H_h$</th>
<th>$\Delta L$</th>
<th>McDaniel’s $A_h$ $\Delta H_m$</th>
<th>$\Delta H_h$</th>
<th>$\Delta L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>-3.7</td>
<td>0.9</td>
<td>2.8</td>
<td>-3.3</td>
<td>0.9</td>
<td>2.5</td>
<td>-3.1</td>
<td>0.2</td>
<td>2.9</td>
<td>-2.9</td>
<td>-0.7</td>
<td>3.6</td>
</tr>
<tr>
<td>Canada</td>
<td>1.5</td>
<td>-1.0</td>
<td>-0.5</td>
<td>-2.0</td>
<td>-0.4</td>
<td>2.4</td>
<td>-2.0</td>
<td>0.1</td>
<td>2.0</td>
<td>-1.9</td>
<td>-0.3</td>
<td>2.2</td>
</tr>
<tr>
<td>Finland</td>
<td>-6.3</td>
<td>-1.1</td>
<td>7.4</td>
<td>-4.5</td>
<td>-1.5</td>
<td>6.0</td>
<td>-4.6</td>
<td>0.4</td>
<td>4.2</td>
<td>-4.4</td>
<td>-1.0</td>
<td>5.4</td>
</tr>
<tr>
<td>France</td>
<td>-11.2</td>
<td>-2.8</td>
<td>13.9</td>
<td>-6.1</td>
<td>-2.1</td>
<td>8.2</td>
<td>-6.5</td>
<td>0.7</td>
<td>5.8</td>
<td>-6.1</td>
<td>-0.8</td>
<td>7.0</td>
</tr>
<tr>
<td>Germany</td>
<td>-7.9</td>
<td>-3.5</td>
<td>11.3</td>
<td>-6.2</td>
<td>-1.9</td>
<td>8.2</td>
<td>-6.6</td>
<td>0.8</td>
<td>5.8</td>
<td>-6.3</td>
<td>-0.8</td>
<td>7.1</td>
</tr>
<tr>
<td>Italy</td>
<td>-5.4</td>
<td>2.4</td>
<td>3.0</td>
<td>-5.5</td>
<td>2.3</td>
<td>3.2</td>
<td>-5.1</td>
<td>0.8</td>
<td>4.3</td>
<td>-5.0</td>
<td>0.5</td>
<td>4.5</td>
</tr>
<tr>
<td>Japan</td>
<td>-4.4</td>
<td>-0.4</td>
<td>4.8</td>
<td>-7.4</td>
<td>-0.4</td>
<td>7.8</td>
<td>-7.0</td>
<td>1.0</td>
<td>6.0</td>
<td>-6.9</td>
<td>-1.0</td>
<td>7.9</td>
</tr>
<tr>
<td>Nether.</td>
<td>5.3</td>
<td>-2.9</td>
<td>-2.4</td>
<td>1.1</td>
<td>-1.9</td>
<td>0.8</td>
<td>0.7</td>
<td>-0.5</td>
<td>-0.2</td>
<td>0.7</td>
<td>-0.5</td>
<td>-0.2</td>
</tr>
<tr>
<td>Norway</td>
<td>1.1</td>
<td>-1.3</td>
<td>0.2</td>
<td>-1.8</td>
<td>-1.1</td>
<td>3.0</td>
<td>-2.3</td>
<td>0.2</td>
<td>2.1</td>
<td>-1.9</td>
<td>-0.5</td>
<td>2.5</td>
</tr>
<tr>
<td>Sweden</td>
<td>2.8</td>
<td>-3.1</td>
<td>0.3</td>
<td>1.8</td>
<td>-1.2</td>
<td>-0.6</td>
<td>1.7</td>
<td>-0.9</td>
<td>-0.8</td>
<td>1.8</td>
<td>-1.3</td>
<td>-0.4</td>
</tr>
<tr>
<td>U.K.</td>
<td>-6.2</td>
<td>-2.0</td>
<td>8.1</td>
<td>-4.5</td>
<td>-2.6</td>
<td>7.1</td>
<td>-4.5</td>
<td>0.0</td>
<td>4.4</td>
<td>-4.3</td>
<td>-2.0</td>
<td>6.3</td>
</tr>
</tbody>
</table>

Table 9 reports the changes in hours in the data and in our model under the headline “Data” and “Our $A_h$”. Our model does remarkable well, in particular with regards to changes in household hours. In all 9 non–targeted countries in which household hours fell in the data, our model predicts that they fall. In the two countries in which household hours increased – Austria and Italy – the model predicts that they increase. Table 15 in the Appendix shows that the model also matches many levels reasonably well, although there is no guarantee whatsoever that the initial and final sample years are on trend.19

Given that Prescott (2004) already showed that taxes can account for most of the differences between market and non–market hours in the U.S. and other OECD countries, critical readers might think that understanding the forces behind changes in the composition of non-market hours is of secondary interest only, and so our findings add relatively little to the existing literature. This view overlooks that the composition of non–market hours is crucial for understanding the welfare implications of policies that affect the allocation of time. How the margin between

19The biggest exception to the previous statements is Japan: for some reason, the Japanese work much less in the household than people in the other OECD countries, which the model misses. Interestingly, Bridgman et al. (2017) find a similar fact for other Asian countries. They document that the main reason is that in these countries men hardly do any household work. We leave it to future work to understand the reasons for this.
total hours worked (in the market and at home) and leisure operates is arguably more relevant for welfare calculations than the distinction between market and non-market hours because leisure yields utility whereas hours worked at home are typically assumed to yield dis-utility, or at least less utility than leisure. Olovsson (2009) argued that this point is important in the context of optimal taxation with home and market production. To be concrete, if one wants to quantify the welfare implications of the increase in, say, French labor-income taxes, then it is not sufficient to know that the tax increase led to a decrease in market hours and an increase in non-market hours, which was what Prescott focused one. Instead, one also needs to know what happened to home production and leisure. The consensus in the existing literature is that the French reduction in market hours primarily was accompanied by an increase in French home hours. This is not only counterfactual, it also exaggerates the welfare costs of the increase in the labor-income tax. In contrast, our model implies that the French reduction in market hours was accompanied by an increase in French leisure without increasing French household hours. This is consistent with the evidence and implies smaller welfare costs of the increase in labor-income taxes than in the existing literature.

5.2 Intuition

In this subsection, we build some intuition for why household hours decrease in our model when effective labor income taxes increase. To this end, we restate the first-order conditions (7) and (8) for the convenience of the reader:

\[
\frac{H_m}{H_h} = \left( \frac{\alpha_c}{1 - \alpha_c(1 - \tau)} \right)^{\sigma} \left[ \frac{A_m}{A_h} \right]^{\sigma - 1}
\]

\[
L = \frac{1 - \alpha_u}{\alpha_u(1 - \tau) + (1 - \alpha_u)} \left[ 1 - \tau H_h - \frac{\bar{C}}{w/P} \right]
\]

We start with the U.S. Since the U.S. is fairly rich already in 1970, real wages are high and \( \bar{C}/(w/P) \) is low. Further increases in \( w/P \) therefore lead to a quantitatively small decrease in hours worked (in the market and the household) and a quantitatively small offsetting increase in leisure. Given total hours worked, the increase in the labor productivity of market production relative to household production implies that hours worked are reallocated from market to household production. This effect is relatively small quantitatively because the exponent is small, \( 1 - \sigma = 0.18 \). Since effective labor income taxes do not change much, they do not affect the allocation of time. The net effects are as follows: leisure increases somewhat, market hours decrease somewhat, and household hours hardly change. The latter happens because the opposing effects from changes in \( \bar{C}/(w/P) \) and \( A_m/A_h \) turn out to offset each other quantitatively.

Now, consider a country in which effective taxes increased a lot. For concreteness, we think of France. In 1970, real wages were still considerably lower than in the U.S. The subsequent
increases in $w/P$ therefore lead to a quantitatively sizeable decrease in hours worked and an offsetting sizeable increase in leisure. The fact that the increases in the labor productivities of market and household production are similar implies there is hardly any reallocation between market and household hours resulting from this margin. The increase in labor income taxes implies that hours are reallocated from market to non–market uses, so both household work and leisure increase as a result. This effect is sizeable quantitatively, as the elasticity of substitution between market and household consumption is close to one. The net effects are that leisure increases strongly, market hours decrease strongly, and household hours decrease somewhat. The last effect again is small in comparison, because the opposing effects from $\bar{C}/(w/P)$ and $\tau$ largely offset each other quantitatively.

5.3 Counterfactuals

To establish that using our measured household labor productivities is critical for our results, we conduct several counterfactual exercises. For each of the 11 countries other than the U.S., we feed into the calibrated model just the country–specific taxes, just the labor productivities of household and market production, and the labor productivity of household production that have been assumed in the literature together with the changes in taxes and the labor productivities of market production.

We start by feeding into the calibrated model one by one the effective labor–income tax, the labor productivity in the market, and the labor productivity in the household. We calculate which percentage of the total model–implied changes in market hours, household hours, and leisure each of these three exercises generates. For each exercise, we report the averages over all 11 non–targeted countries. Note that the percentages do not add up to 100 because the different forces may also interact in non–linear ways. Table 10 shows the results. While using our imputed household labor productivity hardly matters for the margin between market hours versus non–market hours, on which Prescott focused, it matters greatly for the margin between household hours and leisure, on which we focus here. To be concrete, feeding into the model just the measured changes in household labor–productivity generates 139% of the average observed changes in household hours but only –2% and 24% of the average observed changes in market hours and in leisure.

| Table 10: Contributions of Taxes and TFPs to Predicted Changes in Hours (in percent) |
|----------------------------------|----------------|----------------|----------------|----------------|
|                                  | $\Delta \tau$ | $\Delta A_m$ | $\Delta A_h$ | $\Delta \tau$ | $\Delta A_m$ | $\Delta A_h$ | $\Delta \tau$ | $\Delta A_m$ | $\Delta A_h$ |
|----------------------------------|----------------|----------------|----------------|----------------|
|                                  | 54             | 52             | –2             | –26            | –9            | 139           | 39            | 41             | 24             |

We continue by assessing what our model would predict if we made the ad hoc assumptions
of the literature about the labor productivity of household production in the 11 countries other than the U.S. We conduct two counterfactual exercises that replicate as closely as possible how Rogerson (2008) and McDaniel (2011) proceeded.

Rogerson (2008) defined Continental Europe as Belgium, Germany, France, Italy, Netherlands and then proceeded as follows:

- He choose units such that $A_h(US, 1950) = 1$ and assumed that in 1950 Continental European labor productivity in the market and in the household were 33 years behind the U.S. He also assumed that during the 33 years before 1950 the annual growth rate of labor productivity in the market and at home was 1.44%.\(^{20}\) Hence:

$$A_h(EUR, 1950) = A_h(US, 1950) \cdot (1 + 0.0144)^{-33} = 1.0144^{-33} = 0.62$$

- His calibration implied that for the U.S. the annual growth rate of household labor productivity equals −0.26%.\(^{*21}\) Hence,

$$A_h(US, 2003) = A_h(US, 1950) \cdot (1 - 0.0026)^{53} = 0.9974^{53} = 0.87$$

- He assumed that in 2003 Continental European labor productivities in the market and the household had caught up with the U.S.:

$$A_h(EUR, 2003) = A_h(US, 2003) = 0.87$$

The implied average growth rate of Continental European household labor productivity then is:

$$\left( \frac{A_h(EUR, 2003)}{A_h(EUR, 1950)} \right)^{\frac{1}{2003-1950}} - 1 = \left( \frac{0.87}{0.62} \right)^{\frac{1}{53}} - 1 = 0.64\%$$

To obtain the household labor productivity for country $i$ in year $t$ according to Rogerson’s calibration method, we start from our imputed level of U.S. household labor productivity, $A_h(US, 2003) = 0.56$, and impose Rogerson’s annual growth rate of 0.64%. The labor productivities for country $i$ in 1970 and 2005 then follow as:

$$A_h(i, 1970) = A_h(US, 2003) \cdot (1 + 0.0068)^{1970-2003} = 0.56 \cdot (1.0064)^{-33} = 0.45$$

$$A_h(i, 2005) = A_h(US, 2003) \cdot (1 + 0.0068)^{2005-2003} = 0.56 \cdot (1.0064)^2 = 0.57$$

---

\(^{20}\)These assumptions imply that in 1950 the GDP per capita in Continental Europe is about half of that in the U.S., which is what we observe.

\(^{21}\)As mentioned above, Table 1 of his paper reports the wrong annual growth rate of household labor productivity of −0.002%; the correct number implied by his calibration is −0.26%.
The columns “Rogerson” in Table 9 reports the predicted changes in hours. The model’s predictions for changes in market hours $H_m$ are similar for our and for Rogerson’s method. However, the model’s predictions for changes in household hours are very different. In six out of the eleven countries, Rogerson’s method predicts that household hours increase when in the data they decreased. This is due to the fact that for most countries his growth rate of household labor–productivity of 0.64% is well below the growth rate that we measure. Since market and household consumption are complements, this implies that the model allocates more labor to the household sector, which leads to counterfactually high household hours.

The second counterfactual follows the methodology of McDaniel (2011). She assumed that the level of household labor productivity in country $i$ is given by

$$ t = ...1993, 1994 : \quad A_h(i, t) = A_m(i, t) \frac{A_h(US, t)}{A_m(US, t)} $$

$$ t = 1995, 1996... : \quad A_h(i, t) = A_m(i, t) \frac{A_h(US, 1995)}{A_m(US, 1995)} $$

The columns McDaniel in Table 9 show the results. Again, the model predictions for market hours $H_m$ do not change much compared to the baseline model. In contrast to the previous counterfactual, the model predictions for changes in $H_h$ are now closer to the data and McDaniel’s method implies the right direction of change in household hours in all countries but Austria (where it predicts a decrease instead of an increase). The magnitudes of the changes are still somewhat off though.

The previous observations are confirmed by the values of the loss function which are reported in the last row of Table 9. Our method does best, McDaniel’s methodology does second–best, and Rogerson’s methodology does worse. It is particularly striking how well our method does for household hours, where we have by far the smallest value of all reported losses including those of our model for market hours and leisure. We think that this is a remarkable success of our measured household labor–productivities.

Table 11: Calibration of McDaniel’s Specification

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_U$</td>
<td>0.53</td>
</tr>
<tr>
<td>$\alpha_C$</td>
<td>0.62</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1.05</td>
</tr>
<tr>
<td>$\bar{C}_m$</td>
<td>0.07</td>
</tr>
</tbody>
</table>

5.4 Subsistence consumption

The intuition that we provided above for our results suggests that the subsistence term in total consumption plays a crucial role in the workings of our model. A different way of bringing home its importance is to consider how the model would perform in its absence. As we mentioned above, it turns out that the model without any subsistence term cannot match the U.S.
Table 12: Changes in Hours — Data, Our Model, and McDaniel’s Model

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Our Model</th>
<th>McDaniel’s Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\Delta H_m)</td>
<td>(\Delta H_h)</td>
<td>(\Delta L)</td>
</tr>
<tr>
<td>Austria</td>
<td>-3.7</td>
<td>0.9</td>
<td>2.8</td>
</tr>
<tr>
<td>Canada</td>
<td>1.5</td>
<td>-1.0</td>
<td>-0.5</td>
</tr>
<tr>
<td>Finland</td>
<td>-6.3</td>
<td>-1.1</td>
<td>7.4</td>
</tr>
<tr>
<td>France</td>
<td>-11.2</td>
<td>-2.8</td>
<td>13.9</td>
</tr>
<tr>
<td>Germany</td>
<td>-7.9</td>
<td>-3.5</td>
<td>11.3</td>
</tr>
<tr>
<td>Italy</td>
<td>-5.4</td>
<td>2.4</td>
<td>3.0</td>
</tr>
<tr>
<td>Japan</td>
<td>-4.4</td>
<td>-0.4</td>
<td>4.8</td>
</tr>
<tr>
<td>Netherl.</td>
<td>5.3</td>
<td>-2.9</td>
<td>-2.4</td>
</tr>
<tr>
<td>Norway</td>
<td>1.1</td>
<td>-1.3</td>
<td>0.2</td>
</tr>
<tr>
<td>Sweden</td>
<td>2.8</td>
<td>-3.1</td>
<td>0.3</td>
</tr>
<tr>
<td>U.K.</td>
<td>-6.2</td>
<td>-2.0</td>
<td>8.1</td>
</tr>
<tr>
<td>U.S.</td>
<td>-2.4</td>
<td>0.5</td>
<td>1.9</td>
</tr>
</tbody>
</table>

\[ \sum_{n=1}^{N} \left| \Delta \text{Data}_n - \Delta \text{Model}_n \right| \]

To assess this alternative, we re-calibrate the model to the same targets as before. This gives the results shown in Table 11. Note that now the elasticity of substitution is larger than one, \(\sigma_C = 1.05\). We perform the same cross-country analysis as before and obtain the results for the time allocation and marketization shown in Table 12. To facilitate comparison, the table includes the data and the results of the baseline model.

Table 12 shows that the alternative model does poorly in matching the evolution of household hours. In fact, the model predicts increasing household hours in almost all the countries, although we are using the estimated increases in household labor productivity. The reason for the poor performance is that the calibrated value of \(\hat{C}_m\) is positive, implying that the income elasticity of household consumption is larger than the income elasticity of market consumption. This force implies that as economies get richer, time is reallocated from market work to household work. To see this formally, note that the ratio between market and household hours is now given by:

\[
\frac{H_m}{H_h} = \left[ \frac{\alpha_C}{1 - \alpha_C} (1 - \tau) \right]^{\sigma} \left[ \frac{A_h}{A_m} \right]^{1-\sigma} + \frac{\hat{C}_m}{A_m H_h}
\]

The last term above is new compared to the baseline model; compare (7). As \(A_m\) grows, it be-
comes less important, which implies an additional reallocation from market hours to household hours. This leads to counterfactual increases in household hours.

6 Conclusion

Using time–use surveys for 12 OECD countries during 1970–2010, we have documented that in response to increases in labor–income taxes people substitute leisure for market work while household hours hardly change. To understand why this happens, we have built a model with household production and have calibrated it to the U.S. We have fed into the calibrated model the household labor productivity along with country–specific taxes and market labor productivities. We have shown that this generates time–allocation patterns that are similar to the observed ones. In this context, it has been crucial that we have measured, instead of calibrated, the labor productivity of household production. The reason is that the measured labor productivity of household production often increases strongly in the countries which increased their labor–income taxes. This is at odds with what the literature has assumed about the behavior of labor productivity of household production.

Moving our focus beyond the narrow confines of our analysis, we think that the production of leisure warrants more attention. While the production of leisure is typically not modeled at all, many of the same considerations as for household production also apply to it. We think that this is a very important and promising topic for future research. Bridgman (2017) made some interesting initial progress on it.

References


27


Appendix

Appendix A: Ramey versus Our Categories of Home Production

The purpose of this part of the appendix is to compare our estimates of the average weekly household production time for the U.S. with those of Ramey (2009). For this Appendix, we will deviate from our usual procedure of reporting shares of disposable time after education, personal care, and sleep. Instead, we will report levels because Ramey reported her numbers as levels as well.

Table 13: Average Weekly Hours of Home Work in the U.S.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Ramey</td>
<td>18–64 AHTUS</td>
<td>26.5</td>
<td>22.5</td>
<td>21.3</td>
<td>22.3</td>
<td>23.2</td>
</tr>
<tr>
<td>2</td>
<td>D–H</td>
<td>15–64 MTUS</td>
<td>41 activ.</td>
<td>20.6</td>
<td>20.0</td>
<td>19.2</td>
<td>20.2</td>
</tr>
<tr>
<td>3</td>
<td>D–H</td>
<td>18–64 MTUS</td>
<td>41 activ.</td>
<td>22.4</td>
<td>21.2</td>
<td>20.3</td>
<td>20.9</td>
</tr>
<tr>
<td>4</td>
<td>D–H</td>
<td>18–64 MTUS</td>
<td>69 activ., our categ.</td>
<td>27.4</td>
<td>23.7</td>
<td>23.0</td>
<td>23.8</td>
</tr>
<tr>
<td>5</td>
<td>D–H</td>
<td>18–64 MTUS</td>
<td>69 activ., Ramey’s categ.</td>
<td>26.8</td>
<td>22.9</td>
<td>21.8</td>
<td>22.4</td>
</tr>
</tbody>
</table>

Data sources. Ramey uses The American Heritage Time Use Study (AHTUS) whereas we use The Multinational Time Use Surveys (MTUS), whereas we use the Multinational Time Use Surveys (MTUS), which is a repository that contains a large number of harmonized time-use surveys for different countries and years. We use MTUS because its data is comparable across countries and time. While ATUS and MTUS are fairly close for the U.S., there are some differences and one should not expect Ramey’s numbers to be exactly equal to our numbers even if we both follow exactly the same steps.

Age groups. Ramey considers the sample of individuals with age between 18 and 64 years. In contrast, we consider the individuals between 15 and 64 years of age, which is the standard definition of the working-age population. The third row in Table 13 shows our series computed for individuals with age between 18 and 64 years. The level of household hours is higher than in our original series and household hours now decline somewhat at the beginning of the time period.

Categorization of activities. MTUS has classification systems with 41 and 69 activities. Since the classification with 69 activities is not available for the 1965 and 1975 surveys for France, we use 41 activities for all countries and years. This ensures the cross-country comparability of our estimates of household production time. Since the MTUS data for the U.S. has both activity classifications, we can check by how much our numbers change if we apply the classification with 69 activities instead of that with 41 activities. The fourth row in the table has the results. Three remarks are in order. First, the constancy of household hours between 1975 and 2005 is largely preserved. Second, the drop in household hours between 1965 and
Table 14: Home Production Activities in the MTUS Classification System

<table>
<thead>
<tr>
<th>41 activities</th>
<th>69 activities – D–H categories</th>
<th>69 activities – Ramey’s categories</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cook, wash up</td>
<td>Food preparation, cooking</td>
<td>Food preparation, cooking</td>
</tr>
<tr>
<td>Housework</td>
<td>Set table, wash/put away dishes</td>
<td>Set table, wash/put away dishes</td>
</tr>
<tr>
<td>Odd jobs</td>
<td>Cleaning</td>
<td>Cleaning</td>
</tr>
<tr>
<td>Maint. of hh or car</td>
<td>Laundry, ironing, clothing repair</td>
<td>Laundry, ironing, clothing repair</td>
</tr>
<tr>
<td>Hh management</td>
<td>Home/vehicle maintenance/improvement</td>
<td>Home/vehicle maintenance/improvement</td>
</tr>
<tr>
<td>Pet care</td>
<td>Other domestic work</td>
<td>Other domestic work</td>
</tr>
<tr>
<td>Gardening</td>
<td>Purchase goods</td>
<td>Purchase goods</td>
</tr>
<tr>
<td>Shopping</td>
<td>Purchase personal care services</td>
<td>Purchase other services</td>
</tr>
<tr>
<td>Child care</td>
<td>Purchase other services</td>
<td>Purchase other services</td>
</tr>
<tr>
<td>Adult care</td>
<td>Pet care (other than walk dog)</td>
<td>Pet care (other than walk dog)</td>
</tr>
<tr>
<td></td>
<td>Physical or medical child care</td>
<td>Physical or medical child care</td>
</tr>
<tr>
<td></td>
<td>Teach child a skill, help with homework</td>
<td>Teach child a skill, help with homework</td>
</tr>
<tr>
<td></td>
<td>Read to, talk or play with child</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Supervise, accompany, other child care</td>
<td>Supervise, accompany, other child care</td>
</tr>
<tr>
<td></td>
<td>Adult care</td>
<td>Adult care</td>
</tr>
<tr>
<td></td>
<td>Child/adult care–related travel</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Travel for shopping, personal/household care</td>
<td>Travel for shopping, personal/household care</td>
</tr>
</tbody>
</table>

1975 is now more pronounced than before and the magnitude of the change is close to what Ramey finds (14% in our data versus 15% in Ramey). Third, the level of household hours is higher than before. The last point is due to the fact that using the finer classification allows us to be more accurate when we classify household production activities. As a result, several activities are now part of household production but were previously included in leisure, market production or personal care time. This applies for instance to several activities related to travel (for instance, child/adult care-related travel and travel for shopping, or household care). The second and the third column in Table 14 show, for both activity classifications, all activities that our definition includes in household production.

**Ramey’s categorization of activities.** Ramey excluded a number of activities from household production that are arguably part of household production. The reason for doing this was that the pre–1965 time–use data that she utilizes do not report these activities, so by excluding them she ensured the consistency of her categorization over time. The last row in Table 13 below shows our estimates for household hours if we exclude the same activities from our definition of household production as Ramey did. These activities include for instance “purchasing personal care services” or “adult care travel”. The third column in Table 14 shows the remaining household production activities included in our definition. As expected, the level of household production time drops further. More importantly, the last series is very close to Ramey’s series indeed.
Appendix B: Derivations for Section 3.2

The Lagrangian for the household problem (6) is:

\[ L = \alpha_u \log(C - \bar{C}) + (1 - \alpha_u) \log(L) + \eta_c [C - (\alpha_c C_m^{\frac{1}{\sigma}} + (1 - \alpha_c) C_h^{\frac{1}{\sigma}})] + \eta_h [A_h H_h - C_h] + \lambda [(1 - \tau)w H_m + T - C_m] + \mu [1 - L - H_m - H_h] \]

The first–order conditions are:

\[
\frac{\partial L}{\partial C} : \quad \frac{\alpha_u}{C} = \eta_c \\
\frac{\partial L}{\partial L} : \quad \frac{1 - \alpha_u}{L} = \mu \\
\frac{\partial L}{\partial C_m} : \quad \eta_c C^{\frac{1}{\sigma}} C_m^{-\frac{1}{\sigma}} = \lambda \\
\frac{\partial L}{\partial C_h} : \quad \eta_c C^{\frac{1}{\sigma}} (1 - \alpha_c) C_h^{-\frac{1}{\sigma}} = \eta_h \\
\frac{\partial L}{\partial H_m} : \quad \lambda (1 - \tau)w = \mu \\
\frac{\partial L}{\partial H_h} : \quad \mu = \eta_h A_h
\]

To derive (8), we rewrite (12)–(13) as:

\[
\eta_c C^{\frac{1}{\sigma}} C_m^{-\frac{1}{\sigma}} = \lambda \\
\eta_c C^{\frac{1}{\sigma}} (1 - \alpha_c) C_h^{-\frac{1}{\sigma}} = \frac{\mu}{A_h} = \lambda \frac{(1 - \tau)w}{A_h} = \lambda \rho
\]

Next, we raise each of these equations to the power $1 - \sigma$ and rearrange:

\[
\eta_c^{1 - \sigma} C^{-\frac{1}{\sigma}} C_m^{\frac{1}{\sigma}} = \lambda^{1 - \sigma} \alpha_c^{\sigma} \\
\eta_c^{1 - \sigma} C^{-\frac{1}{\sigma}} (1 - \alpha_c) C_h^{\frac{1}{\sigma}} = \lambda^{1 - \sigma} (1 - \alpha_c)^{\sigma} \rho^{1 - \sigma}
\]

Adding these two equations and raising the result to the power of $1/(1 - \sigma)$ gives us:

\[ \eta_c = \lambda P \]

Using this equations and the first–order conditions from above, we obtain a consolidated first–order condition for the allocation of $C$ and $L$:

\[ (1 - \alpha_u) P(C - \bar{C}) = \alpha_u (1 - \tau)wL \]
In order to obtain the closed–form solution (8), we need to bring in the budget constraints of the government and the households. Combining them gives:

\[ wH_m = C_m \]

Adding \( p_h C_h \) on each side and using that \( p_h C_h = (1 - \tau)wH_h \), we get:

\[ w(1 - L) - \tau w H_h = C_m + p_h C_h \] (19)

The next step is to show that:

\[ C_m + p_h C_h = PC \]

We begin by rewriting (16)–(17) to:

\[
\frac{1}{\sigma} \alpha_c C^\sigma_m = \frac{\lambda}{\eta_c} C_m \\
\frac{1}{\sigma} (1 - \alpha_c) C^\sigma_h = \frac{\lambda}{\eta_c} p_h C_h
\]

Adding these two equations and using (9) gives:

\[
\frac{1}{\sigma} C^\sigma P = CP = C_m + p_h C_h
\] (20)

Combining (19)–(20), we find:

\[ w(1 - L) - \tau w H_h = PC \] (21)

Equations (18) and (21) imply the closed–form solution (8) for leisure that we used in the main text.
Appendix C: Results for Levels of Hours

Table 15: Time Allocation — Data and Model (all numbers are percentages of weekly time endowment, first column initial year, second column final year in sample)

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th></th>
<th></th>
<th></th>
<th>Model</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$H_m$</td>
<td>$H_h$</td>
<td>$L$</td>
<td>$H_m$</td>
<td>$H_h$</td>
<td>$L$</td>
<td></td>
</tr>
<tr>
<td>Austria</td>
<td>30.5</td>
<td>26.8</td>
<td>22.3</td>
<td>23.3</td>
<td>47.2</td>
<td>50.0</td>
<td>26.7</td>
</tr>
<tr>
<td>Canada</td>
<td>27.2</td>
<td>28.7</td>
<td>23.4</td>
<td>22.4</td>
<td>49.4</td>
<td>48.9</td>
<td>31.3</td>
</tr>
<tr>
<td>Finland</td>
<td>29.1</td>
<td>22.8</td>
<td>24.1</td>
<td>22.9</td>
<td>46.9</td>
<td>54.2</td>
<td>29.4</td>
</tr>
<tr>
<td>France</td>
<td>33.4</td>
<td>22.2</td>
<td>23.7</td>
<td>20.9</td>
<td>42.9</td>
<td>56.9</td>
<td>29.8</td>
</tr>
<tr>
<td>Germany</td>
<td>31.9</td>
<td>24.1</td>
<td>25.1</td>
<td>21.6</td>
<td>43.0</td>
<td>54.3</td>
<td>31.0</td>
</tr>
<tr>
<td>Italy</td>
<td>30.4</td>
<td>25.0</td>
<td>24.3</td>
<td>26.7</td>
<td>45.3</td>
<td>48.3</td>
<td>30.3</td>
</tr>
<tr>
<td>Japan</td>
<td>40.6</td>
<td>36.2</td>
<td>14.9</td>
<td>14.5</td>
<td>44.5</td>
<td>49.3</td>
<td>36.5</td>
</tr>
<tr>
<td>Netherl.</td>
<td>17.6</td>
<td>22.9</td>
<td>26.9</td>
<td>24.0</td>
<td>55.5</td>
<td>53.1</td>
<td>24.6</td>
</tr>
<tr>
<td>Norway</td>
<td>26.1</td>
<td>27.2</td>
<td>24.0</td>
<td>22.7</td>
<td>49.8</td>
<td>50.1</td>
<td>26.5</td>
</tr>
<tr>
<td>Sweden</td>
<td>26.1</td>
<td>28.9</td>
<td>25.7</td>
<td>22.6</td>
<td>48.2</td>
<td>48.5</td>
<td>21.7</td>
</tr>
<tr>
<td>U.K.</td>
<td>31.2</td>
<td>25.1</td>
<td>22.2</td>
<td>20.2</td>
<td>46.6</td>
<td>54.8</td>
<td>32.2</td>
</tr>
<tr>
<td>U.S.</td>
<td>32.6</td>
<td>30.2</td>
<td>19.9</td>
<td>20.4</td>
<td>47.5</td>
<td>49.4</td>
<td>32.6</td>
</tr>
</tbody>
</table>