Structural Transformation of Occupation and Industry Employment

Motivation

Structural Transformation (ST)

- As economies develop, labor is reallocated across broad sectors:
  - agriculture shrinks
  - industry first grows and then shrinks ("hump shape")
  - services grow.
Historical Employment Shares for 10 Countries 1800–2000
(from Herrendorf, Rogerson, Valentinyi, Handbook Chapter, 2014)
• Large literature: ST crucial force behind important economic issues
  ○ Aggregate hours worked
  ○ Aggregate labor productivity
  ○ Urbanization
  ○ Pollution

• This literature focuses on broad categories of industries (“sectors”).
Alternative perspective on structural transformation: occupations

- Occupations play key role for many important economic issues
  - Wages
  - Human capital
  - Technical progress ...

- Occupations not affected by outsourcing
  - Janitorial labor employed by car manufacturer counted as industry employment
  - Janitorial labor purchased by car manufacturer counted as service employment
  - Janitor is a service–producing occupation in both cases

Thus, focusing on occupations rules out that ST is just relabelling.

- Surprisingly little evidence on ST of occupation employment.
Our contribution

• Provide new evidence on ST of occupation and sector employment
  ◦ Representative census data from IPUMS International.
  ◦ Many rich and many poor countries and large part of world population.
  ◦ Information about both occupations and sectors.

• Establish new stylized facts
  ◦ Standard patterns of ST hold for BOTH sector and occupation employment.
  ◦ The employment share of service occupations rises in ALL sectors.
  ◦ This suggests a broader notion of ST between both sectors AND occupations.

• Build a model of ST with sectors AND occupations, which
  ◦ is consistent with the old and new stylized facts of ST;
  ◦ allows us to study the forces behind ST of sector and occupation employment.
Outline

• Data
• Stylized Facts
• Model
• Results
• Applications
• Conclusion
Data

Censuses for with sector and occupation information

- **IPUMS International:** 169 census observations from 64 countries
  (21 American/Caribbean, 18 African, many Sub-Saharan ones, 14 European, 11 Asian).

- **Large part of the world population**
  - At least one observation for seven of the ten most populous countries in 1990

- **Countries of all income levels including the very poorest ones**
  - More than 2/3 of world output in 1990 in 1990 international $’s.
  - GDP per capita difference of more than a factor fifty:
Available census observations

Definition of sectors

- **Agriculture sector**: agriculture, fishing, and forestry.
- **Industry sector**: construction; electricity, gas and water; manufacturing; mining.
- **Goods sector**: agriculture + industry sector.
- **Service sector**: education; financial services and insurance; health and social work; hotels and restaurants; other services; private household services; public administration and defense; real estate and business services; transportation and communications; wholesale and retail trade.
Definition of occupations

- **Agriculture occupations**: elementary agricultural occupations; skilled agricultural and fishery workers.

- **Industry occupations**: elementary industry occupations; crafts and related trades workers; plant and machine operators and assemblers.

- **Goods occupations**:
  - Agriculture + industry occupations.
  - Produce tangible value added.
  - Related, but not equal, to blue–collar or brawn–intensive occupations.

- **Service occupations**:
  - elementary service occupations; armed forces; clerks; legislators, senior officials and managers; professionals; service workers and shop and market sales; technicians and associate professionals.
  - Produce intangible value added.
  - Related, but not equal, to white–collar or brain–intensive occupations.
Examples

- **Service occupation which is also in the goods sector**
  - Legislators, senior officials and managers
  - E.g., manager of manufacturing plant.

- **Goods occupation which is also in the service sector**
  - Crafts and related trades workers
  - E.g., licensed electricians who works as contractor.
Stylized Facts (SF)

- GDP per capita in 1990 international $’s: Maddison’s Groningen database.
- First pass at the data: three categories.
- Second pass at the data: two categories.
Sector ST for Three Categories

Employment Share in Agriculture Sector

Employment Share in Industry Sector

Employment Share in Services Sector

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Occupation ST for Three Categories

Employment Share in Agriculture Occupations

Employment Share in Industry Occupations

Employment Share in Services Occupations
US Sector ST 1840–2010 in comparison (black diamonds are US observations)
US Occupation ST 1840–2010 in comparison (black diamonds are US observations)
SF’s between two Categories

- Goods production: agriculture and industry
- Services.
SF’s for goods versus services

Duernecker and Herrendorf
### Summary statistics sectors and occupations

<table>
<thead>
<tr>
<th>GDP per capita (1,000 of int. $’s)</th>
<th>&lt;2.5</th>
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</table>

- The goods sector is intensive in the goods occupation.
- The service sector is intensive in the service occupation.
- Each sector’s share of service–occupation employment increases with GDP.
Two important implications of our evidence

1. ST does not follow systematically different patterns in currently poor countries

- Bah (Emerging Markets, Finance, and Trade, 2011)
  - ST in some poor countries seems to follow different patterns.
  - ST in some poor countries happens without GDP growth.

- Prominent deviations from standard patterns
  - India is all but skipping industrialization, jumping right to services.
  - South Korea industrialized much more strongly than other countries.

- We find no evidence that poorer countries follow *systematically* different patterns. We do find a lot of *unsystematic* variation in the service and industry shares.
Figures for Africa and Latin America separately ...
2. Outsourcing is not a quantitatively important force behind ST

- Reallocation of resources may reflect relabeling due to outsourcing, as opposed to fundamental shifts of economic activity across sectors.

- This is a worrying prospect for the ST literature.

- Our findings imply quantitatively outsourcing cannot be main force behind ST.
  - Occupation employment not affected by outsourcing (janitors employed by car manufacturers and cleaning firms are both janitors)
  - If outsourcing was main force behind ST, then
    - no increase in service–occupation employment in goods sector
    - little increase in total service–occupation employment.
  - Instead, we find service–occupation employment increases
    - in both sectors
    - in the aggregate
    (and by at least as much as service–sector employment; see on next page).

- Thus, to the extent that outsourcing drives service–sector employment, there are other forces which offset it (see model to follow for details).
Duernecker and Herrendorf
- **Much stronger conclusion than in the literature**
  - Herrendorf, Rogerson, Valentinyi (AER, 2013)
    - Outsourcing does not affect the composition of final expenditure;
    - There is ST in final expenditure, so not all ST is due to outsourcing.
  - Berlingieri (Manuscript, 2014)
    - Changes in input–output structure increase service employment by 36%.
    - Outsourcing of business services crucial for changes in input–output structure.
  - The above evidence is indirect and for the postwar US; in contrast, our evidence is direct and covers 169 censuses from around the world.
Model

General remarks

- The purpose of the model is to highlight the features that are crucial for the patterns of employment in occupations and sectors.
  - We will write down the most simple version with an AK investment sector.
  - In addition, there are two sectors producing consumption goods and services and two occupations used by both sectors.
We want the model to match seven patterns that unfold as economies develop

**Four SF’s from above**
1. labor is reallocated from goods sector to service sector;
2. total labor is reallocated from goods occupations to service occupations;
3. sectoral labor is reallocated from goods to service occupations;
4. service occupations have larger employment share in service sector;
   goods occupations have larger employment share in goods sector.

**Three SF’s from the literature**
5. the expenditure share of services increases and that of goods decreases;
6. the price of services relative to goods increases;
7. labor productivity increases more in the goods than the service sector.

**Note**

- Our model won’t match the patterns of real shares.
- This is a common problem of ST models with CES preferences.
- See Boppart (ECMA, 2014) and Comin et. al. (2015) for recent solutions.
Environment

- Time discrete and runs forever.
- Three commodities in period $t$
  - investment good $X_t$;
  - consumption goods $C_{Gt}$;
  - consumption services $C_{St}$.
- In each period, the investment good is the numeraire.
• Investment–sector technology

\[ Y_{Xt} = A_X K_{Xt} \]  

- where \( A_X \) is the (constant) TFP of producing investment goods from capital \( K_X \).
- Advantage of using \( AK \): labor reallocated only between consumption sectors.
• Consumption–sector technologies

\[ Y_{Jt} = \left( K_{Jt} \right)^{\theta} \left( A_{Jt} L_{Jt} \right)^{1-\theta} \]  

(2)

where

\[ L_{Jt} = \left[ \left( \alpha_J \right)^{\frac{1}{\sigma}} \left( A_{gt} N_{Jgt} \right)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha_J) \frac{1}{\sigma} \left( A_{st} N_{Jst} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \]  

(3)

and

- \( J \in \{ G, S \} \) indexes sectors and \( j \in \{ g, s \} \) occupations  
  (so \( N_{Jj} \) denotes the labor in sector \( J \) from occupation \( j \));
- \( A_J \) is sector–labor–augmenting technical progress;
- \( A_j \) is occupation–labor–augmenting technical progress;
- \( \sigma \in (0, \infty) \) is the elasticity of substitution  
  (\( \sigma \to 0 \) Leontief, \( \sigma = 1 \) Cobb–Douglas, \( \sigma \to \infty \) perfect substitutes).
• Continuum (of measure one) of identical households.

• Present discounted sum of lifetime utility

\[ \sum_{t=0}^{\infty} \beta^t \log(C_t) \]  

where

○ \( \beta \in (0, 1) \) is the discount factor

○ \( C_t \) is a consumption aggregator

\[ C_t = \left[ (\alpha_U)^{\frac{1}{\varepsilon}} C_{Gt}^{\frac{\varepsilon-1}{\varepsilon}} + (1 - \alpha_U)^{\frac{1}{\varepsilon}} C_{St}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \]
• **Endowments**
  - Positive initial capital stock $K_0 > 0$.
  - One unit of labor in each period.
  - Capital and labor can be supplied to both sectors.
  - Labor can be supplied to both occupations.

• **Remark**
  - The last assumption will have to change if we want to study wage differences.
• Resource constraints and market–clearing conditions

\[ K_{t+1} = (1 - \delta)K_t + X_t \]  \hspace{1cm} (6)

\[ K_t = K_{Xt} + K_{Ct} = K_{Xt} + (K_{Gt} + K_{St}) \]  \hspace{1cm} (7)

\[ N_{Jt} \equiv N_{Jgt} + N_{Jst} \]  \hspace{1cm} (8)

\[ N_{jt} \equiv N_{Gjt} + N_{Sjt} \]  \hspace{1cm} (9)

\[ 1 = N_t = N_{Gt} + N_{St} = N_{gt} + N_{st} \]  \hspace{1cm} (10)

\[ Y_{Xt} = X_t, \hspace{0.5cm} Y_{Gt} = C_{Gt}, \hspace{0.5cm} Y_{St} = C_{St} \]  \hspace{1cm} (11)
Solving for the equilibrium

Two–step version of the household problem

Intertemporal problem:

\[
\max_{\{K_{t+1}, C_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log(C_t) \quad \text{s.t.} \quad C_t P_t + K_{t+1} = (1 + r_t - \delta)K_t + w_t \\
\] (12)

Period problem:

\[
\max_{\{G_t, S_t\}} \left[ (\alpha_U) \frac{1}{\varepsilon} (G_t) \frac{\varepsilon-1}{\varepsilon} + (1 - \alpha_U) \frac{1}{\varepsilon} (S_t) \frac{\varepsilon-1}{\varepsilon} \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad \text{s.t.} \quad G_t C_t + S_t C_S = C_t P_t \\
\] (13)

where

\[
P_t = \left[ \alpha_U (P_{G_t})^{1-\varepsilon} + (1 - \alpha_U) (P_{S_t})^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}
\]
FOCs to the household problem

\[
\frac{C_{t+1}P_{t+1}}{C_tP_t} = \beta(1 + r_{t+1} - \delta) \tag{14}
\]

\[
\lim_{t \to \infty} \frac{\beta^t}{C_tP_t}K_{t+1} = 0 \tag{15}
\]

\[
\frac{P_{St}C_{St}}{P_{Gt}C_{Gt}} = \frac{1 - \alpha_U}{\alpha_U} \left( \frac{P_{St}}{P_{Gt}} \right)^{1-\epsilon} \tag{16}
\]

**Remark**

- (16) establishes the link between expenditure shares and relative prices.
- Ngai–Pissarides (AER, 2007): \( \epsilon < 1 \) required to match patterns in the data.
FOC to firm problem in the investment sector

\[ r_t = A_X \]  \hspace{1cm} (17)

- Remark
  - AK is convenient here because it implies a constant real interest rate.
Implications of FOC’s to firm problems in the consumption sectors

\[
\frac{K_{Ct}}{N_t} = \frac{K_{Gt}}{N_Gt} = \frac{K_{St}}{N_{St}} \tag{18}
\]

\[
\frac{Y_{Gt}/N_{Gt}}{Y_{St}/N_{St}} = \left( \frac{A_{Gt}L_{Gt}/N_{Gt}}{A_{St}L_{St}/N_{St}} \right)^{1-\theta} \tag{19}
\]

\[
\frac{Y_{Gt}/N_{Gt}}{Y_{St}/N_{St}} = \frac{P_{St}}{P_{Gt}} \tag{20}
\]

\[
\frac{N_{J_{st}}}{N_{J_{gt}}} = \frac{1 - \alpha_J}{\alpha_J} \left( \frac{A_{gt}}{A_{st}} \right)^{1-\sigma} \tag{21}
\]

\[
\frac{N_{S_{st}}}{N_{G_{st}}} = \frac{1 - \alpha_S}{1 - \alpha_G} \left( \frac{L_{St}}{L_{Gt}} \right)^{1-\sigma} \left( \frac{N_{St}}{N_{Gt}} \right)^{\sigma} \tag{22}
\]

- **Remarks**
  - (20) establishes link between relative productivities and relative prices.
  - (21) is about reallocation of labor within sectors
  - (22) is about reallocation of labor between sectors.
Results

Generalized balanced growth path (GBGP)

• Since our model features reallocation of labor between sectors and occupations, ratios won’t be constant and imposing BGP would be too strong.

• We focus on GBGP, which only requires that the real interest rate be constant.

• That is trivially the case here because of the AK technology in the investment sector.
Proposition 1

- There is a unique GBGP.
- Along the GBGP, capital, capital in consumption sectors, capital in investment sector, and consumption expenditure all grow at factor $\gamma \equiv \beta(1 + A_x - \delta)$. 
Proposition 2

If the parameters satisfy (i)–(v), then the GBGP is consistent with SF’s 1–7.

(i) \( 0 < \alpha_S < \alpha_G < 1 \)
- the service sector is more intensive in the service occupation
- the goods sector is more intensive in the goods occupation

(ii) \( 0 \leq \sigma < 1 \)
- inputs into the production function are complements

(iii) \( 0 \leq \varepsilon < 1 \)
- inputs into the utility function are complements

(iv) \( A_{gt}/A_{st} \) increases
- technical progress is faster in the goods than the service occupation

(v) \( A_{Gt}/A_{St} \) increases
- technical progress is faster in the goods than in the service sector
**Note 1: necessary versus sufficient**

- While (i)–(v) are sufficient, only (i)–(iv) are necessary.
- (v) is not necessary because as long as $A_{gt}/A_{st}$ increases strongly enough, all SF’s may hold even if $A_{gt}/A_{st}$ decreases.

**Note 2: possible identification procedure**

- Identify $A_{g}/A_{s}$ from $N_{sj}/N_{gj}$.
- Given $A_{g}/A_{s}$, identify $A_{G}/A_{S}$ from $N_{s}/N_{g}$.
- The levels are not identified, so we need to normalize one of them, e.g., $A_{s} = 1$. 
Intuition for Proposition 2

- The relative price of services increases for two reasons:
  - sector–labor–augmenting technical change grows more slowly in the service sector;
  - occupation–labor–augmenting technical change grows more slowly in the service occupations
    the service sector is intensive in the service occupations.

- Since goods are complements in the utility function, the increase in the relative price of services implies that expenditures and labor get reallocated from the goods to the service sector.

- Since occupations are complements in the production function, the slower labor–augmenting technical change in the service occupations implies that labor gets allocated from goods to service occupations in each sector.

- Labor gets allocated from goods to service occupations in the whole economy because not only does that happen in each sector but also does labor get reallocated to the service sector which is intensive in service occupations.
In other words, **two forces generate reallocation of labor to service occupations:**

- substitution between occupations within each sector
  (this results from uneven technological progress at the occupation level and it would take place with fixed sectoral employment);
- substitution of labor between sectors
  (here this also results from uneven technological progress but it could also result form even technological progress and nonhomothetic preferences, see Kongsamut, Rebelo, and Xi, REStud, 2001).

- **This suggests broader notion of ST than just reallocation of sectoral employment.**
Applications of the Model

1. Increase in female labor force participation in rich countries

- Women have a comparative advantage in service occupations (Rendall, Manuscript, 2010).
- ST implies the reallocation of labor from goods to service occupations.
- Structural transformation leads to the increase in female labor force participation.
- Ngai and Petrongolo (Manuscript, 2015) build a Roy model in which that is the case.
2. Job polarization

- Job polarization is the decrease in the employment share of the middle-wage occupations and the increase in the employment shares of the low-wage and high-wage occupations.

- Service occupations tend to be both low-wage and high-wage occupation whereas goods occupations tend to middle-wage occupations.

- ST increases the employment of service occupations and decreases the employment of goods occupations.

- Job polarization is a consequence of structural transformation.

- Barrany–Siegel (Manuscript, 2014) build a Roy model in which that is the case.
3. Changes in the degree of unionization

- In the US, labor unions tend to be occupation specific and the degree of unionization is higher in goods than in service occupations.
- ST accounts for the decline in the rate of unionization in recent decades.
- Note that over the last century the degree of unionization in the US is hump–shaped. The share of industry occupations in total employment follows a similar pattern.
Discussion

- Our model captures that ST causes compositional changes of occupation employment. In the three examples, these have consequences for economic issues of interest.

- Our model cannot speak to changes in relative wages because we assumed that everyone can supply every occupation.

- The next step on our agenda is to break this assumption (for example, there may be an entry cost into each occupation).
Conclusion

- **We have:**
  - provided new evidence on ST from international census data;
  - shown that the standard facts of ST hold for both sector and occupation employment;
  - modeled the link between sector and occupation employment;
  - used the model to shed light on important issues related to occupations.

- **We plan to generalize this model in at least two dimensions:**
  - have more than two sectors/occupations;
  - include entry costs into the occupations.
Appendix: Additional Tables and Figures
## Detailed summary statistics sectors and occupations

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<tr>
<th>GDP per capita (1000)</th>
<th>Goods-producing sector</th>
<th>Services sector</th>
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<tr>
<td></td>
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Differences between sector and occupation employment shares

- Employment Share Difference (Sec–Occ): Agriculture
- Employment Share Difference (Sec–Occ): Industry
- Employment Share Difference (Sec–Occ): Services
Differences between sector and occupation employment shares

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Appendix: Proofs

FOC’s to the Firm Problem

- We need to show (18)–(22).
- We drop the time indexes when this does not cause confusion.
- “Raw” FOC’s for profit maximization:

\[
\begin{align*}
  r &= \theta P_J K_J^{\theta-1} (A_J L_J)^{1-\theta} = \theta P_J \left( \frac{K_J}{N_J} \right)^{\theta-1} \left( \frac{A_J L_J}{N_J} \right)^{1-\theta} \\
  w &= (1 - \theta) P_J K_J^\theta A_J^{1-\theta} L_J^{-\theta} L_J^\sigma A_J^\sigma \alpha_J^\sigma A_J^\sigma \alpha_J^\sigma N_J^{-1} \\
  w &= (1 - \theta) P_J K_J^\theta A_J^{1-\theta} L_J^{-\theta} L_J^\sigma (1 - \alpha_J)^{\sigma-1} A_J^{\sigma-1} \alpha_J^{\sigma-1} N_J^{-\sigma} 
\end{align*}
\]

- (21) follows by dividing (24) and (25) by each other.
• Multiplying (24)–(25) with the respective labor and adding up gives that:

\[ w = (1 - \theta)P_J \left( \frac{K_J}{N_J} \right)^\theta \left( \frac{A_J L_J}{N_J} \right)^{1-\theta} \]  

(26)

• Dividing (26) by (23), we find:

\[ \frac{w}{r} = \frac{1 - \theta K_J}{\theta N_J} \]  

(27)

• Hence,

\[ \frac{K_J}{N_J} = \frac{K_C}{N} = K_C \]  

(28)

which was (18).
• Equalized capital–labor ratios imply (19) and (20):

\[
\frac{Y_G/N_G}{Y_S/N_S} = \left(\frac{A_G L_G/N_G}{A_S L_S/N_S}\right)^{1-\theta} = \frac{P_S}{P_G}
\]

• It remains to show (22). Using (25) for \(J = G, S\), we obtain:

\[
P_G K_G^\theta A_G^{1-\theta} L_G^{-\theta} L_G^\sigma (1 - \alpha_G)^{\frac{1}{\sigma}} A_s^{\frac{1}{\sigma}} N_{Gs}^{\frac{1}{\sigma}} = P_S K_S^\theta A_S^{1-\theta} L_S^{-\theta} L_S^\sigma (1 - \alpha_S)^{\frac{1}{\sigma}} A_s^{\frac{1}{\sigma}} N_{Ss}^{\frac{1}{\sigma}}
\]

Using (20) and (28), this can be simplified to (22):

\[
\frac{N_{Ss}}{N_{Gs}} = \frac{1 - \alpha_S}{1 - \alpha_G} \left(\frac{L_S}{L_G}\right)^{1-\sigma} \left(\frac{N_s}{N_G}\right)^\sigma
\]

• QED
Proof of Proposition 1

\begin{itemize}
\item Need to show that there is a unique GBGP.
\item (17) implies that $r_t = r = A_x$.
\item (14) implies that
\[ \frac{C_{t+1}P_{t+1}}{C_tP_t} = \gamma \equiv \beta(1 + A_X - \delta) \]
\item $w_t = w_tN_t = (1 - \theta)C_tP_t$ implies that $w_t$ grows at the same factor as $C_tP_t$, i.e., $\gamma$.
\item $r_tK_{C_t} = A_XK_{C_t} = \theta C_tP_t$ implies that $K_{C_t}$ grows at the same factor as $C_tP_t$, i.e., $\gamma$.
\end{itemize}
• The consumer budget constraint can be rewritten to:

\[
\frac{\theta C_t P_t}{K_t} + \frac{K_{t+1}}{K_t} = 1 + A_x - \delta
\]

Hence, if \( K_t \) grows at a constant factor, then that factor must be \( \gamma \).

• \( K_t = K_{X_t} + K_{C_t} \) implies that \( K_{X_t} \) grows at factor \( \gamma \) too.

• QED
Proof of Proposition 2

Strategy of the Proof

- We first prove that given (i)–(v) hold, we have SF 1.
- Then we prove SF’s 3 and 4.
- Then we prove SF 2.
- Lastly we prove SF’s 5–7.
- Again, we drop the time indexes when this does not cause confusion.
**Derivation of SF 1**

- **Need to show that** $N_S/N_G$ **increases.**

  - (21) implies:

    $$N_J = N_{Jg} + N_{Js} = \left[ \frac{N_{Jg}}{N_{Js}} + 1 \right] N_{Js} = \left[ \frac{\alpha_J}{1 - \alpha_J} \left( \frac{A_g}{A_s} \right)^{\sigma-1} + 1 \right] N_{Js}$$

- **Hence, the ratio of sectoral labor satisfies:**

  $$\frac{N_S}{N_G} = \frac{\left[ 1 + \left[ \alpha_s/(1 - \alpha_s) \right] \left( \frac{A_g}{A_s} \right)^{\sigma-1} \right] \frac{N_{Ss}}{N_{Gs}}}{\left[ 1 + \left[ \alpha_G/(1 - \alpha_G) \right] \left( \frac{A_g}{A_s} \right)^{\sigma-1} \right] \frac{N_{Gs}}{N_{Gs}}}$$  \hspace{1cm} (29)
• Combining (16) and (20) and rearranging gives:

\[
\frac{P_S}{P_G} = \left( \frac{\alpha_U N_S}{1 - \alpha_U N_G} \right)^{\frac{1}{\sigma-1}}
\]

Substituting this into (22), we obtain

\[
\frac{N_{S,s}}{N_{G,s}} = \frac{1 - \alpha_S}{1 - \alpha_G} \left( \frac{1 - \alpha_U}{\alpha_U} \right)^{\frac{1}{(1-\varepsilon)(1-\theta)}} \left( \frac{N_S}{N_G} \right)^{1-\frac{1}{(1-\varepsilon)(1-\theta)}} \left( \frac{A_G}{A_S} \right)^{1-\sigma}
\]

• Substituting this into (29) gives:

\[
\frac{N_S}{N_G} = \left( \frac{1 - \alpha_S}{1 - \alpha_G} \right)^{\frac{(\varepsilon-1)(1-\theta)}{\sigma-1}} \frac{1 - \alpha_U}{\alpha_U} \left[ 1 + \left( \frac{\alpha_s}{1 - \alpha_S} \right) \left( \frac{A_g}{A_s} \right)^{\sigma-1} \right] \left( \frac{A_G}{A_S} \right)^{\frac{(\varepsilon-1)(1-\theta)}{\sigma-1}}
\]

(30)
• Define

\[ f(x) \equiv \left[ \frac{1 + \tilde{\alpha}_S x^{\sigma-1}}{1 + \tilde{\alpha}_G x^{\sigma-1}} \right]^{1/\sigma-1} \]

where \( \tilde{\alpha}_j \equiv \alpha_j/(1 - \alpha_j) \).

• It is straightforward to show that given Assumption (i) we have \( f'(x) < 0 \).

• Fact 1 now follows from (30), \( f'(x) < 0 \), and Assumptions (iii)–(v).

• QED
Derivation of SF 3

- $N_{Js}/N_{Jg}$ increases.
- This follows directly from (21) and Assumptions (ii) and (iv).
- QED

Derivation of SF 4

- $N_{Ss} > N_{Sg}$ and $(N_{Gg} > N_{Gs})$.
- (i) and (21) imply the claim.
- QED
Derivation of SF 2

- $N_s$ increases and $N_g$ decreases.
- To see this, note that:

$$N_s = N_{Gs} + N_{Ss} = N_G \frac{N_{Gs}}{N_G} + N_S \frac{N_{Ss}}{N_S}$$

Hence,

$$\Delta N_s = \Delta N_G \frac{N_{Gs}}{N_G} + N_G \Delta \frac{N_{Gs}}{N_G} + \Delta N_S \frac{N_{Ss}}{N_S} + N_S \Delta \frac{N_{Ss}}{N_S}$$
Using that $N_S = 1 - N_G$, this becomes:

$$
\Delta N_s = N_G\Delta \frac{N_{Gs}}{N_G} + N_S\Delta \frac{N_{Ss}}{N_S} + \Delta N_S \left( \frac{N_{Ss}}{N_S} - \frac{N_{Gs}}{N_G} \right)
$$

- SF 1 implies $\Delta N_S > 0$;
- SF 3 implies $\Delta N_{Js}/N_J > 0$;
- SF 4 implies $N_{Ss}/N_S > N_{Gs}/N_G$.

- Hence, the right–hand side is positive and $N_s$ grows.

- Since $N_g = 1 - N_s$, this implies that $N_g$ falls.

- **QED**
Derivation of SF 5

• We need to show that \(\frac{P_S Y_S}{P_G Y_G}\) grows.

• To see this, note that (20) implies that

\[
\frac{Y_S P_S}{Y_G P_G} = \frac{N_S}{N_G}
\]

• Fact 5 therefore follows from Fact 1.

• QED
Derivation of SF’s 6 and 7

- We need to show that $P_S/P_G$ increases and $(Y_S/N_S)/(Y_G/N_G)$ decreases.
- (20) implies that either one of these statements is true iff the other one is true.
- We therefore only show that $(Y_S/N_S)/(Y_G/N_G)$ decreases.
- (19) implies that this is equivalent to showing that $(A_S L_S/N_S)/(A_G L_G/N_G)$ decreases.
To see that \((L_S/N_S)/(L_G/N_G)\) decreases, rewrite (3) while using (21):

\[
\frac{L_S}{L_G} = \left[ \frac{1 + \left[ \frac{\alpha_S}{1 - \alpha_S} \right] \frac{1}{\sigma} \left[ \frac{(A_g N_{Sg})}{(A_s N_{Ss})} \right]^{\sigma-1}}{1 + \left[ \frac{\alpha_G}{1 - \alpha_G} \right] \frac{1}{\sigma} \left[ \frac{(A_g N_{Gg})}{(A_s N_{Gs})} \right]^{\sigma-1}} \right]^{\frac{\sigma}{\sigma-1}} \left( \frac{1 - \alpha_S}{1 - \alpha_G} \right)^{\frac{1}{\sigma-1}} \frac{N_{Ss}}{N_{Gs}}
\]

\[= \left( \frac{1 - \alpha_S}{1 - \alpha_G} \right)^{\frac{1}{\sigma-1}} \left[ \frac{1 + \left[ \frac{\alpha_S}{1 - \alpha_S} \right] \left( A_g / A_s \right)^{-\sigma}}{1 + \left[ \frac{\alpha_G}{1 - \alpha_G} \right] \left( A_g / A_s \right)^{-\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \frac{N_{Ss}}{N_{Gs}} \]

(31)
• Multiplying both sides by $A_S/A_G$ and dividing the result by (29) gives:

$$\frac{A_S L_S / N_S}{A_G L_G / N_G} = \left(\frac{1 - \alpha_S}{1 - \alpha_G}\right)^{\sigma-1} \left[1 + \left[\frac{\alpha_S}{(1 - \alpha_S)}\right] \left(\frac{A_g/A_s}{\sigma-1}\right) \left(\frac{A_g/A_s}{\sigma-1}\right) \left(\frac{A_g/A_s}{\sigma-1}\right)\right] \frac{A_S}{A_G} \quad (32)$$

• Using that $f'(x) < 0$ and Assumptions (iv)–(v), it follows that $(A_S L_S / N_S)/(A_G L_G / N_G)$ decreases.

• QED
Proof that (i)–(iv) are necessary

- To get the SF 1–7 conditions (i)–(iv) must hold.
- (21) implies that we need (i) to get SF 4.
- (16) implies that given SF 6, we need (iii) to get SF 5.
- To get SF 1 given (iii) holds, we need (iv).
- (21) implies that given (iv), we need (ii) to get SF 3.
- QED
Detailed Intuition for Proposition 2

Recall

\[ Y_{Jt} = (K_{Jt})^\theta \left( \left[ (\alpha_J) \left( \frac{1}{\sigma} \left( A_{gt} N_{Jgt} \right) \right) \right]^{\frac{\sigma-1}{\sigma}} + (1 - \alpha_J) \left( \frac{1}{\sigma} \left( A_{st} N_{Jst} \right) \right) \right)^{\frac{\sigma}{\sigma-1}} \]^{1-\theta}

\[ C_t = \left[ (\alpha_U) \left( C_{Gt} \right) \right]^{\frac{\varepsilon-1}{\varepsilon}} + (1 - \alpha_U) \left( C_{S_t} \right) \]^{\frac{\varepsilon}{\varepsilon-1}}

**SF 3:** Shares of service occupation in sector employment increase:
\( A_{gt}/A_{st} \) increases and \( \sigma < 1 \).

**SF 4:** Service occupation has larger employment share in service sector:
\( \alpha_S < \alpha_G \).

**SF 6:** Price of services relative to goods increases:
\( A_{gt}/A_{st} \) and \( A_{Gt}/A_{St} \) increases and **SF 4**.

**SF 7:** Labor productivity of goods relative to that of services increases:
\( (Y_{Gt}/N_{Gt})/(Y_{S_t}/N_{S_t}) = P_{S_t}/P_{Gt} \) and **SF 6**.
Recall

\[ Y_{Jt} = (K_{Jt})^\theta \left[ \left( \alpha_J \right)^{\frac{1}{\sigma}} (A_{gt} N_{Jgt})^{\frac{\sigma-1}{\sigma}} + (1 - \alpha_J) \frac{1}{\sigma} (A_{st} N_{Jst})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right]^{1-\theta} \]

\[ C_t = \left[ \left( \alpha_U \right)^{\frac{1}{\varepsilon}} (C_{Gt})^{\frac{\varepsilon-1}{\varepsilon}} + (1 - \alpha_U) \frac{1}{\varepsilon} (C_{St})^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \]

**SF 5:** Expenditure share of services increases and of goods decreases: \( \varepsilon < 1 \) and SF 6 (relative price of services increases).

**SF 1:** Labor is reallocated from goods to service sector:

\[ \frac{P_{Gt}}{P_{St}} = \frac{Y_{St}/N_{St}}{Y_{Gt}/N_{Gt}} \implies \frac{N_{St}}{N_{Gt}} = \frac{P_{St} C_{St}}{P_{Gt} C_{Gt}} \]

and SF 5.

**SF 2:** Labor is reallocated from goods to service occupation:

SF 1, 4 imply \( N_{St}/N_{Gt} \) increases while \( N_{S_{st}}/N_{St} > N_{G_{st}}/N_{G1} \);

SF 3 implies \( N_{J_{st}}/N_{J_{gt}} \) increases for both \( J \in \{G, S\} \).