Sectoral Technology and Structural Transformation

Berthold Herrendorf
(Arizona State University)

Christopher Herrington
(University of South Alabama)

Ákos Valentinyi
(Cardiff Business School, Institute of Economics CERSHAS, and CEPR)

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Abstract
We assess how the properties of technology affect structural transformation, i.e. the reallocation of production factors across the broad sectors agriculture, manufacturing, and services. To this end, we estimate sectoral CES and Cobb–Douglas production functions on postwar US data. We find that differences in technical progress across the three sectors are the dominant force behind structural transformation whereas other differences across sectoral technology are of second order importance. Our findings imply that Cobb–Douglas sectoral production functions that differ only in technical progress capture the main technological forces behind the postwar US structural transformation.

Keywords: capital share; CES production function; Cobb–Douglas production function; structural transformation; elasticity of substitution.

JEL classification: O11; O14.

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1 Introduction

The reallocation of production factors across the broad sectors agriculture, manufacturing, and services is one of the important stylized facts of growth and development. As economies develop agriculture shrinks, manufacturing first grows and then shrinks, and services grow. A growing recent literature has studied this so called structural transformation and has shown that it has important implications for the behavior of aggregate variables such as output per worker, hours worked, and human capital.\(^1\)

The current paper is part of a broader research program that asks what economic forces are behind structural transformation. Herrendorf et al. (2013b) addressed the preference aspect of this question and quantified the importance of the effects of changes in income and relative prices for changes in the composition of households’ consumption bundles. In the current paper, we focus on the technology aspect and ask how important for structural transformation are differences across sectors in technical progress, the capital share parameter (i.e., the weight attached to capital), and the substitutability between capital and labor.\(^2\)

There are two different views in the literature about this question. Most papers on structural transformation use sectoral production functions of the Cobb–Douglas form with equal capital–share parameter, which differ only in technical progress. The advantage of this way of proceeding is that under the additional assumptions of perfect competition and profit maximization, the capital–share parameter equals the share of capital in aggregate income (capital share for short), which is easily calculated. The potential disadvantage of this way of proceeding is that it restricts our attention to forces behind structural transformation that result directly from technical progress, namely increases in income and changes in relative prices. These forces lead to structural transformation because the households response to them by changing the sectoral composition of GDP; see Herrendorf et al. (2013b) for further discussion.

Some contributions to the recent literature on structural transformation suggest that sectoral differences in the capital–share parameter and the substitutability between capital and labor

\(^1\)The recent literature started with Echevarria (1997) and Kongsamut et al. (2001); see Herrendorf et al. (2013a) for a review.

\(^2\)Some writers refer to sectors with a high capital–share parameter as having a high capital–to–labor ratio or as being capital intensive. We refrain from using this terminology because the capital–to–labor ratio and the capital intensity are not primitives of the technology but endogenous variables.
may also have important implications for structural transformation. To see how these features of technology may matter for structural transformation, suppose first that labor–augmenting technical progress is even across sectors and compare two sectoral production functions that only differ in the capital–share parameter. If GDP per capita is relatively low, then capital is relatively scarce compared to labor, the rental price of capital relative to the rental price of labor is relatively high, and the relative price of the output of the sector with the higher capital–share parameter is relatively high. As technical progress takes place, GDP per capita increases, capital becomes less scarce compared to labor, and the relative price of the output of the sector with the higher capital–share parameter falls. Given standard preferences, this leads to the reallocation of resources towards this sector. Acemoglu and Guerrieri (2008) emphasized this economic force behind structural transformation.

Suppose instead that the sectoral production functions only differ in the substitutability between capital and labor (and that technical progress again is even). If GDP per capita is relatively low, then capital is again scarce and the relative price of the output of the sector with the low substitutability between capital and labor is relatively high. As technical progress takes place, GDP per capita again increases and the relative price of the output of the sector with low substitutability falls. Given standard preferences, this again leads to the reallocation of resources towards this sector. Alvarez-Cuadrado et al. (2013) emphasized this economic force behind structural transformation.

In order to assess how quantitatively important these two additional features of sectoral technology are in translating given paths of technical progress into paths for structural transformation, we estimate CES production functions for agriculture, manufacturing, and services. Given data limitations, we focus on the postwar US. To measure the contribution of each feature of technology, we also estimate Cobb–Douglas production functions with sector–specific capital shares and with a common capital share. For all estimations, we assume that technical progress is exogenous and grows exponentially, that the growth rates are sector specific and constant, and that technical progress augments capital as well as labor. We then endow competitive stand–in firms in each sector with the estimated technologies and ask how well their optimal choices replicate the observed allocation of labor across sectors, which is the most
widely available measure of sectoral activity, and the changes in sectoral relative prices.

The estimation of the sectoral CES production functions yields the following results. First, labor–augmenting technical progress is quantitatively much more important than capital–augmenting technical progress, and at the aggregate level, capital–augmenting technical progress is not statistically different from zero; labor–augmenting technical progress is fastest in agriculture and slowest in services, and the differences in the growth rates are sizeable. Second, agriculture has the highest capital–share parameter, services have the second–highest capital–share parameter, and manufacturing has the lowest capital–share parameter. The finding that services have a higher capital–share parameter than manufacturing is due to the fact that services include owner–occupied housing. Third, capital and labor are most substitutable in agriculture and least substitutable in services; moreover, in agriculture capital and labor are more substitutable than in the Cobb–Douglas case and in manufacturing and services they are less substitutable. The finding that in agriculture capital and labor are more substitutable than in the Cobb–Douglas case is consistent with the view that after the second world war a mechanization wave led to massive substitution of capital for labor in US agriculture; see for example Schultz (1964).

In order to assess how quantitatively important the different features of the estimated sectoral production functions are for structural transformation, we compare the predicted trends of sectoral labor and relative prices with those of Cobb–Douglas production functions that have different capital shares and Cobb–Douglas production functions that have equal capital shares. In both cases, we feed in the estimated technical progress at the sectoral level. We find that uneven technical progress is the dominant quantitative force behind these trends, whereas sectoral differences in the capital–share parameters and substitution elasticities do not lead to quantitatively important additional effects. As a result, Cobb–Douglas sectoral production functions that differ only in technical progress capture the main forces behind the structural transformation of the postwar US economy that arise on the technology side. These findings lend support to the assumption made by Ngai and Pissarides (2007) who used Cobb–Douglas production functions with equal capital shares and analyzed theoretically the implications of differences in labor–augmenting technical progress across sectors.

Our findings do not mean that always and everywhere the Cobb–Douglas production func-
tion with equal factor shares is the best modeling choice. For example, if one is interested in the
level of employment in agriculture instead of secular changes in employment, then it is impor-
tant to model that the employment share in agriculture is much lower than in the other sectors.
A Cobb Douglas production function with equal shares would overpredict the level of employ-
ment in agriculture considerably, even though it does capture the main changes in employment.
Moreover, one should keep in mind that our results are obtained for the postwar period, dur-
ing which the US was fairly developed and agriculture had a relatively small shares in overall
employment and value added. It would therefore be premature to conclude that Cobb–Douglas
sectoral production functions will do a good job at capturing structural transformation also in
less developed economies in which agriculture has much bigger employment and value added
shares than in the US economy.

This paper belongs to a large literature that estimates CES production functions at the ag-
grate level, the industry level, or the firm level. Antràs (2004), Klump et al. (2007) and
León-Ledesma et al. (2010) are the contributions to this literature which are most closely re-
lated to our work. These authors revisited the question how substitutable capital and labor are
at the level of the aggregate US economy and found that they are less substitutable than in
the Cobb Douglas case. We broadly follow the methodology developed by León-Ledesma et
al. (2010), who argued that the elasticity of substitution is best estimated from the non–linear
system of equations composed of the CES production function and the two associated first–
order conditions for capital and labor. A key ingredient of their argument was to normalize the
CES production function before estimating it such that the capital share parameter equals the
average capital share and can be calibrated directly from the data. We make a methodologi-
cal contribution by deriving a normalization that holds exactly, instead of approximately as in
León-Ledesma et al. (2010). We apply the improved methodology at the disaggregate level of
the three broad sectors that are relevant in the context of structural transformation and at the
level of the aggregate US economy. We demonstrate that at the aggregate level our estimates of
the parameter values are similar as those obtained by the existing literature.

The remainder of the paper is organized as follows. In Section 2 we introduce the concept
of value–added production functions. Section 3 derives the equations to estimate, discusses
the issues that arise in the estimation, and describes the data that we use. In Section 4, we present the estimation results and in Section 5 we compare the performance of CES production function with the performance of Cobb–Douglas production functions. Section 6 discusses the implications of our results for building multi–sector models, and section 7 concludes.

2 Value–added Production Functions

We start with the question of whether to write production functions in gross–output form or in value–added form. Since gross output equals the sum of value added and intermediate inputs (all expressed in current prices), the difference between the two possibilities lies in whether one counts everything that the sector produces (“gross output”) or whether one counts only what the sector produces beyond the intermediate inputs that it uses (“value added”). To appreciate the difference between the two possibilities, it is useful to start with the aggregate production function. In a closed economy, GDP equals value added by definition. Therefore, GDP $G$ is ultimately produced by combining domestic capital $K$ and labor $L$. Many authors therefore specify the aggregate production function as a value–added production function:

$$ G = F(K, L) $$

In an open economy, GDP is in general not equal to domestic value added because some intermediate inputs are not produced domestically but are imported from other countries. Therefore, GDP is ultimately produced with domestic capital, labor, and imported intermediate inputs $Z$:

$$ G = H(K, L, Z) $$

While imported intermediate inputs are often abstracted from, they can be quantitatively important, in particular in small open economies that import most of the resources and many of the agricultural and manufactured intermediate goods that they use.

Turning now to sectoral production functions, the question which type of production functions to use arises even in a closed economy. The reason for this is that a typical sector uses
intermediate inputs from other sectors, and so sectoral output does not equal sectoral value added in general. Therefore, it is natural to start with a production function for gross output and ask under what conditions a production function for value added exists.

Denoting the sector indexes for agriculture, manufacturing, and services by \( i \in \{a, m, s\} \), the production function for sectoral gross output can be written as:

\[
G_i = H_i(K_i, L_i, Z_i)
\]

The question we ask here is under which conditions do value–added production functions \( F_i(K_i, L_i) \) exist such that sectoral value added is given by:

\[
Y_i \equiv \frac{P_{gi}H_i(K_i, L_i, Z_i) - P_{zi}Z_i}{P_{yi}} = F_i(K_i, L_i)
\]

(1)

where \( P_{gi}, P_{zi}, \) and \( P_{yi} \) denote the prices of gross output, intermediate inputs, and value added (all expressed in current dollars).

Sato (1976) showed that a value added production function exists if there is perfect competition and if the other input factors are separable from intermediate inputs, that is, the gross–output production function is of the form

\[
G_i = H_i(F_i(K_i, L_i), Z_i)
\]

(2)

where \( H_i \) and \( F_i \) satisfy the usual regularity conditions, that is, they are positive, finite, twice continuously differentiable, monotonically increasing in both arguments, strictly concave, homogeneous of degree one, and satisfy the Inada conditions. To understand Sato’s argument, consider the problem of a stand–in firm that takes prices and gross output as given and chooses capital, labor, and intermediate inputs to minimize its costs subject to the constraint that it produces the given output:

\[
\min_{K_i, L_i, Z_i} R_i K_i + W_i L_i + P_{zi}Z_i \quad \text{s.t.} \quad H_i(F_i(K_i, L_i), Z_i) \geq G_i
\]

(3)

where \( R_i \) and \( W_i \) denote the rental rates for capital and labor, both expressed in current dollars.
The first–order conditions for an interior solution to this problem imply:

\[
P_{yi} = \lambda_i \frac{\partial H_i(F_i(K_i, L_i), Z_i)}{\partial Y_i} 
\]

(4)

\[
R_i = \lambda_i \frac{\partial H_i(F_i(K_i, L_i), Z_i)}{\partial Y_i} \frac{\partial F_i(K_i, L_i)}{\partial K_i}
\]

(5)

\[
W_i = \lambda_i \frac{\partial H_i(F_i(K_i, L_i), Z_i)}{\partial Y_i} \frac{\partial F_i(K_i, L_i)}{\partial L_i}
\]

(6)

where \(\lambda_i\) is the multiplier on the constraint. Substituting the first equation into the second and third equation gives:

\[
R_i = P_{yi} \frac{\partial F_i(K_i, L_i)}{\partial K_i}
\]

(7)

\[
W_i = P_{yi} \frac{\partial F_i(K_i, L_i)}{\partial L_i}
\]

(8)

The envelope theorem implies that the multiplier on the constraint equals the price of value added \(P_{yi}\). Therefore, these conditions are also the first–order conditions for an interior solution to the problem of a stand–in firm that takes prices and value added as given and chooses capital and labor to minimize its costs subject to the constraint that it produces the given value added:

\[
\min_{K_i, L_i} R_i K_i + W_i L_i \quad \text{s.t.} \quad F_i(K_i, L_i) \geq Y_i
\]

(9)

The question remains if condition (2) holds for the postwar US economy. A sufficient (but not necessary) condition is that the sectoral production function is of the Cobb–Douglas form.

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\(3\)To obtain (4), we start with the interior first–order condition for the optimal choice of \(Z_i\):

\[
P_{zi} = \lambda_i \frac{\partial H_i}{\partial Z_i}
\]

The assumption that \(H\) is homogeneous of degree one implies that

\[
G_i = Y_i \frac{\partial H_i}{\partial Y_i} + Z_i \frac{\partial H_i}{\partial Z_i}
\]

Solving this equation for \(\partial H_i/\partial Z_i\), substituting the result into the first–order condition for \(Z_i\), and rearranging gives:

\[
Y_i = \frac{\lambda_i G_i - P_{zi} Z_i}{\lambda_i \partial H_i / \partial Y_i}
\]

(4) then follows by comparing the denominator of this equation with the denominator of (1).
between value added and intermediate inputs:

\[ G_i = [F_i(K_i, L_i)]^{\eta_i}Z_i^{1-\eta_i} \]  

(10)

In this case, perfect competition implies that the share of intermediate inputs is constant over time. Figure 1 plots the intermediate good shares for the postwar US economy. We can see that none of them has a pronounced long–run trend, which is captured by the Cobb–Douglas form (10). An additional piece of evidence in favor of the Cobb–Douglas form is that when we regress the changes in the intermediate good share of a given sector on the changes in the price of intermediate goods relative to value added in that sector, the regression coefficient is not significant.\(^4\) We interpret these pieces of evidence to mean that the functional form (10) is a reasonable starting point when one is interested in long–run secular trends that the literature on structural transformation focuses on. We will therefore proceed under the assumption that sectoral value–added production functions exist. In the next section, we will discuss the issues involved in estimating them.

\(^4\)For this exercise we use postwar US data from WorldKLEMS. We thank an anonymous referee for suggesting to do this.
3 Estimating Sectoral Production Functions

3.1 Theory

We focus on the class of CES production functions that was introduced to economics by Arrow et al. (1961):

\[ F_i(K_{it}, L_{it}) = A_i \left[ \alpha_i \left\{ \exp(\gamma_{ik} \cdot t)K_{it} \right\}^{\sigma_i^{-1}} + \left(1 - \alpha_i\right) \left\{ \exp(\gamma_{il} \cdot t)L_{it} \right\}^{\sigma_i^{-1}} \right]^{\sigma_i} \]  

where \( i \in \{a, m, s\} \) denotes the sector, \( A_i \) is TFP, \( \sigma_i \) is the (constant) elasticity of substitution between capital and labor, \( \gamma_{ik} \) and \( \gamma_{il} \) are the growth rates of capital– and labor–augmenting technical progress, and \( \alpha_i \) is the capital-share parameter. For \( \sigma_i \to 1 \), the CES production function (11) converges to the Cobb–Douglas production function:

\[ F_i(K_{it}, L_{it}) = A_{it} \left( K_{it} \right)^{\alpha_i} \left( L_{it} \right)^{1-\alpha_i} \]  

where \( A_{it} = A_i \exp([\alpha_i \gamma_{ik} + (1-\alpha_i) \gamma_{il}] \cdot t) \). Note that if we make the standard assumptions that there is perfect competition in factor and product markets and that firms minimize costs, then the capital–share parameter \( \alpha_i \) in (12) equals the income share of value added paid to capital (or capital share for short).

One might worry that assuming constant technical progress is overly restrictive. However, as we will see below, the estimated production functions do a good job at fitting the secular trends of sectoral capital, employment and relative prices. Assuming constant technical progress plays an important role for identifying the parameters of the model. To see why, consider the extreme opposite case where the \( \gamma_i \)'s may change freely over time. We could then fit the data irrespective of the values of \( \sigma_i \). Even in the extreme case of a Leontief production function that allows for no substitutability between capital and labor, we could rationalize years with low capital–to–labor ratios by choosing high \( \gamma_{ik}/\gamma_{il} \) and years with high capital–to–labor ratios by choosing low \( \gamma_{ik}/\gamma_{il} \).  

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5 In contrast, Jorgenson et al. (1987) estimated translog production functions for 45 disaggregate US industries during 1948–79. Since a translog is a Taylor–series approximation of the unknown production function, the parameters of translogs are not “deep” and the elasticity of substitution is not constant. Translogs are therefore not very useful for general equilibrium models that require calibration, although they are often preferred when flexibility is valued in empirical work.
ratios by choosing low $\gamma_{ik}/\gamma_{il}$. In other words, $\sigma_{i}$ would not be identified. This, of course, is an intuitive restatement of the impossibility theorem of Diamond et al. (1978).

We assume that there is perfect competition in product and factor markets and that each sector has a stand–in firm that minimizes costs. The firm takes as given value added, price of value added, and the rental rates for the production factors and chooses capital and labor to minimize its costs subject to the constraint that it produce at least the given value added. We denote the price of value added in sector $i$ by $P_{yit}$ and the rental rates in term of sector $i$’s value added by $r_{it}$ and $w_{it}$:

$$r_{it} \equiv \frac{R_{it}}{P_{yit}}$$

$$w_{it} \equiv \frac{W_{it}}{P_{yit}}$$

The problem of the stand–in firm can then be written as:

$$\min_{K_{it}, L_{it}} r_{it}K_{it} + w_{it}L_{it} \quad \text{s.t.} \quad F_{i}(K_{it}, L_{it}) \geq Y_{it} \quad (13)$$

The first–order conditions for an interior solution to this problem are:

$$r_{it} = \alpha_{i} \exp(\gamma_{ik} \cdot t) \frac{\sigma_{i}^{-1}}{\sigma_{i}} \left( \frac{Y_{it}}{K_{it}} \right)^{\frac{1}{\sigma_{i}}} \quad (14)$$

$$w_{it} = (1 - \alpha_{i}) \exp(\gamma_{il} \cdot t) \frac{\sigma_{i}^{-1}}{\sigma_{i}} \left( \frac{Y_{it}}{L_{it}} \right)^{\frac{1}{\sigma_{i}}} \quad (15)$$

For future reference, note that these first–order conditions imply that the income shares of the production factors are given as:

$$\theta_{it} \equiv \frac{r_{it}K_{it}}{Y_{it}} = \alpha_{i} \left[ \exp(\gamma_{ik} \cdot t) \frac{K_{it}}{Y_{it}} \right]^{\frac{\sigma_{i}^{-1}}{\sigma_{i}}} \quad (16)$$

$$1 - \theta_{it} \equiv \frac{w_{it}L_{it}}{Y_{it}} = (1 - \alpha_{i}) \left[ \exp(\gamma_{il} \cdot t) \frac{L_{it}}{Y_{it}} \right]^{\frac{\sigma_{i}^{-1}}{\sigma_{i}}} \quad (17)$$

For estimation purposes it is advantageous to normalize the CES production function (11)
as follows:

\[
\bar{F}_i(K_{it}, L_{it}) = \bar{Y}_i \left[ \bar{\theta}_i \left( \frac{\exp(\gamma_{ik} \cdot t) K_{it}}{\exp(\gamma_{ik} \cdot \tilde{t}) \bar{K}_i} \right)^{\sigma_i^{-1}} + (1 - \bar{\theta}_i) \left( \frac{\exp(\gamma_{il} \cdot t) L_{it}}{\exp(\gamma_{il} \cdot \tilde{t}) \bar{L}_i} \right)^{\sigma_i^{-1}} \right]^{\sigma_i} \tag{18}
\]

where \(\bar{Y}_i, \bar{K}_i\) and \(\bar{L}_i\) are the geometric averages of output, capital and labor over the sample period; \(\tilde{t}\) is the arithmetic average of the time index; \(\bar{\theta}_i\) and \(1 - \bar{\theta}_i\) are the geometric averages of the income shares of capital and labor, respectively:

\[
\bar{\theta}_i = \alpha_i \left[ \exp \left( \gamma_{ik} \cdot \tilde{t} \right) \frac{\bar{K}_i}{\bar{Y}_i} \right]^{\sigma_i^{-1}}
\]

\[
1 - \bar{\theta}_i = (1 - \alpha_i) \left[ \exp \left( \gamma_{il} \cdot \tilde{t} \right) \frac{\bar{L}_i}{\bar{Y}_i} \right]^{\sigma_i^{-1}}
\]

Since the income shares are observed, the geometric averages are readily calculated and we can calibrate \(\bar{\theta}_i\) and \(1 - \bar{\theta}_i\) before estimating the other parameters. León-Ledesma et al. (2010) demonstrated that in practice this way of proceeding simplifies the estimation of the other parameters.

The first–order conditions (14)–(15) can also be rewritten in normalized form:

\[
r_{it} = \bar{\theta}_i \bar{Y}_i \bar{K}_i \exp \left( \frac{\sigma_i - 1}{\sigma_i} \gamma_{ik}(t - \tilde{t}) \right) \left( Y_{it} K_{it} \right)^{\frac{1}{\sigma_i}} \tag{19}
\]

\[
w_{it} = (1 - \bar{\theta}_i) \bar{Y}_i \bar{L}_i \exp \left( \frac{\sigma_i - 1}{\sigma_i} \gamma_{il}(t - \tilde{t}) \right) \left( Y_{it} L_{it} \right)^{\frac{1}{\sigma_i}} \tag{20}
\]

We are going to estimate the system (18)–(20) for each sector \(i\). We add an error term to each of these equations, which we think of as productivity shocks or measurement error that may be

\[\footnote{Note that while \(\theta_i\) and \(1 - \theta_i\) add up to one, \(\bar{\theta}_i\) and \(1 - \bar{\theta}_i\) do not in general because the bars refer to geometric averages. This is not an issue though because one can rescale the weights in the CES production function such that they add up to one. Moreover, since the exponents of the limiting Cobb–Douglas case turn out to be \(\tilde{\theta}_i/(\bar{\theta}_i + 1 - \tilde{\theta}_i)\) and \(1 - \tilde{\theta}_i/(\bar{\theta}_i + 1 - \tilde{\theta}_i)\), they add up to one irrespective of whether the weights in the CES case do. For further discussion about calibrating and normalizing CES production functions, see Temple (2012).}

\[\footnote{Note that while our normalization uses geometric averages of capital, labor, value added, and the income shares, León-Ledesma et al. (2010) used arithmetic averages. This difference implies that in our paper (18) holds exactly whereas in their paper it holds only approximately.}
correlated over time. Taking logs and rearranging gives:

\[
\log\left(\frac{Y_{it}}{Y_i}\right) = -\frac{\sigma_i}{\sigma_i - 1} \log \left[ \tilde{\theta}_i \left( \exp(\gamma_{ik}(t - \bar{t})) \frac{K_{it}}{K_i} \right)^{\frac{\sigma_i - 1}{\sigma_i}} + \left(1 - \tilde{\theta}_i\right) \left( \exp(\gamma_{il}(t - \bar{t})) \frac{L_{it}}{L_i} \right)^{\frac{\sigma_i - 1}{\sigma_i}} \right] + \epsilon_{yit} \tag{21}
\]

\[
\log(r_{it}) = \log \left( \frac{\tilde{\theta}_i Y_i}{K_i} \right) + \frac{\sigma_i - 1}{\sigma_i} \left[ \gamma_{ik}(t - \bar{t}) + \frac{1}{\sigma_i} \log \left( \frac{Y_{it}}{Y_i} \frac{K_{it}}{K_i} \right) \right] + \epsilon_{rit} \tag{22}
\]

\[
\log(w_{it}) = \log \left( \frac{(1 - \tilde{\theta}_i) Y_i}{L_i} \right) + \frac{\sigma_i - 1}{\sigma_i} \left[ \gamma_{il}(t - \bar{t}) + \frac{1}{\sigma_i} \log \left( \frac{Y_{it}}{Y_i} \frac{L_{it}}{L_i} \right) \right] + \epsilon_{wit} \tag{23}
\]

where \( (\epsilon_{yit}, \epsilon_{rit}, \epsilon_{wit}) \) denote errors. To avoid confusion, note that the right-hand side of the first equation remains well behaved even as \( \sigma_i \to 1 \). The reason for this is that it converges to the log of the Cobb–Douglas limit of (18).

As mentioned above, \( \tilde{\theta}_i \) and \( 1 - \tilde{\theta}_i \) equal the average income shares of capital and labor in sector \( i \), which we calculate directly from the data according to the method of Gollin (2002). Given the values of \( \tilde{\theta}_i \) and \( 1 - \tilde{\theta}_i \), we estimate \( \sigma_i, \gamma_{ik}, \) and \( \gamma_{il} \) from (21)–(23) for the three sectors. In order to tie our work to the literature, we also estimate \( \sigma, \gamma_k, \) and \( \gamma_l \) for the aggregate economy and compare the results with those in the literature. For the aggregate economy this results in a three-equation system and for the sectoral estimation in a nine-equation system with three equations for each of the three sectors. By estimating the equations for the three sectors together, we allow for the possibility that the error terms across equations and sectors are correlated.

The system (21)–(23) has several endogenous variables on the right-hand side. To begin with, \( Y_{it} \) shows up on the left-hand side of (21) and on the right-hand sides of (22) and (23). Moreover, the theory laid out above implies that \( K_{it} \) and \( L_{it} \) are endogenous, as they are chosen by competitive stand-in firms. To address the endogeneity issue, we follow León-Ledesma et al. (2013) and use one-period lagged values (appropriate to each sector or the aggregate economy) of log rental rates of capital and labor, log normalized value added, log normalized capital, log normalized labor and the time trend. To be valid instruments, the lags need to satisfy two conditions: (i) they are correlated with the variable for which they instrument; (ii) they are uncorrelated with the unobserved determinants of the dependent variable. If the one-period lagged values are correlated with the contemporaneous values, then condition (i) is met.
This is the case in our data. However, the one-period lagged values are also correlated with the contemporaneous error terms, because, as it will turn out, the error terms follow an AR(1) processes:

\[ \epsilon_{jit} = \rho_{ji} \epsilon_{jit-1} + v_{jit} \]

where \( \rho \in (-1, 1) \) and \( v \) is a current-period innovation. Since our instruments are lagged values of the right-hand side variables, they are predetermined in the current period and are uncorrelated with \( v_{jit} \) under the standard assumption that \( v_{jit} \) is i.i.d. with mean zero and finite variance. Condition (ii) is then also met and the lagged values of the right-hand side variables are valid instruments.

We estimate the system (21)–(23) via the non-linear, feasible, generalized three-stage least squares estimation routine offered by Eviews. The first stage obtains the instruments by running a linear least squares regression of the endogenous right-hand side variables on their one-period lags and time trends. The second stage is a non-linear least squares regression with the instruments as the right-hand side variables. This stage takes into account the AR(1) structure of the error terms via the Cochrane–Orcutt procedure.\(^8\) The third stage uses the estimated error terms from the previous stage to correct for heteroscedasticity and cross-equation correlation of the error terms using the non-linear, feasible, generalized least square estimator. Since the estimation in the second stage is non-linear, the results are obtained numerically and so may depend on the initial conditions. We vary the initial parameter values widely. If different initial parameter values result in different parameter estimates, then we choose the one with the smallest log determinant of the residual covariance matrix.

The system (21)–(23) features several non stationary variables (\( Y_{it}, K_{it}, L_{it}, \) and \( \log(w_{it}) \)) and two trends governed by \( \gamma_{ki} \) and \( \gamma_{li} \). In general, the assumptions upon which classical regression analysis rests are violated for nonstationary time series. However, standard growth models of structural transformation imply that our non-stationary variables have deterministic trends. Classical inference is still valid if the nonstationary variables are “trend–stationary”, that is, each of them has a deterministic trend and the deviations from trend are stationary. We estimate

\(^8\)Using boldfaced symbols to denote vectors and matrices, the system (21)–(23) can be written as \( y_t = h(x_t) + \epsilon_t \). Eviews estimates the system as \( y_t = h(x_t) + \rho[y_{t-1} - h(x_{t-1})] + v_t \).
our system of equations under the assumption that our variables have deterministic trends. What matters for classical regression analysis is that the innovations to these autoregressive error terms are stationary. We test whether this is the case.

### 3.2 Data

We use annual US data for the period 1948–2010. We start in 1948 because before 1948 hours worked by sector are not available. We use the North American Industrial Classification (NAICS) to the extent possible and define the three broad sectors in the obvious way: agriculture comprises farms, fishing, forestry; manufacturing comprises construction, manufacturing, and mining; services comprise all other industries (i.e. education, government, real estate, trade, transportation, etc.).

We obtain nominal and real value added from the BEA’s “GDP–by–Industry” tables. An issue arises in agriculture because NIPA reports “Rent paid to nonoperator landlords” as value added in the real estate industry although conceptually it is value added generated in agriculture. We therefore add “Rent Paid to Nonoperator Landlords” (as reported by the BEA in NIPA Table 7.3.5 “Farm Sector Output, Gross Value Added, and Net Value Added”) to the value added of agriculture and subtract it from the value added of services. Since the BEA does not publish the quantity of value added at the level of our broad sectors, we have to construct the sectoral quantities from the underlying BEA data ourselves. An additional complication arises when doing this because the reported real quantities are constructed according to the chain–weighted method and thus are not additive. We use the so called cyclical expansion procedure to calculate real quantities of sectoral aggregates; see the Appendix A for the details.

We calculate the capital stocks by sector from the BEA’s “Fixed Asset” tables, which contain the year–end current cost and quantity index in 2005 prices of the net stock of fixed assets. The capital stocks during year $t$ are the geometric average of the year–end capital stocks in $t − 1$ and $t$, again using the cyclical expansion procedure to aggregate real capital stocks to the sectoral level. Since the BEA does not include agricultural land in its fixed assets, we construct

---

9 Although industry might seem a better term for the sector comprising construction, manufacturing, and mining, we use the term manufacturing because industry also refers to a generic production category.
capital in agriculture by aggregating capital and land following the methodology of Jorgenson and Griliches (1967). The data for the quantity of agricultural land in acres are from “Land in Farms” and “Farm Real Estate Values” tables of the “U.S. and State Farm Income and Wealth Statistics” tables from the U.S. Department of Agriculture (USDA). To aggregate capital and land, we use the rental rates for the their services. Note that this does not require them to be perfect substitutes, but requires that aggregate capital in agriculture $K_a$ is separable from labor:

$$Y_a = F_a(K_a, L_a)$$

where $K_a = f_a(K_{1a}, K_{2a})$, $f$ is a production functions with the standard regularity conditions (differentiability, constant returns etc), and $K_{1a}$ and $K_{2a}$ denote reproducible capital and fixed capital (i.e., land) in agriculture. If $K_{1a}$ and $K_{2a}$ are paid their marginal products, which is implied by our maintained assumptions of perfect competition and cost minimization, then constant returns imply that $K_a = R_1K_{1a} + R_2K_{2a}$ where $R_i$ are the corresponding rental rates.\(^{10}\)

We calculate sectoral labor inputs as hours worked by persons engaged. The principle data sources are the BEA’s “Income–and–Employment–by–Industry” Tables, which contain information about hours worked by full– and part–time employees by industry, full–time equivalent employees by industry, self–employed persons by industry, and persons engaged in production by industry. Unfortunately, these tables change the industry classification system: SIC72 applies to 1948–1987, SIC87 to 1987–1997, and NAICS to 1998–2010. Fortunately, the “GDP–by–Industry Tables” tables report full– and part–time employees by industry consistently according to NAICS throughout the whole period. We merge the two data sources using the GDP–by–Industry Tables for full– and part–time employees by industry and using the “Income–and–Employment–by–Industry” Tables for all other statistics. Appendix B contains the details.

Lastly, we calculate the rental prices of the factors of production by sector according to

$$r_{it} = \frac{\theta_i Y_{it}}{K_{it}}$$

$$w_{it} = \frac{(1 - \theta_i)Y_{it}}{L_{it}}$$

where, as before, $\theta_i$ denotes the share of capital income in sector’s $i$ value added. Given that we

\(^{10}\)Appendix C contains a more detailed discussion of how we obtain aggregate capital in agriculture.
Table 1: Estimation Results

<table>
<thead>
<tr>
<th></th>
<th>Aggregate</th>
<th>Agriculture</th>
<th>Manufacturing</th>
<th>Services</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>0.84**</td>
<td>1.58**</td>
<td>0.80**</td>
<td>0.75**</td>
</tr>
<tr>
<td>(0.041)</td>
<td>(0.068)</td>
<td>(0.015)</td>
<td>(0.020)</td>
<td></td>
</tr>
<tr>
<td>$\gamma_k$</td>
<td>-0.010</td>
<td>0.023**</td>
<td>-0.045**</td>
<td>-0.002</td>
</tr>
<tr>
<td>(0.006)</td>
<td>(0.003)</td>
<td>(0.009)</td>
<td>(0.004)</td>
<td></td>
</tr>
<tr>
<td>$\gamma_l$</td>
<td>0.022**</td>
<td>0.050**</td>
<td>0.044**</td>
<td>0.016**</td>
</tr>
<tr>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.007)</td>
<td>(0.002)</td>
<td></td>
</tr>
<tr>
<td>$\tilde{\theta}$</td>
<td>0.33</td>
<td>0.61</td>
<td>0.29</td>
<td>0.34</td>
</tr>
</tbody>
</table>

Standard errors in parentheses; ** $p < 0.01$

have already described the construction of $Y$, $K$ and $L$, we only need to describe the calculation of the factor shares. We split value added reported in the BEA’s “Components–of–Value–Added–by–Industry” Tables in the standard way: “Compensation of Employees” is labor income; “Gross Operating Surplus minus Proprietors’ Income” is capital income; proprietors’ income is split into capital and labor income according to above shares. In the case of agriculture, we add “Rent Paid to Nonoperator Landlords” to “Gross Operating Surplus minus Proprietors’ Income” since it is capital income. An issue arises because again the industry classification in these tables changes twice. We calculate the sectoral capital shares for each subperiod during which the classification remains unchanged and assume that the same share applies to the corresponding NAICS classifications as well. Since our three sectors are fairly broad, this is unlikely to affect our results in a quantitatively important way.

4 Estimation Results

Table 1 reports the estimation results. Appendix C contains further information, which shows that the fit is good, and it reports multivariate Ljung–Box Adjusted Q–statistics, which test for autocorrelation in the residuals. The null hypothesis that the error terms are serially uncorre-
lated is not rejected, i.e. there is no strong evidence against it. To conserve space we only report the test statistics for the second lag, but the existence of higher order autocorrelation is also strongly rejected. We find that capital and labor are most substitutable in agriculture and least substitutable in services. In agriculture capital and labor are more substitutable than in the Cobb–Douglas case, which is consistent with the view that a mechanization wave led to massive substitution of capital for labor in agriculture after World War II. In manufacturing and services capital and labor are less substitutable than Cobb–Douglas. On the aggregate, we find that capital and labor are less substitutable than Cobb–Douglas, which is consistent with the previous results of Antràs (2004), Klump et al. (2007) and León-Ledesma et al. (2010).

Labor–augmenting technical progress is fastest in agriculture and slowest in services and the differences in the growth rates of technical progress are sizeable: in agriculture technical progress grew by 5.0% per year, whereas in manufacturing it grew by 4.4% and in services it grew by just 1.6%; these growth rates result in an average of 2.2% annual growth of aggregate labor–augmenting technical progress. The fact that technical progress is slowest in services while the share of value added produced in services is growing is sometimes referred to as Baumol “disease”. Baumol (1967) was the first to point out that these two facts imply decreasing growth rates of real GDP. Moreover, if the current trends of structural transformation continue, then services will dominate the economy in the limit and aggregate labor-augmenting technical progress will fall to the lower technical progress in services.

We find mixed results regarding capital–augmenting technical progress. At the aggregate it is negative but not significant.\textsuperscript{11} At the sectoral level, capital–augmenting technical progress is significantly different from zero in agriculture and manufacturing and not significantly different from zero in services. Moreover, in agriculture capital–augmenting technical progress is positive and in manufacturing it is negative and the negative growth rate in manufacturing is relatively large. At first sight, negative technical progress in manufacturing is challenging to interpret. However, if one thinks of the decline of sizeable parts of US manufacturing during the postwar period, then negative technical progress in manufacturing may just reflect that the BEA underestimated the depreciation of manufacturing capital. Since this issue is not central

\textsuperscript{11}Antràs (2004) studies the aggregate US production function during the period 1948–1998 and also finds that capital–augmenting technical progress was negative.
to our study, we leave further investigation of it for future research.

The last row of Table 1 reports $\bar{\theta}$, that is, the average capital share in the post war period. We can see that the aggregate capital share comes out as the standard value of $1/3$. The sectoral capital shares differ from the aggregate capital share: while the agricultural capital share is considerably larger than the aggregate capital share, the capital shares in manufacturing and services are fairly close to the aggregate capital share. The capital share in agriculture is much larger than the other two capital shares because agriculture is intensive in both physical capital and land, which have income shares in agricultural value added equal to 0.54 and 0.07, respectively. The capital share in services is larger than in manufacturing because the owner–occupied housing is part of services and owner–occupied housing is very capital intensive.

5 Sectoral Technology and Structural Transformation

5.1 CES versus Cobb–Douglas production functions

In this section, we evaluate the implications of the different features of sectoral production functions for structural transformation. To this end, we compare the unrestricted CES production functions that we have estimated above with two restricted CES production functions: (i) we impose $\sigma_i = 1$ which results in a Cobb–Douglas production function with sector–specific capital share parameters; (ii) we impose $\sigma_i = 1$ and $\bar{\theta}_i = \bar{\theta}$, which results in Cobb–Douglas production functions with a common capital–share parameter. It is convenient to write (18) in these three cases in the following way:

\[
\tilde{F}_i(K_{iit}, L_{iit}) = \left[\theta_i (A_{ikt} K_{iit})^{\sigma_i - 1 \sigma_i} + (1 - \theta_i) (A_{ilt} L_{iit})^{\sigma_i - 1 \sigma_i} \right]^{\sigma_i - 1 \sigma_i}
\] (24)

\[
\tilde{F}_i(K_{iit}, L_{iit}) = (A_{ikt} K_{iit})^{\bar{\theta}_i} (A_{ilt} L_{iit})^{1 - \bar{\theta}_i}
\] (25)

\[
\tilde{F}_i(K_{iit}, L_{iit}) = (A_{ikt} K_{iit})^{\bar{\theta}} (A_{ilt} L_{iit})^{1 - \bar{\theta}}
\] (26)
where $A_{ikt}$ and $A_{ilt}$ are defined as:

$$A_{ikt} = \exp(\gamma_{ik}(t - \bar{t})\frac{\bar{Y}_i}{\bar{K}_i})$$
and

$$A_{ilt} = \exp(\gamma_{il}(t - \bar{t})\frac{\bar{Y}_i}{\bar{L}_i})$$
if $F_i$ is CES

$$A_{ikt} = \exp(\gamma_i(t - \bar{t})\frac{\bar{Y}_i}{\bar{K}_i})$$
and

$$A_{ilt} = \exp(\gamma_i(t - \bar{t})\frac{\bar{Y}_i}{\bar{L}_i})$$
if $F_i$ is Cobb Douglas

(27)

(28)

The reason for the difference between the two rows is that in the Cobb–Douglas case it is not possible to identify $\gamma_{ik}$ and $\gamma_{il}$ separately so that we are left with just the growth factors of TFP $\gamma_i$. In contrast, for the CES production function, the growth factors $\gamma_i$ cannot be obtained, because the rates of capital– and labor–augmenting technical progress cannot be translated into an observationally–equivalent rate of TFP growth.

We calculate the values of $A_{ijt}$ according to the expressions in (27) and (28). We obtain the geometric averages $\bar{Y}_i$, $\bar{K}_i$, and $\bar{L}_i$ directly from the data. We use the values from Table 1 for $\gamma_{ik}$ and $\gamma_{il}$. We estimate $\gamma_i$ from the output equations (21) for the special case of the Cobb–Douglas production function jointly for the three sectors given the values of the exponents and again assuming AR(1) error terms. Table 2 reports the resulting average annual growth rates of TFP. They are somewhat larger than what other studies tend to find; see for example Jorgenson et al. (1987). One possible reason for this is that we have not taken into account improvements in the quality of sectoral labor (e.g., through increases in years of schooling and experience), which in our estimation shows up as technical progress.

To obtain the capital–share parameters for the Cobb–Douglas production function, we use that under our maintained assumptions of perfect competition in factor and product markets and cost minimization the capital share parameter equals the capital share. One can show that the share parameters of the Cobb-Douglas result from the limits of the share parameters of the CES according to:

$$\tilde{\theta} \equiv \frac{\tilde{\theta}}{\tilde{\theta} + (1 - \tilde{\theta})}$$

and

$$1 - \tilde{\theta} \equiv \frac{1 - \theta}{\theta + (1 - \theta)}$$

As a result, the capital–share parameters of the Cobb–Douglas add up to one although the share parameters of the CES don’t necessarily do.
Table 2: Average Annual Growth Rates of TFP (in %)

<table>
<thead>
<tr>
<th></th>
<th>Aggregate</th>
<th>Agriculture</th>
<th>Manufacturing</th>
<th>Services</th>
</tr>
</thead>
<tbody>
<tr>
<td>CD with $\tilde{\theta}_i$</td>
<td>1.1</td>
<td>3.3</td>
<td>1.5</td>
<td>1.0</td>
</tr>
<tr>
<td>CD with $\tilde{\theta}$</td>
<td>1.1</td>
<td>3.9</td>
<td>1.4</td>
<td>1.0</td>
</tr>
</tbody>
</table>

5.2 Sectoral labor allocations

We now turn to the sectoral labor allocations that result from the optimal choices of stand–in firms which are endowed with the production functions (24)–(26). Solving the first-order conditions to the firm problem for sectoral labor, we obtain for each functional form:

$$L_{it} = \left( \tilde{\theta}_i \left( \frac{\tilde{\theta}_i}{1 - \tilde{\theta}_i} A_{ikt} w_{it} \right)^{1-\sigma_i} + (1 - \tilde{\theta}_i) \right)^{\frac{\sigma_i}{1-\sigma_i}} \frac{Y_{it}}{A_{ilt}}$$ (29)

$$L_{it} = \left( \tilde{\theta}_i \left( \frac{A_{ikt} w_{it}}{1 - \tilde{\theta}_i A_{ilt} r_{it}} \right)^{-\tilde{\theta}_i} \right)^{\frac{\sigma_i}{1-\sigma_i}} \frac{Y_{it}}{A_{ilt}}$$ (30)

$$L_{it} = \left( \tilde{\theta} \left( \frac{A_{ikt} w_{it}}{1 - \tilde{\theta} A_{ilt} r_{it}} \right)^{-\tilde{\theta}} \right)^{\frac{\sigma_i}{1-\sigma_i}} \frac{Y_{it}}{A_{ilt}}$$ (31)

It is worth taking a moment to build intuition for how the different features of technology affect the allocation of labor across the three broad sectors. The term $Y_{it}/A_{ilt}$ is common to the right–hand sides because more labor–augmenting technical progress implies that less labor is needed to produce the given quantity $Y_{it}$ of sectoral value added. The other right–hand–side terms differ among the different functional forms. It is easiest to start with the Cobb–Douglas cases. The term $[\tilde{\theta}/(1 - \tilde{\theta}_i)]^{-\tilde{\theta}_i}$ has a local maximum at $\tilde{\theta}_i = 0.22$, and so it is decreasing $\tilde{\theta}_i \in (0.22, 1)$ which includes the standard value $1/3$. This captures that for empirically relevant values of the capital share, a sector with a larger capital share receives less labor than a sector with a smaller capital share. The term $[(A_{ikt} w_{it})/(A_{ilt} r_{it})]^{-\tilde{\theta}_i}$ captures that an increase in the relative rental rate of capital to labor (where both rental rates are expressed relative to the relevant $A$) leads to a decrease in the sectoral capital–labor ratio and an increase in sectoral labor, which is larger when the sectoral capital share is larger. To see how these features of tech-
nology may matter for structural transformation, suppose first that labor–augmenting technical progress is even across sectors and compare two sectoral production functions that only differ in the capital–share parameter. If GDP per capita is relatively low, then capital is relatively scarce compared to labor, the rental price of capital relative to the rental price of labor is relatively high, and the relative price of the output of the sector with the higher capital–share parameter is relatively high. As technical progress takes place, GDP per capita increases, capital becomes less scarce compared to labor, and the relative price of the output of the sector with the higher capital–share parameter falls. Given standard preferences, this leads to the reallocation of resources towards this sector. Acemoglu and Guerrieri (2008) emphasized this economic force behind structural transformation.

Turning now to the case of the CES production functions, we have an additional substitution effect: if the elasticity of substitution is larger than one, a higher rental rate of capital relative to labor leads to larger reduction of the capital–labor ratio than in the Cobb–Douglas case; if the elasticity of substitution is smaller than one, a higher rental rate of capital relative to labor leads to smaller reduction of the capital–labor ratio than in the Cobb–Douglas case. Hence, if GDP per capita is relatively low, then capital is again scarce and the relative price of the output of the sector with the low substitutability between capital and labor is relatively high. As technical progress takes place, GDP per capita again increases and the relative price of the output of the sector with low substitutability falls. Given standard preferences, this again leads to the reallocation of resources towards this sector. Alvarez-Cuadrado et al. (2013) emphasized this economic force behind structural transformation.¹²

Figure 2 plots the labor allocations that are implied by equations (29)–(31) when we plug in the estimated parameter values for $\sigma_i$ and $\theta_i$, $A_{ikt}$, and $A_{ilt}$ and the data values of the exogenous variables of $Y_{it}$, $r_{it}$, and $w_{it}$. Note that we have divided the hours series from the data and from the model by the hours worked in the data in 1948. This implies that in each sector hours worked in the data in 1948 are equal to one, but hours implied by the model in 1948 are equal to one only if the model gets the level in 1948 right. We can see that all three functional forms

¹²Alvarez-Cuadrado et al. (2013) offer an analytical characterization of the evolution of the capital–labor ratios for the case of two sectors one of which has a Cobb–Douglas production function and the other one has a CES production function.
Figure 2: Hours Worked (Data=1 in 1948)

Cobb Douglas with Different Capital Shares

Cobb Douglas with Same Capital Shares
do a reasonable job at capturing the secular changes in sectoral hours worked.

The main differences between them are that the CES form does marginally better at mimicking the short-run fluctuations in the service sector whereas the Cobb-Douglas production function with unequal shares does somewhat better in mimicking the labor allocations in agriculture and manufacturing.\(^{13}\) The Cobb–Douglas production function with equal shares does very similar to the one with unequal shares in the service and manufacturing sector. Moreover, it does a reasonable job at capturing the secular change in agriculture, but overpredicts the level of employment in agriculture. The root-mean-squared percentage deviations in Table 5 in Appendix D confirm these observations. The reason for the differences in the performance of the two Cobb–Douglas production functions in agriculture is that the one with equal shares misses that agriculture has a much smaller labor share than the aggregate, and so it systematically allocates too much labor to agriculture. Nonetheless, even the Cobb Douglas with equal shares gets the main changes in hours worked mostly right.\(^{14}\) The reason why the CES production function does not outperform the other production functions in agriculture is that it has both by far the largest capital–share parameter and the largest elasticity of substitution. Hence, the effects on structural transformation of the relatively large capital–share parameter and the relatively large elasticities of substitution in agriculture work in opposite directions and largely cancel each other, leaving the effects of uneven labor–augmenting technical progress as the dominating force. All three production functions capture that force.

5.3 Relative prices

We continue by assessing how well we can match relative prices of sectoral value added with the three production functions. Relative prices are of interest in the context of structural transformation because they influence the sectoral composition of value added that is determined by households. To be concrete, the fact that the relative price of agriculture to manufacturing has

\(^{13}\)To avoid confusion, we should emphasize that there is nothing strange about the finding that a Cobb Douglas production function outperforms CES production function regarding sectoral employment. The reason for this is that when we estimated the production function, we did not target the labor allocations.

\(^{14}\)In a different context, Herrendorf and Valentinyi (2012) obtained a similar finding: conducting a growth accounting exercise at the sectoral level, they found that Cobb–Douglas production functions with equal capital shares imply similar values for sectoral TFPs as Cobb–Douglas production functions with sector–specific capital shares.
fallen during the postwar period implies that, everything else the same, the share of agricultural value added has gone up. Herrendorf et al. (2013b) assessed in detail the preference aspect of structural transformation and quantified the importance of the effects of changes in income and relative prices for changes in the composition of household consumption. The goal of this subsection is to assess how well each of the three functional forms does in terms of the implied prices of agriculture and services relative to manufacturing compared to those in the data. We proceed under the maintained assumption that the sectoral stand–in firm behaves competitively and minimizes its costs subject to a production constraint.

The first–order conditions to the firm problem (13) imply that the real wage $w_{it}$ expressed in units of sector $i$’s value added equals the marginal product of labor. Hence, the price of sector $i$’s value added relative to manufacturing is given by:

$$\frac{P_{it}}{P_{mt}} = \frac{W_{it}}{W_{mt}} \frac{MPL_{mt}}{MPL_{it}}$$

(32)

We observe the nominal wages $W_{it}$ and $W_{mt}$ in the data and the model implies the values of the marginal products $MPL_{it}$ and $MPL_{mt}$ as functions of the observed factor prices. Given these, it is straightforward to calculate the implied relative prices from equation (32).

Figure 3 reports the implied relative prices for the three functional forms. In plotting the figure, we have chosen 1948 as the base year, and so by construction the relative prices equal one in the data as well as in the model. We can see that a similar conclusion as for the labor allocation emerges: all three functional forms do reasonably well with respect to changes in the relative prices. In particular, there is little difference among the implied prices of agricultural value added relative to manufacturing value added. The root–mean–squared percentage deviations reported in Table 5 of Appendix D confirm this impression. Interestingly, they are largest for the CES and smallest for the Cobb Douglas with equal capital shares. A similar picture emerges for the implied relative prices of service value added relative to manufacturing value added, except that now all functional forms overpredict the relative price of services after 1970. The root–mean–squared percentage deviations reported in Table 5 of Appendix D show that again the CES does slightly worse than the two Cobb Douglas which are now very
Figure 3: Sectoral Prices Relative to Manufacturing (Data and Model =1 in 1948)

Cobb Douglas with Different Capital Shares

Cobb Douglas with Same Capital Shares
close. Moreover, the Cobb Douglas with unequal capital shares slightly outperforms the one with equal capital shares.\textsuperscript{15}

These findings confirm our conclusion that a Cobb Douglas production function with equal capital shares captures the main technological forces behind structural transformation in the postwar US economy.

6 Implications for Building Multi–sector Models

6.1 Equalizing marginal value products

Many builders of multi–sector models assume that the marginal value products of each primary factor of production are equalized across sectors. A set of assumptions that implies this in multi–sector model is: (i) competitive firms rent each factor of production in a common factor market at a common nominal rental rate; and (ii) each factor of production can be moved across sectors without frictions or costs. Unfortunately, it turns out that in the US the nominal rental rates are not equalized across sectors. Figure 4 shows that the marginal value product of labor is somewhat higher in manufacturing than in services, and is much lower in agriculture than in the other two sectors. Given this evidence, our estimation strategy of system (21)–(23) has been to use the \textit{observed} nominal rental rates and prices of sectoral value added instead of imposing that nominal rental rates are equalized across sectors.

The previous paragraph raises the question, in which way our estimated sectoral production functions may be used for building multi–sector models that equalize marginal value products across sectors. In order to incorporate our estimated production functions in such models, one needs to add a reason for the difference in the marginal value products across sectors. In the case of labor, the most obvious reason is that there are difference in sectoral human capital like in Jorgenson et al. (1987) or Herrendorf and Schoellman (2012). The latter paper, for example, found that average sectoral human capital is lower in agriculture than in the rest of the US economy, and that the difference accounts for almost all of the difference in nominal wages.

\textsuperscript{15}To avoid confusion, we should emphasize that there is nothing strange about the finding that a Cobb Douglas production function outperforms CES production function regarding relative prices. The reason for this is that when we estimated the production function, we did not target the relative prices.
This implies that per efficiency unit of labor the average nominal wages were roughly equal in agriculture and the rest of the US economy during the last thirty years. In the case of capital, the reasons for the difference in the marginal value products across sectors include unmeasured quality differences in the measured stock of sectoral capital and unmeasured parts of the stock of capital; see Jorgenson et al. (1987) and McGrattan and Prescott (2005) for further discussion.

### 6.2 Value–added versus final–expenditure production functions

So far, we have focused on value–added production functions. While this is a natural starting point when one studies the forces behind structural transformation on the technology side, Herrendorf et al. (2013b) pointed out that one can also interpret the sectoral outputs as final goods that are consumed or invested. In this subsection we discuss the implications of our results for models of structural transformation that interpret sectoral outputs as final goods.

Before we delve into the details, an example may be helpful. Consider a household that derives utility from the three consumption categories: agriculture, manufacturing, and services. Herrendorf et al. (2013b) pointed out that one can take two different perspectives on what these categories are: the value–added perspective and the final–goods perspective. The value–added perspective breaks the household’s consumption into the value–added components from the three sectors and assigns each value–added component to a sector. For example, if the house-
hold consumes a cotton shirt, then the value added of producing raw cotton goes to agriculture, the value added of processing to manufacturing, and the value added of distribution to services. This means that the consumption categories in the utility function of the household are the value added that is produced in the three sectors agriculture, manufacturing, and services. In contrast, the final–goods perspective assigns each consumption good to one of the three consumption categories. The cotton shirt, for example, would typically be assigned to manufacturing. This means that the consumption categories in the utility function of the household become final–goods categories. This changes the meaning of the three sectors, as the manufacturing sector now produces the entire cotton shirt, implying that it combines the value added from the different industries that is required to produce the cotton shirt.

Although the sectoral production functions under the two perspectives are very different objects, we emphasize that they are two representations of the same underlying data, which are linked through intricate input–output relationships. To see the implications of this, it is useful to think of the sectoral output under the final–goods perspective as a weighted average of the sectoral value added from the value–added perspective. This implies that the properties of the production function under the final–goods perspective are a weighted average of the properties of the production functions under the value–added perspective. Valentinyi and Herrendorf (2008) showed that as a result the capital shares of industry gross output tend to be closer to the aggregate capital share than the capital shares of industry value added. This implies that the sectoral capital shares under the final–goods perspective are closer to the aggregate capital share than the sectoral capital shares under the value–added perspective. Following the same logic, we conjecture that the differences among the elasticities of substitution of the different sectoral production functions are smaller under the final–goods perspective than under the value–added perspective.

These arguments suggest that under the final–goods perspective the sectoral production functions are at least as close to the Cobb–Douglas production function with a common capital share as under the value–added perspective. Since we have shown above that the Cobb–Douglas production functions with a common capital share do a reasonable job at capturing sectoral employment and relative prices under the value–added perspective, this suggests that they will
also do a reasonable job under the final–goods perspective. Note that since the aggregate cap-
ital share is the same under both perspectives, it is straightforward to parameterize the Cobb–
Douglas production functions with a common capital–share parameter under the final–goods
perspective.

7 Conclusion

In this paper, we have assessed the technical forces behind the reallocation of production factors
across agriculture, manufacturing, and services. In particular, we have asked how important
for structural transformation are sectoral differences in labor–augmenting technical progress,
the capital–share parameter, and elasticity of substitution between capital and labor. We have
estimated CES production and Cobb–Douglas functions for agriculture, manufacturing, and
services on postwar US data and have compared their implications for labor allocations and
relative prices. We have found that differences in labor–augmenting technical progress are
the predominant force behind structural transformation. As a result, sectoral Cobb–Douglas
production functions with equal capital shares (which by construction abstract from differences
in the elasticity of substitution and in capital shares) do a reasonably good job of capturing the
main trends of US structural transformation.

In this paper, we have restricted our attention to the postwar US economy. It is also of
interest to extend this analysis to a larger set of countries, in particular to situations which
feature a larger range of real incomes and a higher share of agricultural employment and value
added. This will be useful in assessing the extent to which one can account for the process of
structural transformation with stable sectoral technologies.

References

Acemoglu, Daron and Veronica Guerrieri, “Capital Deepening and Non–Balanced Economic


Appendix A: Aggregation of Chained Quantity Indices according to the Cyclical Expansion Method

Chain indices relate the value of an index number to its value in the previous period. In contrast, fixed–base indices relate the value of an index number to its value in a fixed base period. While chain indices are preferable to fixed–base indices when relative prices change considerably over time, using them leads to the problem that real quantities are not additive, that is, the real quantity of an aggregate does not equal the sum of the real quantities of its components except in the base year or if relative prices don’t change. In practice, this becomes relevant when one needs to calculate the real quantity of an aggregate, but the statistical agency only reports the real quantities of the components of this aggregate. This appendix explains how to construct the real quantity of the aggregate according to the so called cyclical expansion procedure.\(^{16}\)

Let \(Y_{it}\) be the nominal value, \(y_{it}\) the real value, \(Q_{it}\) the chain–weighted quantity index, and \(P_{it}\) the chain–weighted price index for variable \(i \in \{1, \ldots, n\}\) in period \(t\). Let \(t = b\) be the base year for which we normalize \(Q_{ib} = P_{ib} = 1\). The nominal and real values of variable \(i\) in period \(t\) are then given by:

\[
Y_{it} = P_{it} \frac{Q_{it}}{Q_{ib}} Y_{ib} = P_{it} Q_{it} Y_{ib},
\]

\[
y_{it} = \frac{Y_{it}}{P_{it}} = Q_{it} Y_{ib}.
\]

Let \(Y_t = \sum_{i=1}^{n} Y_{it}\) and suppose that the statistical agency reports \(y_{it}, Q_{it}\) and \(P_{it}\) for all components \(i\) but not \(y_t, Q_t\) and \(P_t\). Since in general \(y_t \neq \sum_i y_{it}\), we need to find a way of calculating \(y_t\).

\(^{16}\)For a more detailed discussion of the practical issues arising from the non–additivity of chain indexes, see the excellent discussion in Whelan (2002).
We start by constructing $Q_t$ using the “chain–summation” method:\footnote{Conceptually this formula is exact. In practice, it is an approximation because the statistical agency typically uses more disaggregate categories when calculating sums like $\sum_i P_{it-1}y_{it}$ than is available to us.}

$$\frac{Q_t}{Q_{t-1}} = \sqrt{\frac{\sum_i P_{it-1}y_{it}}{\sum_i P_{it-1}y_{it-1}}} \cdot \frac{\sum_i P_{it}y_{it}}{\sum_i P_{it}y_{it-1}}. $$

Using this expression iteratively, we obtain $Q_t$ as:

$$Q_t = \frac{Q_t}{Q_{t-1}} \cdot \frac{Q_{t-1}}{Q_{t-2}} \cdots \frac{Q_{b+1}}{Q_b} = \frac{Q_t}{Q_{t-1}} \cdot \frac{Q_{t-1}}{Q_{t-2}} \cdots \frac{Q_{b+1}}{Q_b},$$

where the last step used the normalization $Q_b = 1$. The real value and the price in period $t$ then follow as:

$$y_t = Q_t Y_b,$$

$$P_t = \frac{Y_t}{Q_t Y_b}.$$

**Appendix B: Construction of Hours by Persons Engaged**

In this appendix we describe how we combine the “Income–and–Employment–by–Industry” tables with the “GDP–by–Industry Tables” in order to obtain hours by persons engaged by sector. Recall that the ‘Income–and–Employment–by–Industry” contain information about full–time equivalent employees, self–employed persons, and persons engaged in production but change classification from SIC to NAICS; the “GDP–by–Industry Tables” tables contain only full– and part–time employees by industry, but use NAICS throughout the whole period. We combine...
them as follows:

\[
\text{full–time–equiv empl} = \frac{\text{full–time equiv empl}_{\text{SIC}}}{\text{part & full–time empl}_{\text{SIC}}} \quad \text{part & full–time empl}_{\text{NAICS}}
\]

\[
\text{hours full–time equiv empl} = \frac{\text{full–time equiv empl}_{\text{SIC}}}{\text{part & full–time empl}_{\text{SIC}}} \quad \text{full–time–equiv empl}
\]

\[
\text{self–empl} = \frac{\text{self–empl}_{\text{SIC}}}{\text{part & full–time empl}_{\text{SIC}}} \quad \text{part & full–time empl}_{\text{NAICS}}
\]

\[
\text{hours persons engaged} = \text{hours full–time equiv empl} + \frac{\text{hours full–time equiv empl}}{\text{full–time equiv empl}} \quad \text{self–empl}
\]

**Appendix C: Aggregating Reproducible Capital and Land**

To aggregate reproducible capital and fixed capital (i.e., land) to total capital in agriculture, we use the rental rates for the services that they provide. The rental rates of services from reproducible and fixed equal the real rates of return. To calculate them, we calculate the income shares of agricultural value added that are paid to reproducible capital and fixed capital, \(\theta_1\) and \(\theta_2\). We only observe \(\theta_1 + \theta_2 = 1 - \theta_{L_t}\). To calculate \(\theta_1\) and \(\theta_2\), we will impose that a risk–neutral investor be indifferent between holding them. Assuming that both assets face the same tax treatments, we obtain:

\[
(1 - \delta_{1t} + r_{1t}) \frac{P_{1t}}{P_{1t-1}} = (1 - \delta_{2t} + r_{2t}) \frac{P_{2t}}{P_{2t-1}}
\]

where \(r_{1t}\) and \(r_{2t}\) denote the rates of return on and \(P_{1t}\) and \(P_{2t}\) are the price levels of reproducible and fixed capital. Using the factor incomes and the asset stocks, this condition can be rewritten as:

\[
\left(1 - \delta_{1t} + \theta_{1t} \frac{P_{Y_t} Y_t}{P_{1t} K_{1t}} \right) \frac{P_{1t}}{P_{1t-1}} = \left(1 - \delta_{2t} + \theta_{2t} \frac{P_{Y_t} Y_t}{P_{2t} K_{2t}} \right) \frac{P_{2t}}{P_{2t-1}}
\]

where \(K_{1t}\) denotes the stock of reproducible capital in real terms, \(K_{2t}\) denotes the stock of fixed capital measured in acres, \(Y_t\) denotes agricultural value added in real terms, and \(P_{1t}\), \(P_{2t}\) and \(P_{Y_t}\) denote the price levels of reproducible capital, fixed capital, and agricultural value added.

---

18 We drop the index for agriculture to economize on notation, keeping in mind that everything in Appendix C refers to agriculture.
Using the fact that $\theta_2 = 1 - \theta_{Lt} - \theta_{Lt}$, we can rewrite the above equation as

$$\theta_{Lt} \left( \frac{P_1 Y_t}{P_1 K_{1t}} \frac{P_{1t}^2}{P_{2t}^2} + \frac{P_1 Y_t}{P_2 K_{2t}} \right) = \left( 1 - \delta_2 + \frac{(1 - \theta_{Lt}) P_1 Y_t}{P_2 K_{2t}} \right) - \left( 1 - \delta_1 \right) \frac{P_1}{P_{1t-1}} \frac{P_{2t-1}}{P_{2t}}$$

implying

$$\theta_{Lt} = \frac{1 - \delta_2 + \frac{(1 - \theta_{Lt}) P_1 Y_t}{P_2 K_{2t}} - \left( 1 - \delta_1 \right) \frac{P_1}{P_{1t-1}} \frac{P_{2t-1}}{P_{2t}}}{\frac{P_1 Y_t}{P_2 K_{2t}} + \frac{P_1}{P_{1t-1}} \frac{P_{2t-1}}{P_{2t}} \frac{P_1 Y_t}{P_2 K_{2t}}}$$

We calculate $P_1 K_{1t}$ as the current–cost fixed assets in agriculture from the BEA Standard Fixed Assets tables, $P_2 K_{2t}$ as the value of land from USDA calculated as the price of land per acre multiplied by the quantity of land measured in acres, and $\theta_{Lt}$ as labor share. We assume that $\delta_1 = 0.06$ and $\delta_2 = 0$, which are standard values.

Since in the real world the previous indifference condition holds only in expectations, we cannot assume that it holds ex post in all periods. To deal with this problem, we replace the price of capital relative to land $P_1 / P_2$ with its HP filtered version where the prices both of capital and land are normalized to 1 in 2005. We use a relatively high value of 1800 for the smoothing parameter so as to ensure that the nominal land rent per acre implied by our model is close to the one observed in the data.

### Appendix D: Estimation Results

**Table 3: Standard Errors of Regression Equations (21)–(23)**

<table>
<thead>
<tr>
<th>Specification</th>
<th>(21)</th>
<th>(22)</th>
<th>(23)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Agr</td>
<td>Man</td>
<td>Ser</td>
</tr>
<tr>
<td>CES</td>
<td>0.078</td>
<td>0.026</td>
<td>0.010</td>
</tr>
<tr>
<td>C-D (unequal)</td>
<td>0.078</td>
<td>0.025</td>
<td>0.010</td>
</tr>
<tr>
<td>C-D (equal)</td>
<td>0.077</td>
<td>0.026</td>
<td>0.010</td>
</tr>
</tbody>
</table>

*The standard error of an equation is the unbiased estimator of the root–mean–squared errors of the equation.*
### Table 4: Multivariate Ljung–Box Q–Statistics

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<thead>
<tr>
<th>Specification</th>
<th># of Lags</th>
<th>Degrees of freedom</th>
<th>Adj. Q–stat</th>
<th>p–value</th>
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</thead>
<tbody>
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<tr>
<td>CD (unequal)</td>
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<tr>
<td>CD (equal)</td>
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<td>18</td>
<td>20.079</td>
<td>0.328</td>
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</table>

### Table 5: Root–Mean–Squared Percentage Deviations

<table>
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<tr>
<th>Specification</th>
<th>Labor Allocation</th>
<th>Relative Prices</th>
</tr>
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<tr>
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<td>Man</td>
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<tr>
<td>CES</td>
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<tr>
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