9-8-03 CSE571 notes

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- $AnsProlog^{-not}$: $a_0 \leftarrow a_1 ... a_m.$
- $AnsProlog$: $a_0 \leftarrow a_1 ... a_m, nota_{m+1} ... nota_n.$
- Minimal Model - there is no smaller model, there is no proper subset
- Least Model - there is no smaller model or model of the same size
- A model is an interpretation that satisfies all rules
- A rule is satisfied if the RHS is true, or we don't need to prove the LHS.
- Examples:
  - Ex1.
    $a \leftarrow b.$
    $HB = \{a, b\}$
    $HI = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$
    $HMs = \{\emptyset, \{a\}, \{a, b\}\}$
    $MMs = \{\emptyset\}$
    $LM = \{\emptyset\}$
  - Ex2.
    $a \leftarrow notb.$
    $b \leftarrow nota.$
    $HB = \{a, b\}$
    $HI = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$
    $HMs = \{\{a\}, \{b\}, \{a, b\}\}$
    $MMs = \{\{a\}, \{b\}\}$
    $LM = none$
  - Ex3.
    $a \leftarrow notb.$
    $b \leftarrow nota.$
    $b \leftarrow c.$
    $c \leftarrow b.$
    $HB = \{a, b, c\}$
    $HI = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$
    $HMs = \{\{a\}, \{b, c\}\}$
    $MMs = \{\{a\}, \{b, c\}\}$
    $LM = \{a\}$

- Ex4.
  $a \leftarrow notb.$
  $HB = \{a, b\}$
  $HI = \{\oslash, \{a\}, \{b\}, \{a, b\}\}$
  $HMs = \{\{a\}, \{b\}\}$
  $MMs = \{\{a\}, \{b\}\}$
  $LM = none$
  Here even though$\{b\}$ is a minimal model, it is not stable.

- For $AnsProlog^{-not}$, we can define the notion of stable models, which is equivalent to the following terms:
  - answer set
  - minimal models
  - least model
  - least fixpoint of T^0_Π operator

- Ex5.
  $a \leftarrow b.$
  $b \leftarrow c, d.$
  $d \leftarrow c.$
  $c \leftarrow e.$
  $f \leftarrow g$
  $e \leftarrow .$
  $HB = \{a, b, c, d, e, f, g\}$
  $HI = \{\oslash, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}....\}$

- The fixpoint is found by applying all of the applicable rules at each iteration until two iterations are identical.

- The fixpoint procedure for Ex5. is as follows:
  $\oslash$
  e
  e, c
  e, c, d
  e, c, d, b
  e, c, d, b, a
  e, c, d, b, a

- If nothing can be derived from $\oslash$, then $\oslash$ is the least model.

- Formally $T^0_\Pi(I) = \{L_0 \in HB_\Pi$ s.t. $L_0 \leftarrow L_1...L_m$ is a rule in Π and $\{L_1...L_m\} \subseteq I\}$

- For AnsProlog, $T^1_\Pi = \{L_0 s.t. \exists$ a rule with L_0 in the head and the body is true w.r.t. I$\}$

- Why finding fixed points is not reasonable in AnsProlog: Consider Ex2, starting with $\oslash$, you derive $\{a, b\}$, from which you derive $\oslash$. There is no fix point because it will cycle forever. The next best idea is to take an interpretation and check if it is a stable model by converting it to a $AnsProlog^{-not}$ program.

- Converting AnsProlog to $AnsProlog^{-not}$: Given an AnsProlog program Π and an interpretation I, Π^I is an $AnsProlog^{-not}$ program obtained from Π by:
  1. (i) Removing rules of the form $l_0 \leftarrow l_1...l_m, notl_{m+1}...notl_n$ from Π if for some $j : m+1 \leq j \leq n, l_j \in I$.
  2. (ii) Removing, for all x, the parts of rules stating 'not x'.
- I is an answer set of an AnsProlog program Π is I is the least model of the $AnsProlog^{-not}$ program Π^I.
- Ex6.

  $\Pi = a \leftarrow notb.$

  $b \leftarrow nota.$

  $HB = \{a, b\}$

  $HI = \{\oslash, \{a\}, \{b\}, \{a, b\}\} = \{I_1, I_2, I_3, I_4\}$

  Find the answer sets for each Π^{I_j}:

  Initially $\forall_j \Pi = \Pi^{I_j}$

  After rule (i)

  $\Pi^{I_1} = \Pi$

  $\Pi^{I_2} = a \leftarrow notb.$

  $\Pi^{I_3} = b \leftarrow nota.$

  $\Pi^{I_4} = \oslash$

  After rule (ii)

  $\Pi^{I_1} = a \leftarrow .$

  $b \leftarrow .$

  $\Pi^{I_2} = a \leftarrow .$

  $\Pi^{I_3} = b \leftarrow .$

  $\Pi^{I_4} = \oslash$

  The least models are:

  $lm^{I_1} = \{a, b\}$

  $lm^{I_2} = \{a\}$

  $lm^{I_3} = \{b\}$

  $lm^{I_4} = \oslash$

  Of these, only lm^{I_2} and lm^{I_3} are answer sets because the least model of the $AnsProlog^{-not}$ program is equal to the interpretation of the AnsProlog program.