(1) Consider an AnsProlog language with constants: \( a \), \( b \), and \( c \), variables: \( X \), \( Y \), and \( Z \), predicates: \( p \) and \( q \) of arity one, and functions: \( f \) and \( g \) of arity one. For the following expressions label them as: Belongs to Herbrand base, Belongs to Herbrand Universe, Belongs to both, Belongs to neither. (16 pts)

(i) \( p(a) \)  (ii) \( p(X) \)  (iii) \( p(f(g(a))) \)  (iv) \( f(a) \)  (v) \( f(p(b)) \)  (vi) \( b(a) \)  (vii) \( q(p(c)) \)  (viii) \( a \)

(2) Consider the following program and answer the questions following it. (12 pts)

\[
\begin{align*}
q(a) & \leftarrow p(b). \\
q(b) & \leftarrow p(a). \\
p(b) & \leftarrow . \\
p(c) & \leftarrow .
\end{align*}
\]

1. Is \( \{p(b), p(c), q(a), q(c)\} \) a model of this program? Is it a minimal model? Explain briefly.
2. Is \( \{p(b), p(c), p(a), q(b)\} \) a model of this program? Is it a minimal model? Explain briefly.
3. Is \( \{p(b), p(c), q(a)\} \) a model of this program? Is it a minimal model? Explain briefly.
(3) Consider the following program:

\[
\begin{align*}
    p & \leftarrow \text{not } p. \\
    p & \leftarrow \text{not } q. \\
    q & \leftarrow \text{not } r. \\
    r & \leftarrow \text{not } q.
\end{align*}
\]

Split the above program so that the first two rules belong to the top and the last two rules belong to the bottom. Compute the answer sets of this program and clearly say what they are. (Show your calculation steps.) (10 pts)

(4) Write the following in AnsProlog so that you can conclude Mike has a Ph.D and Carol does not. (12 pts)

Normally instructors have a Ph.D. Instructors who are TAs are exceptions. Mike is an instructor. Carol is an instructor and a TA.
(5) Express the following in AnsProlog or Smodels. There are 10 students: s1, s2, \ldots, s10. They need to be assigned into 4 dormitory rooms a, b, c, and d, so that each room has at most three students assigned and no student is assigned to more than one rooms. (You may assume that your program has facts such as student(s1), \ldots, student(s12), and room(a), \ldots, room(d).) Give intuitive meaning of your predicates.

Now add the following constraints In AnsProlog (cardinality constructs from Smodels are not allowed.)

1. s3 and s4 are in different rooms.
2. s5 and s6 are in the same room.
3. Each room has at least 2 students.
4. Room c has exactly two students assigned.
5. We have a predicate friend(X, Y) meaning that X and Y are friends.
   (a) For the student s7 at least one of his friends is assigned the same room as s7.
   (b) For the student s9 all his friends are assigned the same room as s9.