Proof sketch of R3

- R3: Deciding if (D, O) is consistent is NP-complete.
- Deciding if (D, O) is consistent is in NP.
  - From R_1 deciding whether Φ_D exists is polynomial.
  - Guess an initial state σ_0.
  - From R2, verifying if σ_0 is an initial state with respect to (D, O) can be done in polynomial time.
- We reduce the known NP complete problem 3SAT to R3.
  - I.e., given a 3SAT formula F we will construct (in polynomial time) a pair of D and O such that F is satisfiable iff (D, O) is consistent.
  - Let F be of the form $(f_{11} \vee f_{12} \vee f_{13}) \wedge \ldots \wedge (f_{n1} \vee f_{n2} \vee f_{n3})$.
  - D consists of the following.
    * a **causes** g_1 **if** f_{11}; a **causes** g_1 **if** f_{12}; a **causes** g_1 **if** f_{13}
      $\vdots$
    * a **causes** g_n **if** f_{n1}; a **causes** g_n **if** f_{n2}; a **causes** g_n **if** f_{n3}

    - a' **causes** g **if** $g_1, \ldots, g_n$.
- O consists of the following.
    - **initially** $\neg g_1, \ldots, \neg g_n, \neg g$.
    - g **after** a, a'.
- Argue that if F is satisfiable then (D, O) is consistent.
- Argue that if (D, O) is consistent then F is satisfiable.

Proof sketch of R4

- R4: Deciding if $(D, O) \models f$ **after** $a_1, \ldots, a_n$ is coNP complete.
- Lemma: $(D, O) \models f$ **after** $a_1, \ldots, a_n$ iff $(D, O \cup \{\neg f$ **after** $a_1, \ldots, a_n\})$ is inconsistent.

  i.e., not $((D, O) \models f$ **after** $a_1, \ldots, a_n)$ iff $(D, O \cup \{\neg f$ **after** $a_1, \ldots, a_n\})$ is consistent.
- So the converse of of R4 is an NP complete problem.
- Therefore R4 is coNP complete.

Proof sketch of R5

- R5: Given a D, a complete set of observations about the initial state O, and a fluent g, finding a polynomial sequence of actions $a_1, \ldots, a_m$ such that $(D, O) \models g$ **after** $a_1, \ldots, a_m$ is NP-complete. (i.e., Feasible planning in $\mathcal{A}_0$ with a complete initial state, and simple goal, is NP-complete.)
- R5 is in NP.
  - Given D, O and g, guess a sequence of actions $a_1, \ldots, a_m$.
  - Let $\sigma_0 = \{f \ : \ f$ is a fluent and **initially** f is in $O\}$.
  - Check if g holds in $\Phi_D(a_m, \Phi_D(a_{m-1}, \ldots \Phi_D(a_1, \sigma_0) \ldots))$.
- We now reduce 3SAT to R5.
  - Let F be of the form $(f_{11} \vee f_{12} \vee f_{13}) \wedge \ldots \wedge (f_{n1} \vee f_{n2} \vee f_{n3})$.
  - D consists of the following.

- * Let $g_1, \ldots, g_m, g$ be new propositions not in the language of F.
  a **causes** g_1 **if** f_{11}; a **causes** g_1 **if** f_{12}; a **causes** g_1 **if** f_{13}
  $\vdots$
  a **causes** g_n **if** f_{n1}; a **causes** g_n **if** f_{n2}; a **causes** g_n **if** f_{n3}
  a' **causes** g **if** $g_1, \ldots, g_n$.
- * For all propositions p in the language of F we have an action a_p and the effect proposition a_p **causes** p.

– O consists of the following.

- * For all propositions p in the language of F we have **initially** $\neg p$
- * **initially** $\neg g_1, \ldots, \neg g_n, \neg g$.

– Goal is g.

– Argue that if F is satisfiable then there is a polynomial length plan for g with respect to (D, O).

– Argue that if there is a polynomial length plan for g with respect to (D, O) then F is satisfiable.