Proof sketch of R3

- R3: Deciding if \((D, O)\) is consistent is NP-complete.
- Deciding if \((D, O)\) is consistent is in NP.
  - From R1 deciding whether \(\Phi_D\) exists is polynomial.
  - Guess an initial state \(\sigma_0\).
  - From R2, verifying if \(\sigma_0\) is an initial state with respect to \((D, O)\) can be done in polynomial time.
- We reduce the known NP complete problem 3SAT to R3.
  - I.e., given a 3SAT formula \(F\) we will construct (in polynomial time) a pair of \(D\) and \(O\) such that \(F\) is satisfiable iff \((D, O)\) is consistent.
  - Let \(F\) be of the form \(\left( f_{11} \lor f_{12} \lor f_{13} \right) \land \ldots \land \left( f_{n1} \lor f_{n2} \lor f_{n3} \right)\).
  - \(D\) consists of the following.
    * a \texttt{causes} \(g_1\) if \(f_{11}\); a \texttt{causes} \(g_1\) if \(f_{12}\); a \texttt{causes} \(g_1\) if \(f_{13}\)
    :
    * a \texttt{causes} \(g_n\) if \(f_{n1}\); a \texttt{causes} \(g_n\) if \(f_{n2}\); a \texttt{causes} \(g_n\) if \(f_{n3}\)
Handout 1.

Representing and reasoning about actions and their impact

- $a'$ causes $g$ if $g_1, \ldots, g_n$.

- $O$ consists of the following.
  - * initially $\neg g_1, \ldots, \neg g_n, \neg g$.
  - * $g$ after $a, a'$.

- Argue that if $F$ is satisfiable then $(D, O)$ is consistent.
- Argue that if $(D, O)$ is consistent then $F$ is satisfiable.
Proof sketch of R4

- R4: Deciding if $(D, O) \models f \text{ after } a_1, \ldots, a_n$ is coNP complete.

- Lemma: $(D, O) \models f \text{ after } a_1, \ldots, a_n$ iff $(D, O \cup \{ \neg f \text{ after } a_1, \ldots, a_n \})$ is inconsistent.
  i.e., not $(D, O) \models f \text{ after } a_1, \ldots, a_n$ iff $(D, O \cup \{ \neg f \text{ after } a_1, \ldots, a_n \})$ is consistent.

- So the converse of R4 is an NP complete problem.

- Therefore R4 is coNP complete.
Proof sketch of R5

• R5: Given a \( D \), a complete set of observations about the initial state \( O \), and a fluent \( g \), finding a polynomial sequence of actions \( a_1, \ldots, a_m \) such that \((D, O) \models g \text{ after } a_1, \ldots, a_m \) is NP-complete. (i.e., Feasible planning in \( A_0 \) with a complete initial state, and simple goal, is NP-complete.)

• R5 is in NP.
  - Given \( D, O \) and \( g \), guess a sequence of actions \( a_1, \ldots, a_m \).
  - Let \( \sigma_0 = \{ f : f \text{ is a fluent and initially } f \text{ is in } O \} \).
  - Check if \( g \) holds in \( \Phi_D(a_m, \Phi_D(a_{m-1}, \ldots \Phi_D(a_1, \sigma_0) \ldots)) \).

• We now reduce 3SAT to R5.
  - Let \( F \) be of the form \((f_{11} \lor f_{12} \lor f_{13}) \land \ldots \land (f_{n1} \lor f_{n2} \lor f_{n3}) \).
  - \( D \) consists of the following.
Let \( g_1, \ldots, g_m, g \) be new propositions not in the language of \( F \).

\[ a \text{ causes } g_1 \text{ if } f_{11}; \quad a \text{ causes } g_1 \text{ if } f_{12}; \quad a \text{ causes } g_1 \text{ if } f_{13} \]
\[ : \]
\[ a \text{ causes } g_n \text{ if } f_{n1}; \quad a \text{ causes } g_n \text{ if } f_{n2}; \quad a \text{ causes } g_n \text{ if } f_{n3} \]
\[ a' \text{ causes } g \text{ if } g_1, \ldots, g_n. \]

For all propositions \( p \) in the language of \( F \) we have an action \( a_p \) and the effect proposition \( a_p \text{ causes } p \).

- \( O \) consists of the following.
  - For all propositions \( p \) in the language of \( F \) we have \textit{initially} \( \neg p \)
  - \textit{initially} \( \neg g_1, \ldots, \neg g_n, \neg g \).

- Goal is \( g \).

- Argue that if \( F \) is satisfiable then there is a polynomial length plan for \( g \) with respect to \((D, O)\).

- Argue that if there is a polynomial length plan for \( g \) with respect to \((D, O)\) then \( F \) is satisfiable.