Hypothetical reasoning in initial state complete $\mathcal{A}_0$: $tr\_do_1$ and $tr\_q_1$

1. For a fluent $f$ suppose the domain description $D$ contains only the following effect axioms that either make $f$ true or make it $false$.

- $a_1$ causes $f$ if $p_{l1}, \ldots, p_{lr_l}$
- $b_1$ causes $¬f$ if $q_{l1}, \ldots, q_{ls_l}$

For an integer $n$, formulas obtained by instantiating $N$ below with $1...n$ are in $tr\_do_1(D, O, n)$.

$$f[N + 1] \iff (a_1[N] \land p_{l1}[N] \land \ldots \land p_{lr_l}[N]) \lor \ldots \lor (a_l[N] \land p_{l1}[N] \land \ldots \land p_{lr_l}[N]) \lor (f[N] \land (¬b_1[N] \lor q_{l1}[N] \lor \ldots \lor q_{ls_l}[N])) \land \ldots \land (¬b_m[N] \lor q_{m1}[N] \lor \ldots \lor q_{ms_m}[N])$$

2. for each initially $f$ in $O$ the proposition $f[1]$ is in $tr\_do_1(D, O, n)$.

3. A query $Q$ given by $f$ after $a_1, \ldots, a_t$ is translated to:

$$(a_1[1] \land ¬a_2[1] \ldots \land ¬a_r[1]) \land \ldots \land (¬a_1[t] \land \ldots \land ¬a_{t−1}[t] \land a_t[t] \land ¬a_{t+1}[t] \ldots \land ¬a_r[t])$$

where $a_1, \ldots, a_r$ are all the actions in $D$. We will refer to the above as $tr\_q_1(Q)$.
4. Proposition 1: Given a consistent domain description $D$, a complete set of initial state observations $O$ and a query $Q$ of the form $f$ after $a_1, \ldots, a_t$, $(D, O) \models Q$ iff $\text{tr}_\text{do}_1(D, O, t) \cup \text{tr}_\text{q}_1(Q) \models f[t + 1]$.

5. Lemma 1: For a consistent domain description $D$ and a complete set of initial state observations $O$, $(D, O)$ has a unique model.

6. Lemma 2: Given a consistent domain description $D$, a complete set of initial state observations $O$ and a query $Q$ of the form $f$ after $a_1, \ldots, a_t$, let $M = (\sigma_0, \Phi)$ be a unique model of $(D, O)$.

The propositional theory $\text{tr}_\text{do}_1(D, O, t) \cup \text{tr}_\text{q}_1(Q)$ has a unique model $M$ given by

$$\{f[k + 1] : f \in [a_1, \ldots, a_k]\sigma_0, 0 \leq k \leq t\} \cup \{a[i] : 1 \leq i \leq t\}.$$ 

**Proof-sketch** (by induction on $t$)

Inductive step: $f \in [a_1, \ldots, a_k]\sigma_0$ if

(i) $f \in E_D^+(a_k, [a_1, \ldots, a_{k-1}]\sigma_0)$ or

(ii) $f \in [a_1, \ldots, a_{k-1}]\sigma_0$ and $f \notin E_D^-(a_k, [a_1, \ldots, a_{k-1}]\sigma_0)$.

(i) Let $f \in E_D^+(a_k, [a_1, \ldots, a_{k-1}]\sigma_0)$. This means there must exist an effect propositions of the form $a$ causes $f$ if $p_1, \ldots, p_n$ such that $p_1, \ldots, p_n$ hold in $[a_1, \ldots, a_{k-1}]\sigma_0$. By the induction hypothesis if $p_i$ is a positive fluent literal then $p_i[k]$ must be true in $M$ and otherwise $p_i[k]$ must be false in $M$. Thus to satisfy the
formula in \( tr\_do_1(D, O, t) \), \( f[k + 1] \) must be true in \( M \).

7. A linear regression algorithm: Start from \( f[t + 1] \), use \( tr\_do_1(D, O, t) \) to obtain what should be true at \( t \). Use \( tr\_q_1(Q) \) to use the information about what action occurs at \( t \) and simplify the regressed formula. Pursue each branch of the simplified regressed formula as above until the initial state.
Hypothetical reasoning in initial state complete $A_0$: \( tr\_do_2 \) and \( tr\_q_2 \)

- Given $D$, $O$ and $n$ as mentioned in the previous slide, \( tr\_do_2(D, O, n) \) consists of the formulas in \( tr\_do_1(D, O, n) \) and the following:
  
  - For all $1 \leq i \leq t$, $a_1[i] \lor \ldots \lor a_r[i]$ is in $\text{tr}_\text{do}_2(D, O, n)$.
  
  - For all $1 \leq i \leq t$ and $1 \leq j, k \leq r$ where $j \neq k$, $- (a_j[i] \land a_k[i])$ is in $\text{tr}_\text{do}_2(D, O, n)$.

- A query $Q$ given by $f$ after $a_1, \ldots, a_t$ is translated to:
  
  $a_1[1] \land \ldots \land a_t[t]$
  
  which we will refer to as $\text{tr}_\text{q}_2(Q)$.

- Proposition 2: Given a consistent domain description $D$, a complete set of initial state observations $O$ and a query $Q$ of the form $f$ after $a_1, \ldots, a_t$, $(D, O) \models Q$ iff $\text{tr}_\text{do}_2(D, O, t) \cup \text{tr}_\text{q}_2(Q) \models f[t + 1]$. 
1. Proposition 3: Let $D$ be a consistent domain description, $O$ be a complete set of observations about the initial state, and $g$ be the goal. A sequence of actions $a_1, \ldots, a_t$ is a plan iff there is a model of $tr_{do_2}(D, O, t) \cup g[t + 1]$ containing $\{a_1[1], \ldots, a_t[t]\}$ as the only action occurrences.

2. The above suggests that to find a plan, if we know that a plan of length $t$ exists then we should give a propositional solver the propositional theory $tr_{do_2}(D, O, t) \cup tr_{q_2}(Q) \cup g[t + 1]$ and ask it to find a model. Each model (if exists) will encode a plan.

If we do not know the plan length but have an idea of an upper bound, then

- we can have a dummy action which does not affect the world; or
- we can replace $g[t + 1]$ by $g[1] \lor \ldots \lor g[t + 1]$. 
Some good SAT solvers

