CSE 591 - FALL 04. HANDOUT 4. VERSION 0.1

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October 19, 2004

DESCRIBING GOALS AND DIRECTIVES

Some Motivating examples of directives and goals

1. Goto Room 5 and stay there.
2. Goto Room 5, but don't get hit.
3. Goto Room 5, but make sure you immediately put everything back the way they were. (Immediately Close the door you open, etc.)
4. Goto Room 5, but make sure you eventually put everything back the way they.
5. Goto Room 5, but don't open closed doors.
6. Goto location 6 in a highway network.
7. Goto location 6 in a highway network, but always be within two hops from a gas station.
8. Try your best to get to Room 5.

Goal representation using LTL

- LTL formulas are made up of propositions, propositional connectives $\vee$, $\wedge$, and $\neg$, and future temporal connectives $\bigcirc$, $\Box$, $\Diamond$ and U.
- Truth value of temporal formulas with respect to trajectories.
- Let σ given by $s_0, s_1, \ldots, s_k$, s_{k+1}, ... be a trajectory, p denote a propositional formula, and f and f_is denote LTL formulas.
  - $(s_j, \sigma) \models p$ iff p is true in s_j.
  - $(s_j, \sigma) \models \neg f$ iff $(s_j, \sigma) \not\models f$.
  - $(s_j, \sigma) \models f_1 \vee f_2$ iff $(s_j, \sigma) \models f_1$ or $(s_j, \sigma) \models f_2$.
  - $(s_j, \sigma) \models f_1 \wedge f_2$ iff $(s_j, \sigma) \models f_1$ and $(s_j, \sigma) \models f_2$.
  - $(s_j, \sigma) \models \bigcirc f$ iff $(s_{j+1}, \sigma) \models f$
  - $(s_j, \sigma) \models \Box f$ iff $(s_k, \sigma) \models f$, for all $k \geq j$.
  - $(s_j, \sigma) \models \Diamond f$ iff $(s_k, \sigma) \models f$, for some $k \geq j$.
  - $(s_j, \sigma) \models f_1 \;\mathsf{U}\; f_2$ iff there exists $k \geq j$ such that $(s_k, \sigma) \models f_2$ and for all i, $j \leq i < k$, $(s_i, \sigma) \models f_1$. $\Box$

- Let s be a state designated as the initial state, let $a_1, \ldots, a_n$ be a sequence of deterministic actions whose effects are described by a domain description, and Φ be transition function that defines transitions between states due to actions.
- *The trajectory corresponding to s and $a_1, \ldots, a_n$* is the sequence $s_0, s_1, \ldots,$ that satisfies the following conditions: $s = s_0$, $s_{i+1} = \Phi(a_{i+1}, s_i)$, for $0 \leq i \leq n-1$, and $s_{j+1} = s_j$, for $j \geq n$.
- We then say that the sequence of actions $a_1, \ldots, a_n$ is a *plan* from the initial state s for the LTL goal f, if $(s, \sigma) \models f$, where σ is the trajectory corresponding to s and $a_1, \ldots, a_n$.

Some goals and their LTL representation

1. Pass by Room 5. $\Diamond at(room5)$
2. Go to Room 5 and stay there. $\Diamond\Box at(room5)$
3. Go to Room 5 and stay there, but don't ever get hit. $\Diamond\Box at(room5) \ \wedge \ \Box\neg hit$
4. Go to Room 5 and stay there, but don't get hit until then. $\neg hit \ \mathsf{U} \ \Diamond\Box at(room5)$
5. Pass by Room 5 but don't get hit until then. $\neg hit \ \mathsf{U} \ \Diamond at(room5)$
6. Go to Room 5 and stay there, and any time if the door is closed and you open it then you must immediately close it.

   $\Diamond\Box at(room5) \ \wedge \ \Box(closed \wedge \bigcirc\neg closed \Rightarrow \bigcirc\bigcirc closed)$.
7. Go to Room 5 and stay there, and *on the way* if the door is closed and you open it then you must immediately close it.

   $(closed \wedge \bigcirc\neg closed \Rightarrow \bigcirc\bigcirc closed) \ \mathsf{U} \ \Diamond\Box at(room5)$
8. Go to Room 5 and stay there, and any time if the door is closed and you open it then you must eventually close it.

   $\Diamond\Box at(room5) \ \wedge \ \Box(closed \wedge \bigcirc\neg closed \Rightarrow \bigcirc\Diamond closed)$.

9. Go to Room 5 and stay there, and *on the way* if the door is closed and you open it then you must eventually close it.

   $(closed \wedge \bigcirc\neg closed \Rightarrow \bigcirc\Diamond closed)\ \mathsf{U}\ \Diamond\Box at(room5)$

10. Go to Room 5 and stay there and never open the door if it is closed.

    $\Diamond\Box at(room5)\ \wedge\ \Box(closed \Rightarrow \bigcirc closed)$.

11. Go to Room 5 and stay there and *on the way* do not open the door if it is closed.

    $(closed \Rightarrow \bigcirc closed)\ \mathsf{U}\ \Diamond\Box at(room5)$

MITL: Metric interval temporal logic

- Syntax: Each LTL operator has an interval associated with it. Intervals are of the form, $(a, b), (a, b], [a, b), [a, b]$.
- Semantics
  - There is a timing function $\mathcal{T}$ which maps each state to a non-negative real number, such that $\mathcal{T}(s_i) \leq \mathcal{T}(s_{i+1})$.
  - $\bigcirc_I\ \phi$ and $\bigcirc\ \phi$ have the same meaning.
  - $\Box_I\ \phi$ is true in state s_t, if ϕ is true in all states whose time lies in the upcoming interval I.
  - $\Diamond_I\ \phi$ is true in state s_t, if ϕ is true in some state whose time lies in the upcoming interval I.
  - $\phi_1\ \mathsf{U}_I\ \phi_2$ is true in state s_t, if ϕ_2 is true in some state whose time is in the upcoming interval I and ϕ_1 is true until then.

    For example, $\phi_1\ \mathsf{U}_{[2,4]}\ \phi_2$ is true in state s_t, if ϕ_2 is true in some state whose time is 2 to 4 units into the future (from the time of s_t) and ϕ_1 is true until then.
- Examples:

– Recall that "achieve goal ϕ" is expressed as $\Diamond \Box \phi$.
  The directive "achieve goal ϕ within 10 time units" is expressed as $\Box_{[10,\infty)} \phi$, which is also written as $\Box_{\geq 10} \phi$.
– ψ must be made true within 5 time units of ϕ being true all through the trajectory.
  $\Box (\phi \Rightarrow \Diamond_{\leq 5} \psi)$.

Simple branching temporal logic: CTL*

- In the Branching Temporal Logic CTL* in addition to LTL operations, we have two additional operators E and A which describe possible futures.

  Besides the notion of a trajectory we have an evolution relation $R \subseteq S \times S$. If $(s, s') \in R$ it means that the world can go from state s to state s' in one step. This could be because of the agent's action or because of the environment.

- Given a transition relation R and a state s, a path in R starting from s is a sequence of states $s_0, s_1, \ldots$ such that $s_0 = s$, and $R(s_i, s_{i+1})$ is true.

- For example, the requirement that, no matter what action we apply to the state s, a fluent f will always stay true, can be described as $\mathsf{A}(\bigcirc f)$.

Syntax and Semantics of CTL*

- There are two kinds of formulas in CTL*: state formulas and path formulas.
- Normally state formulas are properties of states while path formulas are properties of paths.
- Syntax of state and path formulas. ($\langle p \rangle$ denote an atomic proposition, $\langle sf \rangle$ denote state formulas, and $\langle pf \rangle$ denote path formulas)
  $\langle sf \rangle ::= \langle p \rangle \mid \langle sf \rangle \wedge \langle sf \rangle \mid \langle sf \rangle \vee \langle sf \rangle \mid \neg \langle sf \rangle \mid \mathsf{E}\langle pf \rangle \mid \mathsf{A}\langle pf \rangle$
  $\langle pf \rangle ::= \langle sf \rangle \mid \langle pf \rangle \, \mathsf{U} \, \langle pf \rangle \mid \neg \langle pf \rangle \mid \langle pf \rangle \wedge \langle pf \rangle \mid \langle pf \rangle \vee \langle pf \rangle \mid \bigcirc \langle pf \rangle \mid \Diamond \langle pf \rangle \mid \Box \langle pf \rangle$
- Semantics – truth of state formulas (p denotes a propositional formula sf_is are state formulas and pf_is are path formulas)
  - $(s_j, R) \models p$ if p is true in s_j.
  - $(s_j, R) \models sf_1 \wedge sf_2$ if $(s_j, R) \models sf_1$ and $(s_j, R) \models sf_2$.
  - $(s_j, R) \models sf_1 \vee sf_2$ if $(s_j, R) \models sf_1$ or $(s_j, R) \models sf_2$.
  - $(s_j, R) \models \neg sf$ if $(s_j, R) \not\models sf$.

– $(s_j, R) \models \mathsf{E}\, pf$ if there exists a path σ in R starting from s_j such that $(s_j, R, \sigma) \models pf$.

– $(s_j, R) \models \mathsf{A}\, pf$ if for all paths σ in R starting from s_j we have that $(s_j, R, \sigma) \models pf$.

- Semantics: Truth of path formulas
  – $(s_j, R, \sigma) \models sf$ if $(s, R) \models sf$.
  – $(s_j, R, \sigma) \models pf_1 \,\mathsf{U}\, pf_2$ iff there exists $k \geq j$ such that $(s_k, R, \sigma) \models pf_2$ and for all i, $j \leq i < k$, $(s_i, R, \sigma) \models pf_1$.
  – $(s_j, R, \sigma) \models \neg pf$ iff $(s_j, R, \sigma) \not\models pf$.
  – $(s_j, R, \sigma) \models pf_1 \wedge pf_2$ iff $(s_j, R, \sigma) \models pf_1$ and $(s_j, R, \sigma) \models pf_2$.
  – $(s_j, R, \sigma) \models pf_1 \vee pf_2$ iff $(s_j, R, \sigma) \models pf_1$ or $(s_j, R, \sigma) \models pf_2$.
  – $(s_j, R, \sigma) \models \bigcirc pf$ iff $(s_{j+1}, R, \sigma) \models pf$
  – $(s_j, R, \sigma) \models \Box pf$ iff $(s_k, R, \sigma) \models pf$, for all $k \geq j$.
  – $(s_j, R, \sigma) \models \Diamond pf$ iff $(s_k, R, \sigma) \models pf$, for some $k \geq j$.
- We say a sequence of actions $a_1, \ldots, a_n$ is a plan with respect to the initial state s_0 and a goal G if $(s_0, R, \sigma) \models G$, where σ is the trajectory corresponding to s_0 and $a_1, \ldots, a_n$.

- Although state formulas are also path formulas, since the evaluation of state formulas do not take into account the trajectory suggested by a prospective plan, often the overall goal of a planning problem is better expressed as a path formula which is not a state formula.
- Similar to an LTL goal which is just a propositional formula, a CTL* goal which is a state formula either leads to no plans (if the initial state together with R does not satisfy the goal) or leads to the plan with no actions (if the initial state together with R satisfies the goal).
- But unlike propositional goals in LTL, a state formula in CTL* is useful in specifying the existence of a plan.

Examples of planning goals in CTL*

1. Goal: Get to B such that from any where in the path we can get to a state where p holds in at most two steps.

   $(p \vee \mathsf{E} \bigcirc p \vee \mathsf{E} \bigcirc \mathsf{E} \bigcirc p)\ \mathsf{U}\ at\_B$

   vs

   $\mathsf{E}\ ((p \vee \mathsf{E} \bigcirc p \vee \mathsf{E} \bigcirc \mathsf{E} \bigcirc p)\ \mathsf{U}\ at\_B)$

2. Goal: Find a path to home, such that from every point in the path there is a path to a telephone booth.

   $(\mathsf{E}\Diamond\ has\_telephone\_booth)\ \mathsf{U}\ at\_home$

3. Goal: Find a path that travels through ports until a port is reached from where there are paths to a fort and a hill.

   $is\_a\_port\ \mathsf{U}\ (is\_a\_port \wedge (\mathsf{E}\Diamond\ has\_fort) \wedge (\mathsf{E}\Diamond\ has\_hill))$

4. Find a path to a place with a hotel such that from any point in the path there is a path to a garage, until we reach a shopping center from where there is a path to the hotel.

   $((\mathsf{E}\Diamond\ has\_garage)\ \mathsf{U}\ (shopping\_center \wedge \Diamond\ has\_hotel)) \wedge \Diamond\Box\ has\_hotel$

5. Goal of a robot: Reach a state satisfying the property h such that the states on the path are obstacle free and at least one immediate successor state has a power socket.

   $(obstacle\_free \wedge (\mathsf{E} \bigcirc has\_power\_socket))\ \mathsf{U}\ h$

6. A slight change to the last specification: Agent has to make sure that all (instead of at least one) immediate successor state has a power socket.

   $(obstacle\_free \wedge (\mathsf{A} \bigcirc has\_power\_socket))\ \mathsf{U}\ h$