Chitta Baral
Department of Computer Science and Engineering
Arizona State University
Tempe, AZ 85287-5406 USA
chitta@asu.edu
http://www.public.asu.edu/~cbaral/

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DESCRIBING GOALS AND DIRECTIVES
Some Motivating examples of directives and goals

1. Goto Room 5 and stay there.
2. Goto Room 5, but don’t get hit.
3. Goto Room 5, but make sure you immediately put everything back the way they were. (Immediately Close the door you open, etc.)
4. Goto Room 5, but make sure you eventually put everything back the way they.
5. Goto Room 5, but don’t open closed doors.
7. Goto location 6 in a highway network, but always be within two hops from a gas station.
8. Try your best to get to Room 5.
Goal representation using LTL

- LTL formulas are made up of propositions, propositional connectives $\lor$, $\land$, and $\neg$, and future temporal connectives $\Diamond$, $\Box$, $\Diamond$ and $\bigcup$.
- Truth value of temporal formulas with respect to trajectories.
- Let $\sigma$ given by $s_0, s_1, \ldots, s_k, s_{k+1}, \ldots$ be a trajectory, $p$ denote a propositional formula, and $f$ and $f_i$s denote LTL formulas.

  - $(s_j, \sigma) \models p$ iff $p$ is true in $s_j$.
  - $(s_j, \sigma) \models \neg f$ iff $(s_j, \sigma) \not\models f$.
  - $(s_j, \sigma) \models f_1 \lor f_2$ iff $(s_j, \sigma) \models f_1$ or $(s_j, \sigma) \models f_2$.
  - $(s_j, \sigma) \models f_1 \land f_2$ iff $(s_j, \sigma) \models f_1$ and $(s_j, \sigma) \models f_2$.
  - $(s_j, \sigma) \models \Diamond f$ iff $(s_{j+1}, \sigma) \models f$.
  - $(s_j, \sigma) \models \Box f$ iff $(s_k, \sigma) \models f$, for all $k \geq j$.
  - $(s_j, \sigma) \models \Diamond f$ iff $(s_k, \sigma) \models f$, for some $k \geq j$.
  - $(s_j, \sigma) \models f_1 \bigcup f_2$ iff there exists $k \geq j$ such that $(s_k, \sigma) \models f_2$ and for all $i$, $j \leq i < k$, $(s_i, \sigma) \models f_1$. □
Let $s$ be a state designated as the initial state, let $a_1, \ldots, a_n$ be a sequence of deterministic actions whose effects are described by a domain description, and $\Phi$ be transition function that defines transitions between states due to actions.

The trajectory corresponding to $s$ and $a_1, \ldots, a_n$ is the sequence $s_0, s_1, \ldots$, that satisfies the following conditions: $s = s_0$, $s_{i+1} = \Phi(a_{i+1}, s_i)$, for $0 \leq i \leq n - 1$, and $s_{j+1} = s_j$, for $j \geq n$.

We then say that the sequence of actions $a_1, \ldots, a_n$ is a plan from the initial state $s$ for the LTL goal $f$, if $(s, \sigma) \models f$, where $\sigma$ is the trajectory corresponding to $s$ and $a_1, \ldots, a_n$. 
Describing goals and directives

Some goals and their LTL representation

1. Pass by Room 5. \( \Diamond at(room5) \)
2. Go to Room 5 and stay there. \( \Diamond \Box at(room5) \)
3. Go to Room 5 and stay there, but don’t ever get hit. \( \Diamond \Box at(room5) \land \Box \neg hit \)
4. Go to Room 5 and stay there, but don’t get hit until then. \( \neg hit \lor \Diamond at(room5) \)
5. Pass by Room 5 but don’t get hit until then. \( \neg hit \lor \Diamond at(room5) \)
6. Go to Room 5 and stay there, and any time if the door is closed and you open it then you must immediately close it.
\( \Diamond \Box at(room5) \land \Box (closed \land \Box \neg closed \Rightarrow \Box \Box closed) \).
7. Go to Room 5 and stay there, and on the way if the door is closed and you open it then you must immediately close it.
\( (closed \land \Box \neg closed \Rightarrow \Box \Box closed) \lor \Diamond \Box at(room5) \)
8. Go to Room 5 and stay there, and any time if the door is closed and you open it then you must eventually close it.
\( \Diamond \Box at(room5) \land \Box (closed \land \Box \neg closed \Rightarrow \Box \Diamond closed) \).
9. Go to Room 5 and stay there, and on the way if the door is closed and you open it then you must eventually close it.

\[(closed \land 
\lnot\neg closed \Rightarrow \diamond\Diamond closed) \cup \Diamond\Box at\left(room5\right)\]

10. Go to Room 5 and stay there and never open the door if it is closed.

\[\Diamond\Box at\left(room5\right) \land \Box\left(closed \Rightarrow \Diamond closed\right)\]

11. Go to Room 5 and stay there and on the way do not open the door if it is closed.

\[(closed \Rightarrow \Diamond closed) \cup \Diamond\Box at\left(room5\right)\]
MITL: Metric interval temporal logic

- Syntax: Each LTL operator has an interval associated with it. Intervals are of the form, \((a, b), (a, b], [a, b), [a, b]\).

- Semantics
  - There is a timing function \(T\) which maps each state to a non-negative real number, such that \(T(s_i) \leq T(s_{i+1})\).
  - \(\bigcirc_I \phi\) and \(\bigcirc \phi\) have the same meaning.
  - \(\Box_I \phi\) is true in state \(s_t\), if \(\phi\) is true in all states whose time lies in the upcoming interval \(I\).
  - \(\Diamond_I \phi\) is true in state \(s_t\), if \(\phi\) is true in some state whose time lies in the upcoming interval \(I\).
  - \(\phi_1 U_I \phi_2\) is true in state \(s_t\), if \(\phi_2\) is true in some state whose time is in the upcoming interval \(I\) and \(\phi_1\) is true until then.
    For example, \(\phi_1 U_{[2,4]} \phi_2\) is true in state \(s_t\), if \(\phi_2\) is true in some state whose time is 2 to 4 units into the future (from the time of \(s_t\)) and \(\phi_1\) is true until then.

- Examples:
– Recall that “achieve goal $\phi$” is expressed as $\Diamond \Box \phi$.
    The directive “achieve goal $\phi$ within 10 time units” is expressed as $\Box_{[10,\infty)} \phi$,
    which is also written as $\Box_{\geq 10} \phi$.
– $\psi$ must be made true within 5 time units of $\phi$ being true all through the trajectory.
  $\Box (\phi \Rightarrow \Diamond_{\leq 5} \psi)$. 
Simple branching temporal logic: CTL*

• In the Branching Temporal Logic CTL* in addition to LTL operations, we have two additional operators $E$ and $A$ which describe possible futures. Besides the notion of a trajectory we have an evolution relation $R \subseteq S \times S$. If $(s, s') \in R$ it means that the world can go from state $s$ to state $s'$ in one step. This could be because of the agent’s action or because of the environment.

• Given a transition relation $R$ and a state $s$, a path in $R$ starting from $s$ is a sequence of states $s_0, s_1, \ldots$ such that $s_0 = s$, and $R(s_i, s_{i+1})$ is true.

• For example, the requirement that, no matter what action we apply to the state $s$, a fluent $f$ will always stay true, can be described as $A(\Box f)$. 
There are two kinds of formulas in CTL*: state formulas and path formulas.

Normally state formulas are properties of states while path formulas are properties of paths.

Syntax of state and path formulas. (\langle p \rangle \) denote an atomic proposition, \langle sf \rangle denote state formulas, and \langle pf \rangle denote path formulas)

\[
\langle sf \rangle ::= \langle p \rangle | \langle sf \rangle \land \langle sf \rangle | \langle sf \rangle \lor \langle sf \rangle | \neg \langle sf \rangle | \mathbf{E} \langle pf \rangle | \mathbf{A} \langle pf \rangle
\]

\[
\langle pf \rangle ::= \langle sf \rangle | \langle pf \rangle \lor \langle pf \rangle | \langle pf \rangle \land \langle pf \rangle | \langle pf \rangle \lor \langle pf \rangle | \Diamond \langle pf \rangle | \Box \langle pf \rangle
\]

Semantics – truth of state formulas (\( p \) denotes a propositional formula \( sf \)'s are state formulas and \( pf \)'s are path formulas)

- \((s_j, R) \models p\) if \( p \) is true in \( s_j \).
- \((s_j, R) \models sf_1 \land sf_2\) if \((s_j, R) \models sf_1\) and \((s_j, R) \models sf_2\).
- \((s_j, R) \models sf_1 \lor sf_2\) if \((s_j, R) \models sf_1\) or \((s_j, R) \models sf_2\).
- \((s_j, R) \models \neg sf\) if \((s_j, R) \models sf^c\).
\[ (s_j, R) \models \mathbb{E} pf \text{ if there exists a path } \sigma \text{ in } R \text{ starting from } s_j \text{ such that } (s_j, R, \sigma) \models pf. \]
\[ (s_j, R) \models \mathbb{A} pf \text{ if for all paths } \sigma \text{ in } R \text{ starting from } s_j \text{ we have that } (s_j, R, \sigma) \models pf. \]

- **Semantics: Truth of path formulas**

\[ (s_j, R, \sigma) \models sf \text{ if } (s, R) \models sf. \]
\[ (s_j, R, \sigma) \models pf_1 \lor pf_2 \text{ iff there exists } k \geq j \text{ such that } (s_k, R, \sigma) \models pf_2 \text{ and for all } i, j < k, (s_i, R, \sigma) \models pf_1. \]
\[ (s_j, R, \sigma) \models \neg pf \text{ iff } (s_j, R, \sigma) \not\models pf. \]
\[ (s_j, R, \sigma) \models pf_1 \land pf_2 \text{ iff } (s_j, R, \sigma) \models pf_1 \text{ and } (s_j, R, \sigma) \models pf_2. \]
\[ (s_j, R, \sigma) \models pf_1 \lor pf_2 \text{ iff } (s_j, R, \sigma) \models pf_1 \text{ or } (s_j, R, \sigma) \models pf_2. \]
\[ (s_j, R, \sigma) \models \Box pf \text{ iff } (s_{j+1}, R, \sigma) \models pf \]
\[ (s_j, R, \sigma) \models \Diamond pf \text{ iff } (s_k, R, \sigma) \models pf, \text{ for all } k \geq j. \]
\[ (s_j, R, \sigma) \models \Diamond pf \text{ iff } (s_k, R, \sigma) \models pf, \text{ for some } k \geq j. \]

- We say a sequence of actions \(a_1, \ldots, a_n\) is a plan with respect to the initial state \(s_0\) and a goal \(G\) if \((s_0, R, \sigma) \models G\), where \(\sigma\) is the trajectory corresponding to \(s_0\) and \(a_1, \ldots, a_n\).
• Although state formulas are also path formulas, since the evaluation of state formulas do not take into account the trajectory suggested by a prospective plan, often the overall goal of a planning problem is better expressed as a path formula which is not a state formula.

• Similar to an LTL goal which is just a propositional formula, a $\text{CTL}^*$ goal which is a state formula either leads to no plans (if the initial state together with $R$ does not satisfy the goal) or leads to the plan with no actions (if the initial state together with $R$ satisfies the goal).

• But unlike propositional goals in LTL, a state formula in $\text{CTL}^*$ is useful in specifying the existence of a plan.
Examples of planning goals in CTL*

1. Goal: Get to B such that from any where in the path we can get to a state where $p$ holds in at most two steps.

\[(p \lor E \circ p \lor E \circ E \circ p) U at_B\]

vs

\[E ((p \lor E \circ p \lor E \circ E \circ p) U at_B)\]

2. Goal: Find a path to home, such that from every point in the path there is a path to a telephone booth.

\[(E \Diamond has\_telephone\_booth) U at\_home\]

3. Goal: Find a path that travels through ports until a port is reached from where there are paths to a fort and a hill.

\[is\_a\_port U (is\_a\_port \land (E \Diamond has\_fort) \land (E \Diamond has\_hill))\]
4. Find a path to a place with a hotel such that from any point in the path there is a path to a garage, until we reach a shopping center from where there is a path to the hotel.

\[(\text{E}\Diamond \text{has\_garage}) \cup (\text{shopping\_center} \land \Diamond \text{has\_hotel})) \land \Diamond \Box \text{has\_hotel}\]

5. Goal of a robot: Reach a state satisfying the property $h$ such that the states on the path are obstacle free and at least one immediate successor state has a power socket.

\[(\text{obstacle\_free} \land (\text{E} \bigcirc \text{has\_power\_socket})) \cup h\]

6. A slight change to the last specification: Agent has to make sure that all (instead of at least one) immediate successor state has a power socket.

\[(\text{obstacle\_free} \land (\text{A} \bigcirc \text{has\_power\_socket})) \cup h\]