

## CSE 355 HOMEWORK THREE

DUE 21 OCTOBER 2016, START OF CLASS

- (1) Let  $\Sigma = \{0, 1\}$ . A string  $w \in \Sigma^*$  of length  $n = |w|$  is called *sparse* if every two 1s in the string have at least  $\lceil \log_2 n \rceil$  0s between them. Let  $L_S \subseteq \Sigma^*$  be the language of sparse strings. Show that  $L_S$  is not regular.
- (2) Let  $\Sigma = \{0, 1\}$ . A string  $w \in \Sigma^*$  is called *prefix-dense* if every prefix of  $w$  contains at least twice as many 1s as 0s. Let  $L_P \subseteq \Sigma^*$  be the language of prefix-dense strings.
  - (a) Show that  $L_P$  is not regular.
  - (b) Show that  $L_P$  is context-free.
- (3) Let  $X$  be the language  $\{a^n b^m a^\ell b^\ell a^m b^n : n, m, \ell \geq 0\}$ .
  - (a) Show that  $X$  is not regular.
  - (b) Show that  $X$  is context-free.
- (4) Let  $L$  be a context-free language generated by the context-free grammar  $G$ . Let  $L'$  be the language that contains exactly the strings that both belong to  $L$  and have length a multiple of 3. Show that  $L'$  must be context-free by devising and describing a method for producing a context-free grammar for  $L'$  starting with the grammar  $G$ .