CSE 355 Test 1, Fall 2016

30 September 2016, 8:35-9:25 a.m., LSA 191

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Regrading of Midterms

If you believe that your grade has not been added up correctly, return the entire paper to the instructor with a short note indicating what you believe to be the error.

Other than for that reason, test grades are almost never changed. If you believe that you did not receive the proper credit, first **read these sample solutions carefully** to see if you can understand the answer to your concern. If that does not resolve it, write a clear explanation of why you believe the grade is in error and submit that, along with the entire test paper, to the instructor. Please do not discuss in your explanation how your solution is like that of another student, as FERPA legislation makes it impossible for me to discuss one student's work with another. Please take into account that more than 350 papers were graded, and it is quite unfair to change the grade on one paper without giving every other student the same opportunity. If you nevertheless want the paper regraded, be advised that the entire paper will be regraded and the grade may go up, stay the same, or go down. The new grade will be final.

It is a violation of the Academic Integrity Policy to request a grade change simply because you need or want a higher grade.

If you require a clarification of the sample solutions (not a grade change or review as discussed above), ask in recitations, or in office hours of the TA or instructor. You will be asked whether you have read the sample solution and to indicate what precisely is unclear to you about it, so **read these sample solutions carefully** first. Note that under no circumstances can anyone change a grade other than the instructor, so do not ask the TAs to do so – they are not able to.

Grade change requests, whether submitted as described above or not, will not be considered if received after 26 October 2016.

Instructions

Do not open the exam until you are instructed to do so. The exam will be submitted in THREE pieces: "Multiple Choice Questions", "Answers to Multiple Choice", and "Long Answer". *You **must** write your name and student number on each and every sheet indicated; failure to do so may result in your test not being properly graded.* Write legibly – we must be able to read your name and number. You must turn in **all sheets** including the multiple choice questions. You have 50 minutes to complete the exam. No books, notes, electronic devices, or other aids are permitted. Turn off all wireless devices and place them away from your work space. Write all answers on the examination paper itself. BUDGET YOUR TIME WELL! SHOW ALL WORK!

Some General Comments

1. As stated in class, papers written in pencil are not eligible for regrading.
2. Some students continued to write after it was announced that the time is over. This is an academic integrity violation and may result in your receiving no credit for your test.
3. We noticed a group of students discussing the test after the time was over but before they handed in their tests. This is an academic integrity violation. Hand in your test before you start your post mortem.
4. Many people did not write their name and/or student number on all sheets.This created a substantial amount of extra work for the TAs and instructor. You were asked to do this at the start of the test; do not wait until it is time to hand it in and hold up 300+ people trying to hand in the test.
5. Many people wrote a name that is not in agreement with the name under which they are listed in the ASU system. Use the name by which ASU knows you. In at least one case, in trying to read the name I did not guess a single letter correctly, nor did I guess the number of letters correctly. Please write your name clearly; use BLOCK letters if there is any chance it will be misread.
6. Sixteen people did not fill in the answers to the multiple choice in the “Answers to Multiple Choice”. In two cases there was no corresponding question sheets on which to locate the answers. The “Answers to Multiple Choice” sheet was displayed in class on 28 September precisely to avoid this problem, and the instructions on the test itself are clear. Follow the instructions.
7. I know that the time is severely limited to do each question. There is no need to write “out of time” or “no time left”. On the other hand, kudos to the students who said “If I had time, I would find a long enough string in the language and show there is no way it can be pumped to always stay in the language” or a similar statement.

Answers to Multiple Choice [17 marks in total]

Enter each response (one of `a, b, c, d, e`) for the questions on the “Multiple Choice” pages. Giving 0, or 2 or more, responses to a question is incorrect. Illegible or blank responses are incorrect.

1 D	2 D	3 E	4 B	5 E	6 A	7 D	8 D
9 E	10 C	11 C	12 A	13 E	14 B	15 D	16 B
17 A	TOTAL						

Multiple Choice [17 marks in total]

Select the most appropriate answer for each, and enter each response (one of `a, b, c, d, e`) on the “Answers to Multiple Choice” page.

1. The powerset method
   d constructs a DFA from an NFA
2. Which of the following is **false**?
   d Languages are defined to have a finite number of strings.
3. On input string $w \in \Sigma^\star$, the number of computations of an NFA $(Q, \Sigma, \delta, q_0, F)$ is always
   e at least 1 if the NFA accepts w
4. If DFA $M = (Q, \Sigma, \delta, q_0, F)$ accepts input string $w \in \Sigma^\star$ with $|w| = n$, a computation of M on w
   b is a sequence of exactly $n + 1$ states
5. The pumping lemma for regular languages can be proved by
   e showing that a DFA computation must repeat a state.
6. DeMorgan's Laws ensure that
   a Closure under intersection and complementation imply closure under union.
7. The regular operations are
   d star, union, and concatenation.
8. To show that a language is regular, one could give a DFA for it. One could also
   d give a regular expression or use closure properties
9. To show that a language is **not** regular, one could
   e use the pumping lemma for regular languages or use closure properties
10. The pumping lemma for regular languages implies that
    c a regular language is infinite if and only if it contains a string that can be pumped
11. A generalized NFA, or GNFA,
    c can have transitions labelled with $\emptyset$ or $\emptyset^\star$
12. To rip a state q_{rip} in the GNFA method, when there is a transition from q to q_{rip} labeled A, a transition from q_{rip} to q_{rip} labeled B, a transition from q_{rip} to q' labeled C, and a transition from q to q' labeled D, we make a transition from q to q' labeled
    a $(A(B)^\star C) \cup D$
13. The GNFA method is used to show that
    e Every regular language is described by a regular expression.
14. Which of these does not imply that L is regular?
    b L is closed under the regular operations: i.e, $L = LL = L \cup L = L^\star$.
15. The *agreement* A of two languages L_1 and L_2 is a language consisting of all strings that are in both or neither of the two. Suppose that L_1 and L_2 are regular.
    d A is regular because we could construct a DFA using the product construction
16. For an NFA $M = (Q, \Sigma, \delta, q_0, F)$ and a subset $S \subseteq Q$, which of the following is *not always true* about the ε closure $E(S)$?
    b there is no ε transition from a state not in $E(S)$ to a state in $E(S)$
17. $\{a^n b^m : n > m \geq 0\}$ is
    a not regular because $a^{p+1} b^p$ cannot be pumped

Question 1. [11 marks] Let $L \subset \{a, b\}^\star$ be the language

$$L = \{a^n b^m : n < \sqrt{m} \text{ or } m < \sqrt{n}\}.$$

State whether or not L is regular, and show that your answer is correct.

L **is not regular.** To show this, suppose to the contrary that L is regular, and let p be its pumping length from the pumping lemma for regular languages.

Choose $w = a^p b^{p^2+1}$. Note that $w \in L$ because $p < \sqrt{p^2+1}$, and $|w| = p^2+p+1 \geq p$. Therefore by the pumping lemma, we can write $w = uvx$ with v not empty and $|uv| \leq p$ so that $uv^ix \in L$ for all $i \geq 0$. Because uv is a prefix of w of length at most p, it contains only as. So without loss of generality, $u = a^\alpha$, $v = a^\beta$, and $x = a^{p-\alpha-\beta} b^{p^2+1}$, with $\alpha \geq 0$, $\beta \geq 1$, and $\alpha + \beta \leq p$.

Now $uv^2x = a^{p+\beta} b^{p^2+1}$, and $p + 1 \leq p + \beta \leq 2p$. Because $p + 1 > \sqrt{p^2+1}$, uv^2x can only belong to L if $p^2+1 < \sqrt{p+\beta} \leq \sqrt{2p}$. But for every positive integer p, $p^2+1 > \sqrt{2p}$. We conclude that $uv^2x \notin L$, which is a contradiction. So L is not regular.

Other strings you might have considered:

$w = a^{p^2+1} b^p$ good choice, by considering uv^0x, i.e. pumping down.

$w = a^p b^{p+p!}$ good choice, using the p! trick.

$w = a^{p^2} b^p$ **or** $w = a^p b^p$ bad choices, because they are not in L.

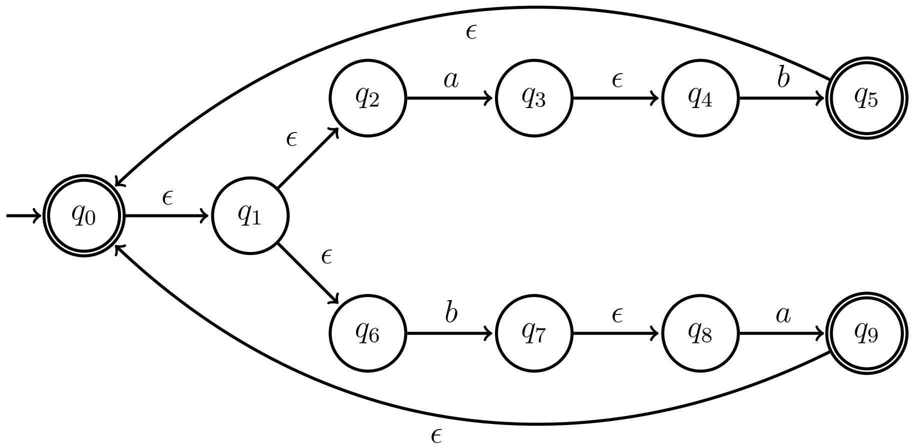
$w = a^{\sqrt{p}-1} b^p$ bad choice because we cannot be sure that $\sqrt{p}$ is an integer.

$w = a^{\lceil\sqrt{p}\rceil-1} b^p$ **or** $w = a^{p-1} b^{p^2}$ bad choice because it can be pumped in the bs.

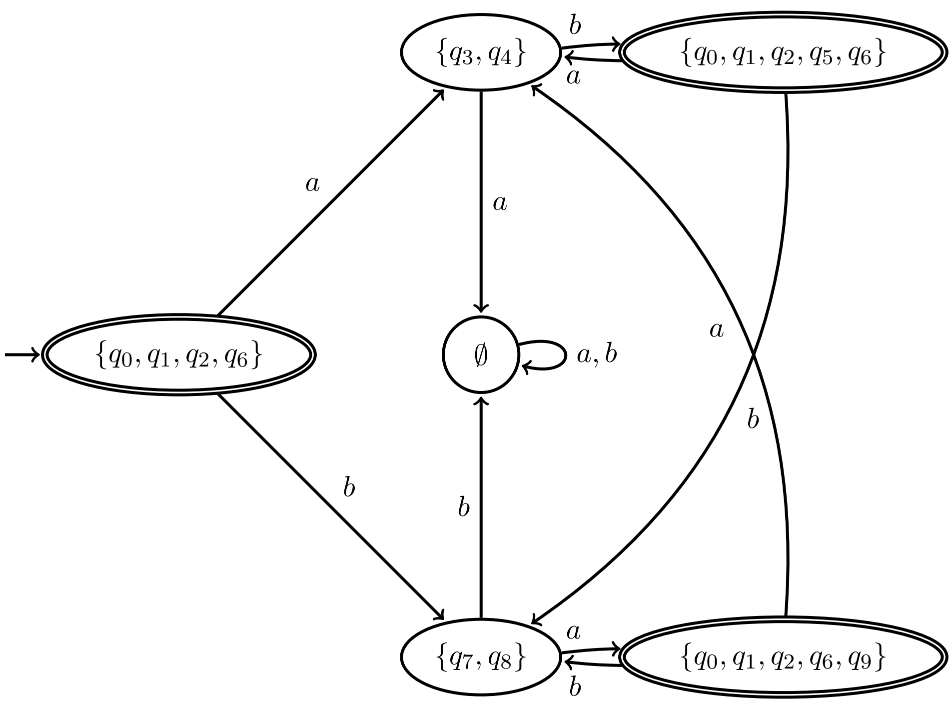
$w = abbb$ bad choice because the string may be too short.

Some students wrote $L = L_1 \cup L_2$ with $L_1 = \{a^n b^m : n < \sqrt{m}\}$ and $L_2 = \{a^n b^m : m < \sqrt{n}\}$. Then they argued that neither L_1 nor L_2 is regular, which is correct. But then they wanted to conclude that therefore L is not regular. But this is not true in general. Indeed let N be any language that is not regular. Then $\overline{N}$ is not regular either. However $N \cup \overline{N} = \Sigma^\star$, which definitely is regular. So this 'argument' is fallacious.

Question 2. [11 marks]
(a) [5 marks] *Using the method from class*, produce an NFA for the regular expression $(ab \cup ba)^\star$. Do not simplify. (You can give a transition diagram or a transition table, and do not need both.)



(b) [6 marks] *Using the powerset method*, produce a DFA for the NFA from the (a) part. Do not simplify the NFA first. (You can give a transition diagram or a transition table, and do not need both.)



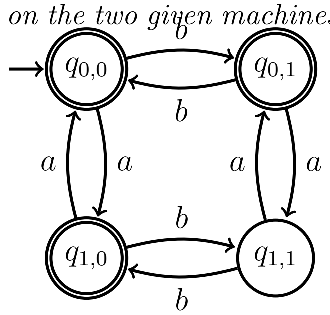
Question 3. [11 marks] DFAs with alphabet $\{a, b\}$ and transition functions

State $\downarrow$ Symbol $\rightarrow$	a	b
q_0 – start, final	q_1	q_0
q_1	q_0	q_1

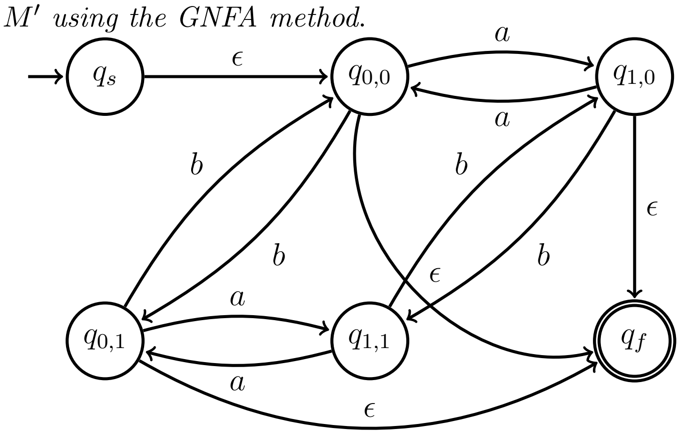
State $\downarrow$ Symbol $\rightarrow$	a	b
q_0 – start, final	q_0	q_1
q_1	q_1	q_0

recognize languages of strings with an even number of as, and an even number of bs, respectively.

(a) [5 marks] Let $L \subset \{a, b\}^\star$ consist of strings having an an even number of as *or* an even number of bs. *Using the product construction on the two given machines*, produce a DFA M that recognizes L.



(b) [6 marks] Show how to turn M into a GNFA M'. Then show the GNFA obtained after one state is removed from M' *using the GNFA method*.



Rip state $q_{1,1}$: