CSE355 Fall 2016–Recitation Quiz 13 (Solutions)



1. Show that $SUB_{DFA} = \{ \langle M_1, M_2 \rangle \mid M_1, M_2 \text{ are DFAs and } L(M_1) \subseteq L(M_2) \}$ is decidable. (Hint: what's another way of writing $L(M_1) \subseteq L(M_2)$?)

We create a decider D for SUB_{DFA} :

D = "On input $\langle M_1, M_2 \rangle$ where M_1, M_2 are DFAs:

- (a) Construct a DFA M'_2 with $L(M'_2) = \overline{L(M_2)}$.
- (b) Construct a DFA C with $L(C) = L(M_1) \cap L(M'_2)$.
- (c) Run the decider for E_{DFA} on input $\langle C \rangle$.
- (d) If it accepts, accept; otherwise, reject."

The reasoning is that $L(M_1) \subseteq L(M_2)$ if and only if $L(M_1) \cap \overline{L(M_2)} = \emptyset$.

- 2. Answer True or False and give a brief explanation as to your answer:
 - (a) Decidable languages are not closed under complement. False, they are closed under complement. The reasoning is that we can just swap q_{accept} and q_{reject} here, since on every input the decider will always halt in one of these states.
 - (b) The problem of checking whether a DFA accepts some palindrome (a string that is equal to its reverse) is decidable. (Hint: is the "palindromes" language a CFL?) True. Suppose the DFA is D. We can make a CFG G for the set of palindromes over the alphabet Σ. Consider L(G) ∩ L(D): this will be non-empty if and only if D accepts some palindrome. Since the intersection of a context-free language and a regular language is context-free, then we can make a CFG G' for L(G) ∩ L(D). Then we just run the decider for E_{CFG} on input ⟨G'⟩, and output what it outputs.