CSE355 Fall 2016–Recitation Quiz 14 (Solutions)

Name: ______________________________ ASU ID: ____________________

Monday 9:40AM	Monday 10:45AM	Tuesday 7:30AM	Tuesday 4:35PM	Wednesday 9:40AM
☐	☐	☐	☐	☐

Wednesday 10:45AM	Thursday 7:30AM	Thursday 4:35PM	Friday 9:40AM	Friday 10:45AM
☐	☐	☐	☐	☐

1. Show that $INFINITE_{TM} = \{\langle M \rangle \mid M \text{ is a TM and } L(M) \text{ is infinite}\}$ is undecidable by a reduction from A_{TM}.

   Suppose $INFINITE_{TM}$ were decided by I. Then we can make a decider D for A_{TM} as follows:

   D = "On input $\langle M, w \rangle$ where M is a TM:

   (a) Construct a TM M' = "On input x:

      i. If $x = \epsilon$, accept.

      ii. If $x \neq \epsilon$, run M on w and accept x only if M accepts w."

   (b) Run I on $\langle M' \rangle$.

   (c) If I accepts, accept; otherwise, reject."

   If M accepts w, then $L(M') = \Sigma^\star$, which is an infinite language; if M does not accept w, then $L(M') = \{\epsilon\}$, which is a finite language. Therefore, since A_{TM} is undecidable, then so is $INFINITE_{TM}$.

   In fact, this actually shows something more important: given *any* language $L \subseteq \Sigma^\star$, it is undecidable to check that an arbitrary TM M has $L(M) = L$.

2. Answer True or False and give a brief explanation as to your answer:

   (a) *Undecidable* languages are closed under complement.

   True. If they were not closed under complement, then some undecidable language L has $\overline{L}$ being decidable. But then since decidable languages are closed under complement, we have L being decidable, a contradiction.

   (b) If $A \leq_m B$ and B is not decidable, then A is not decidable.

   False. We cannot say anything if B is not decidable and $A \leq_m B$. However, we *can* say something if instead of B being undecidable, A was undecidable. So therefore, the reduction is "in the wrong direction."

   (c) If $A \leq_m \overline{A}$ and A is recognizable, then A is decidable.

   True. We also then have $\overline{A} \leq_m A$ with the same reduction. Since A is recognizable, then because of the result of $A \leq_m B$ with A being recognizable implies B recognizable, then $\overline{A}$ is recognizable. But we know that if a language and its complement are both recognizable, then it is decidable.

TA use only! Quiz has been recorded: ☐