A resolvable Steiner triple system

CSE/MAT 591 Homework # 4

Due 13 February

Here is a construction for an STS(15):

- elements are $(\mathbb{Z}_7 \times \{0,1\}) \cup \{\infty\}$;
- blocks are $\{\{(i,a),(i+1,b),(i+3,c)\} : i \in \mathbb{Z}_7, a,b,c \in \{0,1\}, a+b+c \equiv 0 \pmod 2\}$ and $\{\{\infty,(i,0),(i,1)\} : i \in \mathbb{Z}_7\}$.

Convince yourself that this is indeed an STS(15).

I claim that this design is resolvable, and want you to find a resolution. It has 35 blocks, and each parallel class needs $\frac{15}{3} = 5$ blocks, so a resolution needs 7 parallel classes. As presented, this system is preserved by adding 1 and reducing modulo 7 to the first coordinate of the elements in $\mathbb{Z}_7 \times \{0,1\}$. Each block produces a set of 7 distinct blocks by such repeated addition of 1. Use these facts to find a resolution of the STS(15), and explain.

(You might wonder whether this is isomorphic to the projective system, the Bose system, or neither from homework #2. It is another way of writing the projective system, and it is a fun exercise to convince yourself of that. No need to write anything about this for the homework.)

This problem was made famous in 1844 by the Reverend Thomas Penyngton Kirkman in a query in the *Lady's and Gentleman's Diary*, in which he asked for a way to parade 15 schoolgirls in ranks of three so that over the seven days, each two girls walked together exactly once.