NEURAL ACTIVITY TRACKING USING SPATIAL COMPRESSIVE PARTICLE FILTERING

Lifeng Miao *Jun Jason Zhang*[†] *Antonia Papandreou-Suppappola* *Chaitali Chakrabarti* [*]

School of Electrical, Computer and Energy Engineering, Arizona State University, Tempe, AZ
[†]Department of Electrical and Computer Engineering, University of Denver, Denver, CO

ABSTRACT

We investigate and demonstrate the sparsity of electroencephalography (EEG) signals in the spatial domain by incorporating grid spacing in the area of the head enclosing the brain volume. We exploit this spatial sparsity and propose a new approach for tracking neural activity that is based on compressive particle filtering. Our approach results in reducing the number of EEG channels required to be stored and processed for neural tracking using particle filtering. Simulations using both synthetic and real EEG signals illustrate that the proposed algorithm has tracking performance comparable to existing methods while using only a reduced set of EEG channels.

Index Terms— Compressive sensing, EEG, dipole model, multiple particle filter.

1. INTRODUCTION

Advances in scanning technology during the last few decades have extended our understanding of the human brain pathology. Magnetoencephalography (MEG) and electroencephalography (EEG) offer temporal resolutions below 100 ms, allowing studies on the dynamics of basic neural activities [1]. Although MEG/EEG measurements provide high temporal resolutions, hundreds of sensors need to be placed on the scalp in order to provide high spatial resolution and localize neuronal activity. One of the biggest challenges with MEG/EEG data collection and analysis is that huge amounts of data need to be stored and processed; MEG/EEG data reduction or compression has thus become an important issue.

Compressive sensing (CS) is a method used to recover a signal from a small number of projections onto a basis, provided that the signal has a sparse representation in another basis, that is incoherent with the first basis [2]. Recently, CS methods have been investigated for the efficient acquisition of EEG signals. In [3], it was shown that EEG signals are sparse when represented using Gabor basis functions and chirped Gabor basis functions, and this property is utilized to recover multiple channel, multiple trial EEG data from a small number of measurements. In [4], Bayesian CS (BCS) techniques were developed by exploiting the sparsity of EEG signals in spatio-temporal dictionaries. In [5], an EEG sensor design method was proposed to generate high fidelity EEG measurements using a CS approach. In [6], the performance of various CS implementations is compared and quantified for scalp EEG signals.

In this paper, we propose an efficient spatial domain EEG CS technique, which results in a significant reduction in the amount of EEG data that needs to be stored and processed. Sequential Bayesian estimation techniques, such as the particle filter (PF) [7], have been used to track neural activity dipole sources. [8]. However, as the number of dipole sources increases, the computation complexity of the PF tracking algorithm grows proportionally. In order to avoid this high computational complexity, we first analyze the EEG data sparsity in the spatial domain using equivalent current dipole source representations. We then compressively sense the multiple channel EEG signals using an independent and identically distributed Gaussian function basis. Finally, by using the compressive particle filter algorithm [9], we apply the PF on the spatial compressed EEG data to localize the neural activities with reduced computational complexity.

2. COMPRESSIVE SENSING

CS can reduce the number of measurements required to reconstruct a signal since a small set of linear projections of a sparse signal contains enough information for reconstruction. Let $\mathbf{z} \in \Re^{d_z}$ denote a vector with d_z elements; then $\mathbf{z}$ is said to be K-sparse if $\mathbf{z}$ can be represented by $\mathbf{z} = \Psi\boldsymbol{\theta}$, where the columns of $\Psi \in \Re^{d_z \times d_{\boldsymbol{\theta}}}$ constitute a basis function and $\boldsymbol{\theta}$ is a vector with at most K non-zero elements and $K << d_{\boldsymbol{\theta}}$ [9]. CS theory states that it is highly probable that $\boldsymbol{\theta}$ and $\mathbf{z}$ can be exactly recovered from the measurements $\mathbf{y} = \Phi\mathbf{z}$, where $\Phi \in \Re^{d_y \times d_z}$ is a projection matrix and $d_y < d_z$ [2]. It has been shown that the signal can be recovered from its measurements when the projection matrix Φ is incoherent with the basis Ψ that the signal is sparse over. A typical choice for the projection matrix, Φ, is a random matrix with independent and identically distributed Gaussian or Bernoulli entries. The vector $\mathbf{z}$ can be recovered from the measurement $\mathbf{y}$ by solving the following optimization problem [2]

$$\arg\min_{\boldsymbol{\theta}} \; ||\boldsymbol{\theta}||_1 \quad \text{subject to} \quad \Phi\Psi\boldsymbol{\theta} = \mathbf{y}, \qquad (1)$$

[*]This work was supported by NSF under Grant No. 0830799.

where $||\cdot||_i$ denotes the ℓ_i norm. When $d_y << d_z$, CS theory can be used to reduce the dimensionality of vector **z**.

3. SPATIAL SPARSITY OF EEG SIGNAL

At time k, we assume that a small patch of activated cortex can be represented by an equivalent current dipole with three-dimensional (3-D) location $\mathbf{p}_k$ and 3-D moment $\mathbf{m}_k = s_k\,\mathbf{q}_k$, where $\mathbf{q}_k$ is a 3-D orientation vector and s_k is the current amplitude of the dipole [1]. Using the dipole source model, EEG signals acquired by M sensors are represented as

$$\mathbf{z}_k = \mathbf{A}_k\,\mathbf{s}_k + \mathbf{n}_k. \quad (2)$$

where $\mathbf{z}_k = [z_k^1\ z_k^2 \ldots z_k^M]^{\mathcal{T}}$ is the M channel EEG signal at time k, $\mathbf{s}_k = [s_k^1\ s_k^2\ \ldots s_k^{N_d}]^{\mathcal{T}}$ is the amplitude of N_d dipoles at time k, $\mathbf{n}_k$ is the measurement noise, and $\mathbf{A}_k$ is the $M \times N_d$ gain matrix. The jth column of this matrix is given by $\mathbf{a}_k^j = \mathbf{F}(\mathbf{p}_k^j)\mathbf{q}_k^j$, where the lead field $\mathbf{F}(\mathbf{p})$ is represented by a $M \times 3$ matrix and is a nonlinear function of the dipole location **p** [10].

In order to show the spatial sparsity of the EEG signal, we constrained the dipoles to G grids in a 3-D Cartesian coordinate enclosing the brain volume. For the fixed Cartesian coordinate y, the grid is shown in Figure 1. Given this constraint, the M-channel EEG signal can be approximated as,

$$\mathbf{z}_k = \mathbf{\Psi}\,\boldsymbol{\theta}_k + \mathbf{n}_k \quad (3)$$

where $\mathbf{\Psi} = [\mathbf{F}(\mathbf{r}_1)\ \mathbf{F}(\mathbf{r}_2)\ \ldots\ \mathbf{F}(\mathbf{r}_G)]$ is an $M \times 3G$ matrix, $\mathbf{F}(\mathbf{r}_i)$ is the $M \times 3$ lead field of the ith grid located at $\mathbf{r}_i$, $\boldsymbol{\theta}_k$ is a $3G \times 1$ vector with elements $[\mathbf{m}_k^1\ \ldots\ \mathbf{m}_k^G]^{\mathcal{T}}$, and $\mathbf{m}_k^i = [m_k^{i,x}\ m_k^{i,y}\ m_k^{i,z}]$ represents the moment of a dipole concentrated on the ith grid at time k. Since there are N_d dipoles, only N_d grids have non-zero moments. As a result, at most $3 \times N_d$ elements in $\boldsymbol{\theta}_k$ are non-zero, where $N_d \ll G$. Thus, according to CS theory, the EEG signal $\mathbf{z}_k$ is $3 \times N_d$-sparse in the spatial domain.

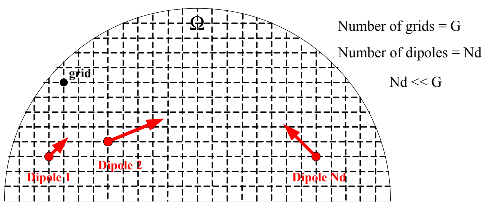


Fig. 1. Sparse current dipole signals.

We define the L-dimensional compressed measurement as

$$\mathbf{y}_k = \mathbf{\Phi}\mathbf{z}_k + \boldsymbol{\epsilon}_k = \mathbf{\Phi}\mathbf{\Psi}\boldsymbol{\theta}_k + \boldsymbol{\epsilon}_k \quad (4)$$

where $L \ll M$, $\mathbf{\Phi}$ is the $L \times M$ projection matrix uncorrelated with $\mathbf{\Psi}$, and $\boldsymbol{\epsilon}$ is the projected noise. CS theory ensures that the compressed measurement $\mathbf{y}_k$ has all the information of $\mathbf{z}_k$ and $\boldsymbol{\theta}_k$. As a result, we can use $\mathbf{y}_k$ as our new measurement in tracking neural dipole sources.

4. SPATIAL COMPRESSED PARTICLE FILTERING

Using the new compressed measurement vector, we can reformulate the state-space model for the neural dipole source tracking problem as

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \mathbf{u}_k, \quad (5)$$
$$\mathbf{y}_k = \Phi h(\mathbf{x}_k) + \mathbf{v}_k, \quad (6)$$

where $\mathbf{x}_k$ is the state of a dipole at time k including its 3-D location $\mathbf{p}_k$ and 3-D moment $\mathbf{m}_k$, $h(\cdot)$ is the nonlinear measurement function obtained from equation (2), $\mathbf{u}_k$ is the modeling error random process, and $\mathbf{v}_k$ is a combination of measurement noise and projection noise ϵ_k. Based on this model, we use a particle filter (PF) to track the state **x** of the neural dipole source. Here, we use the spatial compressed measurements **y** as the PF input, and the steps of the proposed algorithm are described next.

Initialization: The samples $\{\mathbf{x}_0^{(i)}\}_{i=1}^N$ are drawn from the initial density $p(\mathbf{x}_0)$, where N is the number of particles and $p(\mathbf{x}_0)$ is a uniform distribution all over the head. The weights are assigned to be initially equal, $w_0^{(i)} = 1/N$.

Prediction: For $k = 1, \ldots, K$, the particles $\mathbf{x}_k^{(i)}$ are drawn from the probability density $p(\mathbf{x}_k|\mathbf{x}_{k-1}^{(i)})$. Then the prior density $p(\mathbf{x}_k|\mathbf{y}_{1:k-1})$ can be approximated by $p(\mathbf{x}_k|\mathbf{y}_{1:k-1}) = \sum_{i=1}^N w_{k-1}^{(i)}\delta(\mathbf{x}_k - \mathbf{x}_k^{(i)})$.

Update: The weights $w_k^{(i)}$ are updated based on the new compressed measurement y_k using $w_k^{(i)} \propto w_{k-1}^{(i)} p(\mathbf{y}_k|\mathbf{x}_k^{(i)})$. We also normalize the weights $w_k^{(i)} = w_k^{(i)} / \sum_{i=1}^N w_k^{(i)}$.

Resampling: The particles are resampled based on their weights to obtain $\{\mathbf{x}_k^{(i)}, w_k^{(i)}\}_{i=1}^N$. The posterior density $p(\mathbf{x}_k|\mathbf{y}_{1:k})$ can be approximated by

$$p(\mathbf{x}_k|\mathbf{y}_{1:k}) = \sum_{i=1}^N w_k^{(i)}\delta(\mathbf{x}_k - \mathbf{x}_k^{(i)}).$$

Thus, the measurement $\mathbf{y}_k$ is the spatial compressed version of the original measurement $\mathbf{z}_k$, $\mathbf{y}_k = \mathbf{\Phi}\mathbf{z}_k + \mathbf{v}_k$. Typically, the elements of the projection matrix are random variables drawn from independent and identically distributed Gaussian distributions or Bernoulli distributions to make sure that $\mathbf{\Phi}$ is uncorrelated with $\mathbf{\Psi}$. After the spatial compressive sensing, the dimension of the measurement has been reduced from M to L, where $L \ll M$. As a result, the amount of EEG data that is needed to be stored can be reduced from $M \times K$ to $L \times K$ bytes. The likelihood $p(\mathbf{y}_k|\mathbf{x}_k)$ is now an L-dimensional Gaussian density function instead of an M-dimensional density function. This results in a significant reduction in the computational complexity of the signal processing operations that are a function of the number of EEG channels.

This algorithm can be extended to multiple dipole source tracking problems by using the multiple particle filtering framework [8]. In our simulation results, we use one PF for each dipole source. The jth sub-PF uses the same prediction and resampling steps as the ones just just described. However, the weight update step is modified to $w_{j,k}^{(i)} \propto w_{j,k-1}^{(i)} p(\mathbf{y}_k|\mathbf{x}_{j,k}^{(i)}, \tilde{\mathbf{x}}_{-j,k})$, where $\tilde{\mathbf{x}}_{-j,k} = [\tilde{\mathbf{x}}_{1,k}^{\mathcal{T}} \ \ldots \ \tilde{\mathbf{x}}_{j-1,k}^{\mathcal{T}} \ \tilde{\mathbf{x}}_{j+1,k}^{\mathcal{T}} \ \ldots \ \tilde{\mathbf{x}}_{N_d,k}^{\mathcal{T}}]^{\mathcal{T}}$ are the predicted values of all the states, excluding $\tilde{\mathbf{x}}_{j,k}$ and $\tilde{\mathbf{x}}_{j,k} = \sum_{i=1}^{N} w_{j,k-1}^{(i)} \mathbf{x}_{j,k}^{(i)}$.

5. RESULTS

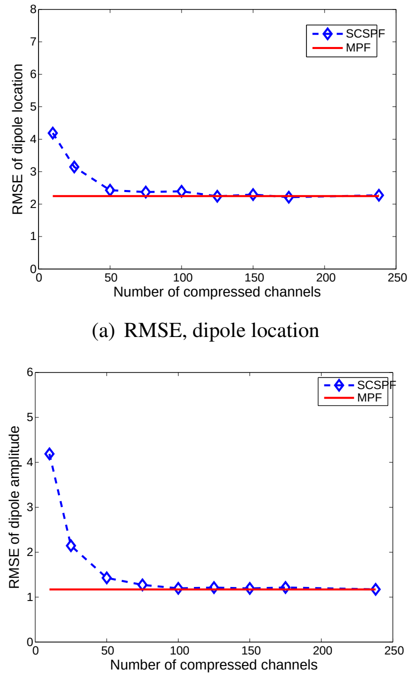
We compared our proposed spatial CS particle filter (SCSPF) with the multiple particle filter (MPF) [8] using simulated and real EEG data. For our simulations, we draw each element of the projection matrix $\mathbf{\Phi}$ from a zero-mean, additive white Gaussian random process with unit variance elements.

Simulated data results. The synthetic data was created by inserting three current dipoles into a sphere head model and calculating the resulting EEG signals using Equation (2) with Gaussian noise. The three dipoles are localized at $V1$ (1.11, 5.34, 4.98), $V5_R$ (4.36, 3.68, 4.44) and $V5_L$ (3.37, 4.85, 4.81) from a previous study. For this simulation, we used 1,000 particles for each dipole. The particles are initially uniformly distributed in the brain volume hemisphere with a radius of 85 mm. The dipole evolution model in (6) is a random walk with Gaussian transition kernel $p_{k+1|k}(\mathbf{p}_{k+1}|\mathbf{p}_k) = \mathcal{N}(\mathbf{p}_k, \sigma_p)$ and $p_{k+1|k}(\mathbf{m}_{k+1}|\mathbf{m}_k) = \mathcal{N}(\mathbf{m}_k, \sigma_m)$, with σ_p=1 cm and σ_m=2 nA, where $\mathbf{p}$ and $\mathbf{m}$ are the 3-D dipole location and moment vector, respectively.

In Figure 2, we show the estimation results, averaged over 50 independent runs, in terms of root mean-squared error (RMSE), for both the proposed SCSPF and the MPF [8]. The MPF used all 238 EEG measurements, and the resulting RMSE for estimating the dipole location and amplitude is shown in red in Figures 2(a) and (b), respectively. The figures also show that the RMSE estimation performance of the SCSPF is comparable to the MPF using only 50 compressed channel signals; this shows is a significant reduction in the use of measurements. Note that when the number of compressed channels is greater than 50, there is no noticeable improvement in the RMSE. Thus, we choose the size of the projection matrix $\mathbf{\Phi}$ to be 50×238. From Figure 2, we can see that the location and amplitude RMSE for SCSPF using 50 compressed channel signals are 2.42 mm (2.25 mm for MPF) and 1.43 nA (1.17 nA for MPF), respectively. Assuming that there are 100 time steps in this simulations, the amount of EEG data needed to be stored has been reduced from $238 \times 100 \approx 24$ k bytes to $50 \times 100 = 5$ k bytes. Table 1 shows a comparison of the two competing methods in terms of computing operations. Note that SCPF has reduced number of additions and multiplications.

Real EEG data results. Next we apply the proposed SCSPF to real EEG data that is publicly available [11]. In this experiment, the subject's screen showed 5 empty boxes arranged horizontally above the screen center. At the screen center, there was a plus sign which was used as the fixation point throughout the experiment. When the task began, a white disc would appear in any one of the boxes for 100 ms. The location of the box is called the attended location. The subject was instructed to press the response button whenever the disc appeared at the attended location. Data were collected from 238 scalp, neck, face and eye locations using the Biosemi Active Two system. Datai is referenced with respect to the electrode located in the right mastoid.

First, we preprocess the real EEG data based on the methods in [12]. The preprocessing steps include bandpass filtering, event extracting and independent component analysis. Next we apply the SCSPF and MPF on the preprocessed EEG data. Here, we choose the projection matrix $\mathbf{\Phi}$ as an 50×238 matrix with elements drawn from standard Gaussian distribution. We estimate the locations and amplitudes of three dipoles and for each dipole we use 1,000 particles for both the SCSPF and MPF algorithms. We also compare the estimation results with the dipole fitting method in [12]. Since the true locations and amplitudes of the dipoles are unknown in the real data case, we set the dipole fitting results in [12] as the *ground truth*. The estimation results are shown in Figure 3



(a) RMSE, dipole location

(b) RMSE, dipole amplitude

Fig. 2. RMSE estimation performance comparison.

Table 1. Computing operators for SCSPF and MPF.

Algorithm	+	$\times$	$\div$	$\sqrt{}$	exp
MPF	323	341	1	2	1
SCPF	142	165	1	2	1

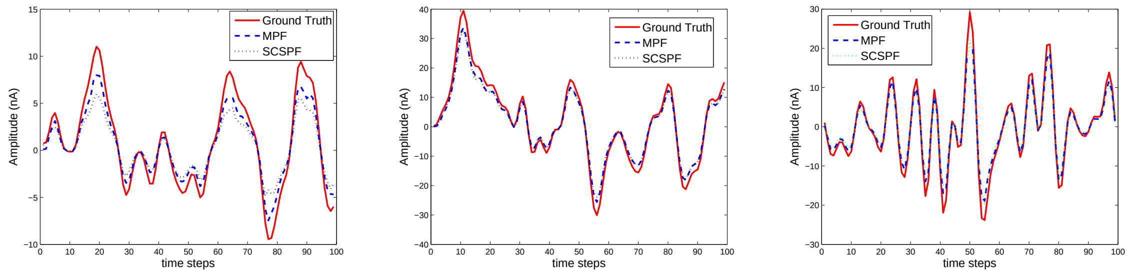


Fig. 3. Estimation of the amplitudes of three dipoles.

and Figure 4 and the RMSE is shown in Table 2. From Figure 3 and Figure 4, we can see that the amplitudes and locations of the dipoles estimated by SCSPF and MPF match with the results given in [12]. Table 2 indicates that the SCSPF can give comparable RMSE performance with the MPF. However, since for SCSPF we use only 50 compressed channel measurements instead of the 238 original channel measurements, the amount of EEG data needed to be stored and processed is significantly reduced. For one hour of EEG data acquisition with 1 kHz sampling rate, the amount of EEG data needed to be stored has been reduced from 857 MB to 180 MB.

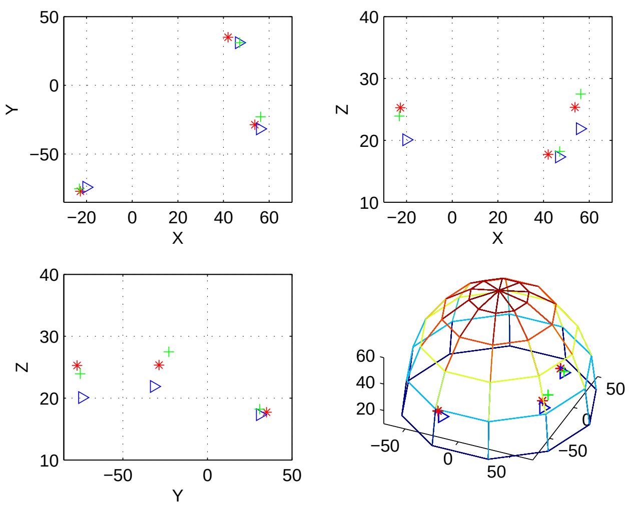


Fig. 4. Estimation of the locations of three dipoles.

Table 2. RMSE comparison: SCSPF (50 channels) and MPF

Algorithm	Location	Amplitude
MPF	5.17 mm	2.63 nA
SCSPF	5.62 mm	2.85 nA

6. CONCLUSION

We first investigated the spatial domain sparsity of EEG signals, and then we proposed its application to compressive sensing of EEG signals. We applied multiple particle filtering on the compressed EEG data to track the locations and amplitudes of the neural activities. The RSME tracking results of the proposed algorithm are comparable with those of conventional methods. However, the number of required EEG channels is now reduced from 238 to 50. Thus, the use of compressive sensing significantly reduces the required storage capacity and computational complexity involved in EEG processing.

7. REFERENCES

[1] S. Baillet, J.C. Mosher, and R.M. Leahy, "Electromagnetic brain mapping," *IEEE Signal Processing Magazine*, vol. 18, no. 6, pp. 14–30, Nov. 2001.

[2] E. Candes and T. Tao, "Decoding by linear programming," *IEEE Transactions on Information Theory*, vol. 51, no. 12, pp. 4203–4215, 2005.

[3] S. Aviyente, "Compressed sensing framework for EEG compression," in *IEEE/SP 14th Workshop on Statistical Signal Processing*, 2007, pp. 181–184.

[4] A. Bolstad, B. V. Veen, and R. Nowak, "Space-time sparse reconstruction for magneto-electroencephalography," *NeuroImage*, vol. 46, pp. 1066–1081, 2009.

[5] Q. Hao and F. Hu, "A compressive eletroencephalography (EEG) sensor design," in *IEEE Sensors*, Nov. 2010, pp. 318–322.

[6] A. M. Abdulghani, A. J. Casson, and E. Rodriguez-Villegas, "Quantifying the performance of compressive sensing on scalp EEG signals," in *Symposium on Applied Sciences in Biomedical and Communication Technologies*, 2010.

[7] M.S. Arulampalam, S. Maskell, N. Gordon, and T. Clapp, "A tutorial on particle filters for online nonlinear/non-Gaussian Bayesian tracking," *IEEE Transactions on Signal Processing*, vol. 50, no. 2, pp. 174–188, 2002.

[8] L. Miao, J. J. Zhang, C. Chakrabarti, and A. Papandreou-Suppappola, "Multiple sensor sequential tracking of neural activity with FPGA implementation," in *Asilomar Conf. Signals, Systems and Computers*, November 2010, pp. 369–373.

[9] E. Wang, J. Silva, and L. Carin, "Compressive particle filtering for target tracking," in *Statistical Signal Processing Workshop*, Sept. 2009, pp. 233–236.

[10] J.C. Mosher, R.M. Leahy, and P.S. Lewis, "EEG and MEG: forward solutions for inverse methods," *IEEE Transactions on Biomedical Eng.*, vol. 46, pp. 245–259, March 1999.

[11] M. Westerfield, T. P. Jung, S. Enghoff, J. Townsend, E. Courchesne, and T. J. Sejnowski, "Dynamic brain sources of visual evoked responses," *Science*, vol. 295, pp. 690–694, Jan. 2002.

[12] "EEGlab," `http://sccn.ucsd.edu/eeglab/`.