Compact Modeling of STT-MTJ for SPICE Simulation

Zihan Xu, Ketul B. Sutaria, Chengen Yang, Chaitali Chakrabarti, Yu Cao
School of Electrical, Computer and Energy Engineering, Arizona State University
Tempe, Arizona 85287, USA
Email: {zihanxu, kbsutaria, chengen.yang, chaitali, ycao}@asu.edu

Abstract—STT-MTJ is a promising device for future high-density and low-power integrated systems. To enable design exploration of STT-MTJ, this paper presents a fully compact model for efficient SPICE simulation. Derived from the fundamental LLG equation, the new model consists of RC elements that are closed-form solutions of device geometry and material properties. They support transient SPICE simulations, providing necessary details beyond the macromodel. The accuracy is validated with numerical results and published data.

INTRODUCTION

As Si technology is scaling toward the 10nm regime, CMOS-based devices may no longer be the technology of choice. Instead, emerging devices, such as spin-transfer torque (STT) based magnetic tunnel junction (MTJ), are more scalable and are viable alternatives for both memory and logic applications in the post-silicon era [1][2]. Indeed, STT-MTJ promises a good combination of high density, fast access, low power consumption, and non-volatile data storage [3]. Figure 1 illustrates the basic structure and the switching behavior of a STT-MTJ device [4]. By programming it with a current pulse, the magnetic moment in the free layer can be switched to be 0° (parallel, P) or 180° (anti-parallel, AP) to that in the fixed layer. This is the essential process to define the digital state.

To explore the design potential of STT-MTJ, compact modeling is a critical bridge between the underlying technology and large-scale design practice. This compact model should be fully compatible with the SPICE simulator, such that designers can mix STT-MTJ with CMOS devices to realize new functions. It should be scalable with process and operation conditions, capturing all dynamic and static properties during the operation. Furthermore, the new compact model should have high simulation efficiency, which is important for statistical design of memory applications.

Such a model is presented in this paper. Previous approaches either only address the macromodel [4], which lacks sufficient flexibility and switching details, or still use complicated equations and iterations in SPICE simulation [3]. Our work transfers the fundamental Landau-Lifshitz-Gilbert (LLG) equation into a passive RC network, in which all components are closed-form solutions of device geometry and material properties. As demonstrated in Fig.1, the new SPICE model efficiently generates the transient behavior under all programming conditions. The physical basis of model derivation further helps gain design insights on STT-MTJ.

Figure 1. An in-plane STT-MTJ is programmed by a current pulse. The magnitude (I) and the width (τ) of the current pulse determine the switching success of the magnetic angle (θ) in the free layer.

COMPACT MODELING OF STT-MTJ

The modeling focus is to solve the LLG equation into an equivalent RC network, for all possible switching conditions.

A. STT-MTJ Basics

Figure 1 shows the three layers in STT-MTJ at the radius r and the dielectric thickness Tox. The magnetic moment (M) of one layer is free while the other is fixed. With I, the spin of the electrons is polarized by the fixed layer, and transferred to the free layer. The state of P or AP affects MTJ resistance [1]. The dynamics of M is defined by the LLG equation [5][6]:

$$\frac{dM}{dt} = -\gamma \mu_0 M \times H_d - \frac{2K}{M_s} (M \times \hat{a}_x) (M \times \hat{a}_y) + \frac{\alpha M_s}{M_s} \frac{dM}{dt} + \eta I + \frac{eV}{2}\int dV$$

The terms (from left to right) represent the Zeeman torque, by both the local field and the thermal fluctuation field, the anisotropic torque, the damping torque, and the spin-transfer torque, in which the efficiency (η) depends on the current direction. Table I defines key model parameters.

<table>
<thead>
<tr>
<th>TABLE I. PARAMETERS ON THE GEOMETRY AND MATERIALS.</th>
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<tr>
<td>Saturation magnetization (Ms)</td>
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<tr>
<td>Ms = (1 - (\frac{1}{4\gamma H_c})) \exp \left[ \frac{25}{3\mu_0 4\gamma H_c} \right]</td>
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<tr>
<td>Ef. magnetic field ((\mu_0 H_{\text{eff}}))</td>
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<td>--------------------------------</td>
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<tr>
<td>1.76x10^11 rad-s^-1/T</td>
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Numerical - Model
B. Dynamic Magnetization Model

The switching of the magnetic moment is the key dynamics in STT-MTJ. In general, it is a three-dimensional movement: the Zeeman torque and the anisotropic torque contribute to the rotation in the plane perpendicular to the easy axis, indicated by an angle \( \phi \); the damping torque and the spin-transfer torque dominate the switching in the easy plane, resulting in the change of \( \theta \). Considering a realistic structure of STT-MTJ (Fig. 1), the change of the magnetic moment can be separated into two planes and thus, the LLG equation reduced to two scalar equations of magnetic angle \( \phi \) and \( \theta \).

\[
M_e \frac{d\phi}{dt} = -\gamma_H M_e H \sin \theta - 2K \sin \theta \cos \theta \tag{2}
\]

\[
M_s \frac{d\theta}{dt} = \alpha M_s \frac{d\phi}{dt} + \eta \frac{\mu_i I}{eV} \tag{3}
\]

Substituting \( \frac{d\phi}{dt} \) from Eq. (2) to Eq. (3):

\[
M_s \frac{d\theta}{dt} = -\gamma_H M_e H \sin \theta - 2K \sin \theta \cos \theta + \frac{\mu_i I}{eV} \tag{4}
\]

The scalar equation of Eq. (4) is the foundation to analyze the switching dynamics of \( \theta \). Based on Eq. (4), Fig. 2 plots \( d\theta/dt \) for different \( I \) when \( \theta \) changes from 0° to 180°. Some critical points are highlighted below in order to obtain the physical map for further model derivation:

- **Threshold current (\( I_{th} \))**: This concept separates two possible switching mechanisms in a STT-MTJ device, precession switching and thermally assisted switching. When \( I > I_{th} \), \( d\theta/dt \) is always > 0 (Fig. 2) and thus, the magnetic angle is able to complete the switching with sufficient \( \tau \). However, when \( I < I_{th} \), \( d\theta/dt \) may be < 0, requiring the assist of thermal fluctuation \( H_T \) to switch the system. \( I_{th} \) can be solved from the minimum of \( d\theta/dt = 0 \), which is associated with the threshold angle \( \theta_{th} \) (Fig. 2).

- **Critical angle \( (\theta_c) \)**: This angle defines a critical value the magnetic moment has to reach at the end of the current pulse; if \( \theta \) at time = \( \tau \) is smaller than \( \theta_c \), the damping torque may pull \( \theta \) back to 0° (Fig. 1). As observed in Fig. 2, when \( I = 0 \), there are three points to satisfy \( d\theta/dt = 0 \): 0° and 180° are two stable solutions, while \( \theta_c \) is a metastable point. This behavior is similar as that in a SRAM cell, and helps us develop the model of \( \theta_c \).

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**Table II. Models of Critical Points \((P \to AP)\).**

| \( \theta_{th} \) | \( \cos^{-1} \left( \sqrt{\mu_i M_s H} \right) + 32K^2 - \mu_i M_s H \) [\( \text{[K]} \)] |
| \( I_{th} \) | \( \eta \gamma_H M_s H \sin \theta + K \sin (2\theta_0) \) |
| \( \theta_c \) | \( \cos^{-1} \left( \frac{-\mu_i M_s H}{|\epsilon|} \right) \) |
| \( I_c \) | \( \left\{ \eta \right\} \left[ 2.1 \times 10^{-5} M_e/\tau + 0.34 M_s H + 4.26 \times 10^{-5} \right] \)

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**Figure 3.** Validation with published STT-MTJ data [4]. (\( \tau = 45\text{nm}, T_{as} = 0.85\text{nm} \)).

- Critical current \( (I_c) \): Given the pulse width \( \tau \), \( I_c \) is the minimum current required to switch the magnetic angle from 0° to \( \theta_c \). \( I > I_c \) ensures a successful precession switching. To solve \( I_c \), Eq. (4) is integrated from 0° to \( \theta_c \) for \( d\theta \) and from 0 to \( \tau \) for \( dt \). A compact solution is obtained (Table II). \( I_c \) is proportional to the inverse of \( \theta_c \), implying a tradeoff between speed and the writing power in design optimization.

Table II summarizes the models for \( P \to AP \), in standard international units. For \( AP \to P \), the formulas remain the same, but with different coefficient values due to the different initial condition. The formulas in Table II have a clear root in physics, and are accurate in the precession switching. They are scalable with process and material parameters, supporting the development of the RC network in Section III. Figure 3 validates our model with the measurement data [4]. Due to the operation nature of STT-MTJ, it requires more current and energy to switch it from \( P \) to \( AP \) (Fig. 3).

C. TMR Model of the Tunnel Junction

The switching of the magnetic angle represents the write process of STT-MTJ. The read of the state is by characterizing the resistance of MTJ. When a read current is delivered to STT-MTJ, the resistance reaches a low value \( (R_P) \) if the magnetic moments in both ferromagnetic layers are in parallel; otherwise a higher resistance \( (R_D) \) is detected. Coupled with the dynamic magnetization procedure in previous section, this property completes the operation of STT-MTJ.

The tunnel magneto-resistance (TMR) of MTJ is defined as \( (R_D - R_P)/R_P \). During the continuous switching of the magnetic angle, the change of MTJ resistance follows [9]:

\[
R(\theta) = 2R_P \left( \frac{1 + \text{TMR}}{2 + \text{TMR} \cos \theta} \right) \tag{5}
\]

The static values of \( R_D \) and \( R_P \) are calculated from the tunneling current through \( T_{as} \) [9]. Eq. (5) is used to model the dynamic resistance during the switching period.
The details of the switching period are important for various design purpose, such as power and yield [3]. In addition, design applications of STT-MTJ usually involve CMOS as the control device. For these reasons, compact model of STT-MTJ needs to be embedded into the SPICE simulator. Different from previous approach that directly implement the LLG equation through complex Verilog-A codes [3][4], this work proposes a simple RC network that is physical, intuitive, and general.

### D. Regional RC Structure for Circuit Simulation

Starting from the fundamental LLG equation (Eq. (4)), \( \sin \theta \) can be approximated as \( \theta \), 1, or -\( \theta \), when \( \theta \) is close 0\(^\circ\), 90\(^\circ\), or 180\(^\circ\), respectively. A similar treatment can be applied to \( \cos \theta \). By expanding \( \sin \theta \) and \( \cos \theta \) in this approach, \( d\theta/dt \) in Eq. (4) is expressed as a linear function of \( \theta \), and thus, the solution of the LLG equation is transferred as a passive RC network for SPICE simulation.

Based on this general principle, four distinct regions are recognized, easing the implementation. Figure 4 shows the network which supports transient SPICE simulations, with the output node representing \( \theta \). \( R \)s are functions of those critical points in Table II, and \( C \) is a constant, as derived below:

- **Region 1**: This is at the beginning of the current pulse, when \( \theta \) is close to 0 and thus, \( \sin \theta \approx 0 \) and \( \cos \theta \approx 1 \). The damping torque resists the change of \( \theta \) while \( \theta \) is close to 0\(^\circ\). Such a fact helps speed up the switching. Therefore, a negative resistance, \( R_1 \), is obtained from Eq. (4), giving an exponential increase in the magnetic angle (Fig. 4). If the current pulse stays long enough, the magnetic angle rapidly reaches 180\(^\circ\), as shown in Fig. 1. However, if \( \tau \) is not long enough to complete the switching, two more regions are needed for time > \( \tau \).
- **Region 2**: As soon as \( \theta \) exceeds the threshold angle \( \theta_0 \), \( \theta \) is close to 90\(^\circ\) so that \( \sin \theta \approx 1 \) and \( \cos \theta \approx 90\(^\circ\). In this region, \( d^2 \theta/dt^2 \) becomes positive, as indicated in Fig. 2. Such a fact suggests that the RC network is a positive feedback: the increase in \( \theta \) helps speed up the switching. Therefore, a negative resistance, \( R_2 \), is obtained from Eq. (4), giving an exponential increase in the magnetic angle (Fig. 4). If the current pulse stays long enough, the magnetic angle rapidly reaches 180\(^\circ\), as shown in Fig. 1. However, if \( \tau \) is not long enough to complete the switching, two more regions are needed for time > \( \tau \).
- **Region 3**: If \( \theta > \theta_0 \) when the current pulse ends, the damping torque helps finish the switching without \( I \), as shown in Figs. 1 and 5. In this case, Eq. (4) can be expanded around 180\(^\circ\) to obtain \( R_3C \).
- **Region 4**: Finally, if \( \theta < \theta_0 \) when the current pulse ends, the damping torque overcomes and pulls the magnetic angle back to the initial state, 0\(^\circ\). The switching fails, under the influence of \( R_4C \).

Table III summarizes all model parameters. They are in closed-form, derived from the LLG equation and parameters in Table II. The proposed RC network is followed by the TMR model (Eq. (5), in Verilog-A) to complete the simulation structure. Working together, they convert the magnetic angle to electrical resistance. As all parameter values are pre-solved before the simulation, this RC network is highly efficient in the SPICE environment.

### E. Demonstration of the SPICE Model

The newly developed models are implemented into SPICE. Two simulation examples are presented in Figs. 1 and 3. Under the same assumptions of \( I \), \( \tau \) and \( T_{\text{on}} \), Fig. 5 further demonstrates the prediction under different pulse width \( \tau \). As expected by the RC network in Fig. 4, different RC components are activated, depending on the switching period. Furthermore, Fig. 6 presents the matching in the prediction of MTJ resistance.
condition. The success of data writing is determined by both the magnitude and the duration of the current pulse. The proposed modeling and simulation method smoothly captures such a behavior for design exploration.

By combining the switching model and the TMR model together, the new solution generates the electrical property of STT-MTJ. Figure 6 validates this approach with the experimental data [10][11]. For a STT-MTJ device, since $P \rightarrow A_P$ starts from $\theta = 0^\circ$ but $AP \rightarrow P$ starts from $\theta = 180^\circ$, these two switching paths experience different switching thresholds, as predicted by the LLG equation. This causes the hysteresis behavior in the resistance, which is well matched by our proposed models. In addition, the new compact model is general enough to describe the data from different processes, as demonstrated in Fig. 6.

**DESIGN BENCHMARKING**

With solid validation with numerical results and published data, the proposed compact model enables all types of design exploration with STT-MTJ. Under the shrinking of device feature size, Fig. 7 shows the minimum programming current, $I_c$. The radius $r$ impacts the density of $I_c$ mainly through saturation magnetization ($M_s$ in Table I), which is a material property [7]. The density of $I_c$ is sensitive to $r$ only when $r < 20\text{nm}$. On the contrary, $I_c$ is highly sensitive to $T_{ox}$ as $T_{ox}$ affects the intrinsic magnetic field in Eq. (1). In addition, $T_{ox}$ has a strong influence on the resistance and the long-term reliability of the tunnel junction [9]. Thus, process control of $T_{ox}$ is extremely important to STT-MTJ based memory design.

From the perspective of design optimization, Fig. 8 investigates the energy consumption during the write process. When the current pulse is narrow for high-speed operations, $I_c$ is high for a successful switching and thus, the energy consumption is high. As $r$ increases, $I_c$ goes down, reducing the energy cost. However, $I_c$ becomes less sensitive to $r$ after $r > 5\text{ns}$ and the energy consumption increases again even with a wider pulse, because of the longer integration time. As observed in Fig. 8, for a low-power design, there exists an optimal programming current that achieves minimum energy consumption of the write process, at $r \sim 3\text{ns}$.

**SUMMARY**

STT-MTJ represents a promising alternative for future logic and memory applications. Compact modeling of STT-MTJ is essential for integrated design, performance analysis and optimization. Unique from previous approaches on macro-modeling or direct implementation of the LLG equation, this work proposes a set of closed-form solutions, which map the physics of the LLG equation into a RC network. The values of RCs are pre-solved from the geometry, material properties, and the appropriate switching region. The new approach supports SPICE simulation with high efficiency, paving the path for further design study of STT-MTJ.

**ACKNOWLEDGEMENT**

This research is sponsored by National Science Foundation (NSF) under CNS-1218183.

**REFERENCES**


