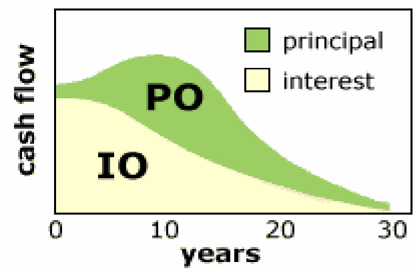
Professor Crocker H. Liu — Revised: December 28, 2007
Real Estate Capital Markets

Stripped MBS, CMOs, and Option Adjusted Spreads

1. **Principal Only (PO) and Interest Only (IO) Stripped MBS:** (25 points): Suppose that the underlying mortgage pass-through which is used to create a stripped MBS (mortgage-backed security) has the following features:

Original Mortgage Balance	$415,000 (in 000s)
Interest Rate	6.125%
Servicing Fee	0.5%
Pass-Through Rate	5.625%
Term	360 months



a. If the PSA is 205%, what are the cash flows to the holder of a PO (Principal-Only) Strip and the holder of an IO (Interest-Only) Strip respectively over 30 years only a monthly basis?

b. If the current price of the PO strip is $249,000 what is the monthly IRR, annual IRR and bond equivalent yield on the PO strip at various PSA speeds ranging from a PSA of 50% to a PSA of 430% (at 10% increments e.g., 50%, 60%, 70%, …., 420%, 430%). To facilitate your calculations use the Data Table command in Excel (pull down **Data** submenu→ **Table..**).

c. If the current price of the IO strip is $166,000 what is the monthly IRR, annual IRR and bond equivalent yield on the IO strip at various PSA speeds ranging from a PSA of 50% to a PSA of 430% (at 10% increments).

d. Prepare a graph showing the bond equivalent yield on both the IO strip and the PO strip at various PSA speeds. Discuss what happens to the BEY on both securities as the interest rate decreases?

2. **Collateralized Mortgage Obligation (CMO) with a Z Tranche** (35 points): Suppose that the underlying mortgage pass-through which is used to create a collateralized mortgage-backed security (CMO) has the following features:

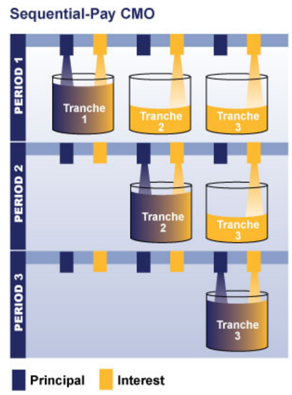
Orig. Mortgage Balance (000s)	$415,000	Pass-Through Rate	5.625%
Weighted Avg Coupon (WAC)	6.125%	Weighted Avg Maturity (WAM)	353 months
Servicing Fee	0. 5%		

We wish to create a CMO with 4 tranches (bond classes) - Class A, Class B, Class C and Class Z having the following attributes:

Class	Par Value	Coupon Rate
A	$179,500	5.25%
B	53,850	5.75%
C	71,800	6.00%
Z	53,850	6.25%

Assumptions:

- The coupon rate of interest is paid currently on tranches A, B, and C but not on tranche Z until principal on the other tranches is repaid. All current amortization of principal and prepayments from the entire mortgage pool is allocated first to tranche A.



- Payments into the pool occur *monthly*.

- No consideration of any reinvestment of interim cash flows.

- Rate of Prepayment is 205% PSA

- Note: After the A, B, and C tranches are all paid off, the outstanding balance of the mortgage pool should equal the outstanding balance of the Z tranche. This means that we will have to make an assumption regarding the fact that a portion of the principal paid to the Z tranche in the month that the C tranche is extinguish needs to be considered as a Net Interest payment.

Questions:

a. What are the cash flows to each class for the full 353 months?

b. Graph the flows of CMO Principal to Classes A, B, C, and Z on the same graph over the full term 353 months (you may have to use the "skipping" feature in your spreadsheet to plot the graph)

3. **Negative Convexity** (20 points): The attached handout by Jeff Berenbaum, Fall 1991, "Negative Convexity", *Secondary Mortgage Markets* 8(3): 32-33, allows you to set up a spreadsheet to show how negative convexity can arise as a result of rapid prepayments.

a. Set up the Negative Convexity spreadsheet using the article and replicate the graph given on page 32 for negative convexity. Note: since the article uses Lotus functions and most of you use Excel, I have provided the Excel formulas directly following the article.

b. Make a duplicate copy of your spreadsheet and "as an experiment, replace the look-up table with one that assumes borrowers react event more strongly to changes in interest rates e.g., double the prepayment speeds for positive spreads and cut in the prepayment speeds for negative spreads". Show that the bottom curve in Figure 1 on page 32 becomes even more negatively convex.

c. What is the expected price of the mortgage pool assuming that yields will vary between 8%-12% with an equal likelihood/probability of occurring e.g. the probability that the yield will be 8% is equal to the probability that the yield will be at 11%.

Hint: "Averaging the results for many different yield assumptions would better approximate the true value of a mortgage compared to using one interest rate."

Negative Convexity

A spreadsheet illustrates that ignoring prepayment options leads to overvaluing mortgages.

by Jeffrey S. Berenbaum

Prepayment options are exercised by borrowers. As a result, mortgage prepayments speed up and slow down exactly when investors would prefer the reverse. For example, when interest rates fall, investors would like their mortgage-backed securities with relatively high coupons to remain outstanding, yet borrowers prepay more rapidly.

This Lotus 1-2-3 spreadsheet can be used to show that adverse prepayments result in "negative convexity," that is, mortgage prices fall by more when interest rates rise than they increase when interest rates fall.

The spreadsheet values a pool of mortgages with the same interest rates, remaining terms and ages by calculating their cash flows for 360 months and discounting them at the yield on new mortgages. A user-entered, look-up table assigns a prepayment rate to the mortgage pool based on the spread between the mortgage interest rate and the current market rate. This percentage of the loans prepays fully; the calculations assume no curtailments.

The prepayment rates in the look-up table are expressed using the Public Securities Association (PSA) standard. Thus, the calculation of prepayments in the spreadsheet depends on the mortgage age as well as the interest rate.

At 100 percent PSA, a pool of mortgages prepays at 0.2 percent (annually) in the first month; this rate increases by 0.2 percent each month for 30 months until it levels off at 6 percent prepayments annually.

In the hypothetical look-up table shown, mortgages at the current rate (spread = 0) prepay at 180 percent PSA, that is 1.8 times the prepayment rates for 100 percent PSA. Mortgages below the current rate (spread < 0) prepay more slowly than 180 percent PSA. The prepayment rate slows down less as the spread gets more negative, reflecting housing turnover and other factors that cause prepayments even for loans with attractive rates.

Mortgages above the current rate (spread > 0) prepay more rapidly than 180 percent PSA. Initially the increase is relatively small; then the spread reaches levels where many borrowers find it worthwhile to refinance.

Figure 1 shows the results of using the spreadsheet to value a pool of one-year-old, 30-year, 10 percent mortgages assuming that current interest rates vary from 7 to 13 percent. The two curves are drawn using different prepayment assumptions.

The bottom curve is based on the look-up table. It exhibits negative convexity. Because of the shape of the curve, the value of the 10 percent mortgage at any point on the curve will be higher than if its value is calculated by taking an average of two possible outcomes, one with higher and one with lower rates. For example, the value of the 10 percent mortgages at 10 percent is par (100), while the average of the values at 9 and 11 percent (104.49 and 95.31) is 99.9.

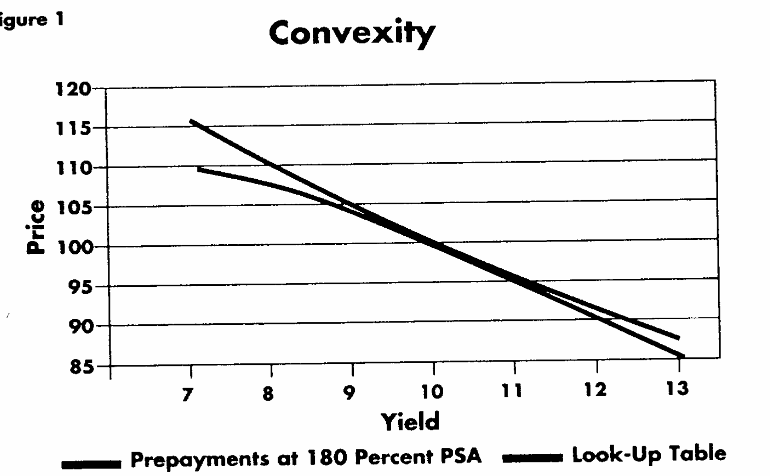
The top curve in Figure 1 was drawn by changing all the PSA speeds in the look-up table to 180 percent PSA, regardless of current market rates. In other words, the investor does not face the adverse choice of the borrower. In this case the curve is positively convex.

As an experiment, this look-up table could be replaced with one that assumes borrowers react even more strongly to changes in interest rates. For example, prepayment speeds could be doubled for positive spreads and be cut in half for negative spreads. The bottom curve in Figure 1 would become even more negatively convex.

Note that the look-up table only has values for spreads between -3 and 3. For lower spreads the value will not be calculated; for higher spreads the calculation uses the PSA for spreads equal to 3. However, the table can be expanded.

Should this spreadsheet be used to value mortgages? Because of negative convexity,

Figure 1



Jeffrey S. Berenbaum is a senior financial analyst in Freddie Mac's financial research department.

	A	B	C	D	E
1	Inputs:				
2					
3		Original Term			360
4		Remaining Term			348
5		Mortgage Coupon			10
6		Current Mortgage/Discount Rate			9
7					
8	Intermediate Computations:				
9					
10		Monthly Payment			0.882
11		PSA			210
12					
13	Output:				
14					
15		PV of Scheduled Amortization			4.85
16		PV of Interest			44.95
17		PV of Prepayments			54.69
18		Total PV (Price)			104.49
19					
20	* *				
21					
22	PSA Look-Up Table				
23	---------------				
24	Spread	PSA			
25	−3.0	140			
26	−2.5	145			
27	−2.0	150			
28	−1.5	155			
29	−1.0	160			
30	−0.5	170			
31	0.0	180			
32	0.5	190			
33	1.0	210			
34	1.5	240			
35	2.0	285			
36	2.5	345			
37	3.0	400			

Formulas

```
E10: @PMT (100,E5/1200,E4)
E11: @VLOOKUP ((E5-E6), $A$25..$B$37,1)
     Note: This function requires the PSA
           look-up table to be located
           in cells A25 through B37.
E15: @NPV ($E$6/1200,$N$2..$N$361)
E16: @NPV ($E$6/1200,$L$2..$L$361)
E17: @NPV ($E$6/1200,$P$2..$P$361)
E18: @SUM (E15..E17)
```

	H	I	J	K	L	M	N	O	P	Q	R
1	Month		Balance		Interest		Sched Am		Ppmts		Survival Rate
2	1		100.0000		0.8333		0.0491		0.4666		1.0000
3	2		99.4843		0.8290		0.0493		0.5009		0.9953
4	3		98.9341		0.8245		0.0495		0.5348		0.9903
.	.		.		.		.		.		.
.	.		.		.		.		.		.
359	358		0.0000		0.0000		0.0000		0.0000		0.0000
360	359		0.0000		0.0000		0.0000		0.0000		0.0000
361	360		0.0000		0.0000		0.0000		0.0000		0.0000

Formulas

```
H2: 1
J2: 100
L2: @IF(H2< = $E$4, - $E$5/1200*J2,0)
N2: @IF(H2< = $E$4, - $E$10*R2 - L2,0)
P2: @IF(H2< = $E$4 - 1,(J2 - N2)*(1 - (1 - $E$11/100*(@IF($E$3 - $E$4 + H2<30,0.002*($E$3 - $E$4 + H2),0.06)))^(1/12)),0)
R2: 1
H3: + H2 + 1
J3: @IF(H3< = $E$4, + J2 - N2 - P2,0)
L3: @IF(H3< = $E$4, + $E$5/1200*J3,0)
N3: @IF(H3< = $E$4, + $E$10*R3 - L3,0)
P3: @IF(H3< = $E$4 - 1,(J3 - N3)*(1 - (1 - $E$11/100*(@IF($E$3 - $E$4 + H3<30,0.002*($E$3 - $E$4 + H3),0.06)))^(1/12)),0)
R3: @IF(H3< = $E$4,(1 - P2/(J2 - N2))*R2,0)
.
.
.
H361: + H360 + 1
J361: @IF(H361< = $E$4, + J360 - N360 - P360,0)
L361: @IF(H361< = $E$4, + $E$5/1200*J361,0)
N361: @IF(H361< = $E$4, + $E$10*R361 - L361,0)
P361: @IF(H361< = $E$4 - 1,(J361 - N361)*(1 - (1 - $E$11/100*(@IF($E$3 - $E$4 + H361<30,0.002*($E$3 - $E$4 + H361),0.06)))^(1/12)),0)
R361: @IF(H361< = $E$4,(1 - P360/(J360 - N360))*R360,0)
```

Note: Columns H, J and R should be copied from row 3 to rows 4 through 361. Columns L, N and P should be copied from row 2 to rows 3 through 361. Column J calculates the balance as of the beginning of the month indicated in column H. Column L calculates the interest owed on the balance for the current month. Column N calculates the scheduled amortization for the month by subtracting the interest payment for the month from the total monthly payment (cell E10). Column P computes the monthly prepayment using the PSA speed (cell E11) selected from the look-up table. Column R calculates the pool survival rate which is used to scale the monthly payment to the appropriate dollar amount.

valuing mortgages assuming their cash flows are known leads to results that are consistently overstated. Averaging the results for many different yield assumptions would better approximate the true value of a mortgage compared to using one interest rate.

This spreadsheet cannot fully capture the option nature of mortgages, however. Complex models allow prepayments to vary based on additional factors, such as "burnout." That is, the effect of the spread may depend on how much the prepayment option has been in-the-money before. In addition, more complete models allow the discount rate to change each month (see "The New Breed: Option Adjusted Spreads," *SMM*, Winter 1988-89). **SMM**

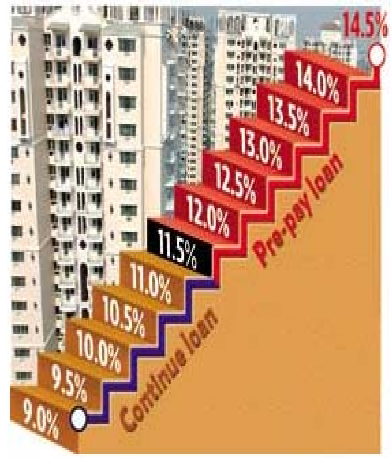
Excel formulas to accompany the Berenbaum (1991) article:

Cell	**Formula**
E10	=PMT(E5/1200,E4,-100)
E11	=VLOOKUP(E5-E6,$A$25:$B$37,2)
E15	=NPV($E$6/1200,$N$2:$N$361)
E16	=NPV($E$6/1200,$L$2:$L$361)
E17	=NPV($E$6/1200,$P$2:$P$361)
E18	=SUM(E15:E17)
L2	=IF(H2<=$E$4,+$E$5/1200*J2,0)
N2	=IF(H2<=$E$4,+$E$10*R2-L2,0)
P2	=IF(H2<=$E$4-1,(J2-N2)*(1-(1-$E$11/100*(IF($E$3-$E$4+H2<30,0.002*($E$3-$E$4+H2),0.06)))^(1/12)),0)
H3	=H2+1
J3	=IF(H3<=$E$4,+J2-N2-P2,0)
L3	=IF(H3<=$E$4,+$E$5/1200*J3,0)
N3	=IF(H3<=$E$4,+$E$10*R3-L3,0)
P3	=IF(H3<=$E$4-1,(J3-N3)*(1-(1-$E$11/100*(IF($E$3-$E$4+H3<30,0.002*($E$3-$E$4+H3),0.06)))^(1/12)),0)
R3	=IF(H3<=$E$4,(1-P2/(J2-N2))*R2,0)
…	
J361	=IF(H361<=$E$4,+J360-N360-P360,0)
L361	=IF(H361<=$E$4,+$E$5/1200*J361,0)
N361	=IF(H361<=$E$4,+$E$10*R361-L361,0)
P361	=IF(H361<=$E$4-1,(J361-N361)*(1-(1-$E$11/100*(IF($E$3-$E$4+H361<30,0.002*($E$3-$E$4+H361),0.06)))^(1/12)),0)
R361	=IF(H361<=$E$4,(1-P360/(J360-N360))*R360,0)

Note: Columns H, J, and R should be copied from row 3 to rows 4 through 361. Columns L, N and P should be copied from row 2 to rows 3 through 361. Column J calculates the balance as of the beginning of the month indicated in column H. Column L calculates the interest owed on the balance for the current month. Column N calculates the scheduled amortization for the month by subtracting the interest payment for the month from the total monthly payment (cell E10). Column P computes the monthly prepayment using the PSA speed (cell E11) selected from the look-up table. Column R calculates the pool survival rate which is used to scale the monthly payment to the appropriate dollar amount.

4. **Option Adjusted Spread** (20 points): Calculate the option-adjusted spread (OAS) and the yield to maturity on a mortgage pass-through given the following assumptions:

- Borrowers only prepay due to refinancing mortgages at a lower rate
- The current zero coupon yield curve for Treasury bonds is flat and the discount rate on Treasury bonds = 3.5%
- The mortgage coupon rate is 7.25% on an outstanding mortgage pool with an outstanding principal balance of $5,000,000. The mortgages have a 3-year maturity and pay principal and interest only once at the end of each year.



- Mortgage loans are fully amortized and there is no servicing fee
- The current mortgage rate (y) is 6.125%. Interest rate movements over time change a maximum of .44% up or -.30% down each year. The time path of interest rates follows a binomial process. For example, in year 1, the new interest rate will either be 6.565% or 5.825%.
- Because of prepayment penalties and other refinancing costs, mortgagees don't begin to prepay until mortgage rates, in any year, fall 1.5% or more below the contract mortgage coupon rate for the pool. In other words, refinancing doesn’t occur unless the current interest rate is at or lower than 5.75% (5.75% = 7.25% - 1.5%).
- With prepayments present, cash flows in any year can be either the promised debt service, the promised debt service + repayment of outstanding principal, or cash flow = 0 if all mortgages have been prepaid or paid off in the previous year

Also, assume that the time path of mortgage interest rates over 3 years with the associated probabilities (p) is as given in the OAS spreadsheet template (highlighted in purple) that you downloaded.

Hints in Calculating OAS:

Recall that

$$P = \frac{E(CF_1)}{(1 + r_1 + OAS)} + \frac{E(CF_2)}{(1 + r_2 + OAS)^2} + \frac{E(CF_3)}{(1 + r_3 + OAS)^3}$$

where P = Price of Mortgage Pass-through
r_1 = Discount rate on 1-year, zero-coupon Treasury bond
r_2 = Discount rate on 2-year, zero-coupon Treasury bond
r_3 = Discount rate on 3-year, zero-coupon Treasury bond
OAS = Option adjusted spread on mortgage pass-through

therefore since you are solving for the unknown variable, you can use the SOLVER algorithm in EXCEL