

Additional Simulation Results for the Comparison of Methods to  
Test the Significance of the Intervening Variable Effect

by

David P. MacKinnon<sup>1</sup>, Chondra M. Lockwood<sup>1</sup>, Jeanne M. Hoffman<sup>2</sup>,

Stephen G. West<sup>1</sup>, and Virgil Sheets<sup>3</sup>

<sup>1</sup>Arizona State University

<sup>2</sup>University of Washington

<sup>3</sup>Indiana State University

Simulation Addendum to MacKinnon, D. P., Lockwood, C. M., Hoffman, J. M., West, S. G., & Sheets, V. (2002). A Comparison of Methods to Test Mediation and Other Intervening Variable Effects. *Psychological Methods*, 7(1), 83-104.

A simulation study was conducted to compare tests of the intervening variable (mediated) effect. Please see MacKinnon, Lockwood, Hoffman, West, & Sheets (2002) for a description of the intervening variable model and each of the tests of the intervening variable effect, as well as a full description of the simulation. This supplemental report includes results for parameter values omitted from the final manuscript due to space constraints for a continuous independent variable. Note that the results for a binary independent variable are not reported here because they were virtually identical to the results described in the paper and the supplemental results described below. Note that the effect size for the case of a binary independent variable were the same as for the continuous independent variable by using corresponding parameter values in the simulation study.

Tables 1 through 3 present type I error rates and statistical power for  $J'=.14$ . Tables 4 through 6 present type 1 error rates and statistical power for  $J'=.39$ . Tables 7 through 9 present type 1 error rates and statistical power for  $J'=.59$ .

Noteworthy in these results are the decreasing power to direct the mediated effect as the direct effect ( $J'$ ) increases for the the Judd and Kenny (1981) method, which requires  $J_{II}$  to not differ significantly from zero. There is also a slight decrease in power for the Olkin and Finn (1995) test of the difference between a simple and partial correlation. The Baron and Kenny (1985) method also has slightly more power as  $J'$  increased.

For further discussion of the differences between methods, and recommendations for use, please see the original article in *Psychological Methods* (MacKinnon, D.P., Lockwood, C. M., Hoffman, J. M., West, S. G., and Sheets, V. (2002), A comparison of methods to test the mediation and other intervening variable effects, *Psychological Methods*, 7(1), 83-104.

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Table 1. Type 1 Error Rates and Statistical Power for Causal Step Methods

Method	Effect Size	Sample Size				
		50	100	200	500	1000
Judd & Kenny (1981)	Zero Effect	0.0000	0.0000	0.0000	0.0000	0.0000
	Small Effect	0.0040	0.0120	0.0740	0.0620	0.0060
	Medium Effect	0.2040	0.5040	0.5100	0.1860	0.0200
	Large Effect	0.7080	0.7720	0.6440	0.2220	0.0360
Baron & Kenny (1986)	Zero Effect	0.0000	0.0000	0.0000	0.0000	0.0020
	Small Effect	0.0040	0.0260	0.1740	0.7200	0.9780
	Medium Effect	0.2520	0.7160	0.9680	1.0000	1.0000
	Large Effect	0.8080	0.9840	1.0000	1.0000	1.0000
Joint Significance of " and \$	Zero Effect	0.0000	0.0040	0.0020	0.0020	0.0020
	Small Effect	0.0180	0.0780	0.2560	0.7740	0.9780
	Medium Effect	0.5320	0.9340	1.0000	1.0000	1.0000
	Large Effect	0.9560	1.0000	1.0000	1.0000	1.0000

Note: For all analyses " = \$ and  $J\beta = .14$ . Small effect size = .14, medium effect size = .36, and large effect size = .51. Tests are 2-tailed,  $\alpha = .05$ .

Table 2. Type 1 Error Rates and Statistical Power for Difference in Coefficients Methods

Method	Effect Size	Sample Size				
		50	100	200	500	1000
J - $J\beta$ Freedman & Shatzkin (1992)	Zero Effect	0.0360	0.0280	0.0200	0.0220	0.0280
	Small Effect	0.1360	0.2480	0.5400	0.8760	0.9920
	Medium Effect	0.7440	0.9700	1.0000	1.0000	1.0000
	Large Effect	0.9840	1.0000	1.0000	1.0000	1.0000
J - $J\beta$ McGuigan & Langholtz (1988)	Zero Effect	0.0000	0.0000	0.0000	0.0000	0.0020
	Small Effect	0.0040	0.0120	0.0740	0.5400	0.9540
	Medium Effect	0.3220	0.8640	0.9980	1.0000	1.0000
	Large Effect	0.9260	1.0000	1.0000	1.0000	1.0000
J - $J\beta$ Clogg et al. (1992)	Zero Effect	0.0500	0.0420	0.0240	0.0260	0.0400
	Small Effect	0.1900	0.3000	0.5460	0.8760	0.9920
	Medium Effect	0.7620	0.9700	1.0000	1.0000	1.0000
	Large Effect	0.9840	1.0000	1.0000	1.0000	1.0000
Simple vs. Partial Correlation Olkin & Finn (1995)	Zero Effect	0.0000	0.0000	0.0000	0.0000	0.0020
	Small Effect	0.0040	0.0240	0.0860	0.5800	0.9540
	Medium Effect	0.4140	0.8820	0.9960	1.0000	1.0000
	Large Effect	0.9100	1.0000	1.0000	1.0000	1.0000

Note: For all analyses " = \$ and  $J\beta = .14$ . Small effect size = .14, medium effect size = .36, and large effect size = .51. Tests are 2-tailed,  $\alpha = .05$ .

Table 3. Type 1 Error Rates and Statistical Power for Product of Coefficients Methods

Method	Effect Size	Sample Size				
		50	100	200	500	1000
First Order Test Sobel (1982)	Zero Effect	0.0000	0.0000	0.0000	0.0000	0.0020
	Small Effect	0.0040	0.0200	0.0840	0.5720	0.9640
	Medium Effect	0.3700	0.8740	0.9980	1.0000	1.0000
	Large Effect	0.9300	1.0000	1.0000	1.0000	1.0000
Second Order Test Aroian (1944)	Zero Effect	0.0000	0.0000	0.0000	0.0000	0.0020
	Small Effect	0.0040	0.0120	0.0740	0.5400	0.9540
	Medium Effect	0.3200	0.8640	0.9980	1.0000	1.0000
	Large Effect	0.9260	1.0000	1.0000	1.0000	1.0000
Unbiased Test Goodman (1960)	Zero Effect	0.0220	0.0080	0.0200	0.0040	0.0120
	Small Effect	0.0100	0.0260	0.1100	0.6160	0.9680
	Medium Effect	0.4160	0.8840	0.9980	1.0000	1.0000
	Large Effect	0.9360	1.0000	1.0000	1.0000	1.0000
Distribution of Products Test $P = Z_1 \cdot Z_2$ MacKinnon et al. (1998)	Zero Effect	0.0620	0.0440	0.0440	0.0360	0.0360
	Small Effect	0.2080	0.3480	0.6900	0.9780	1.0000
	Medium Effect	0.9360	1.0000	1.0000	1.0000	1.0000
	Large Effect	1.0000	1.0000	1.0000	1.0000	1.0000
Distribution of $\sqrt{F_1} \cdot \sqrt{F_2}$ MacKinnon et al. (1998)	Zero Effect	0.0460	0.0420	0.0500	0.0380	0.0360
	Small Effect	0.1920	0.3280	0.6460	0.9640	1.0000
	Medium Effect	0.9040	1.0000	1.0000	1.0000	1.0000
	Large Effect	1.0000	1.0000	1.0000	1.0000	1.0000
Asymmetric Distribution of Products Test $P = Z_1 \cdot Z_2$ MacKinnon & Lockwood (1999)	Zero Effect	0.0000	0.0020	0.0000	0.0020	0.0020
	Small Effect	0.0140	0.0640	0.2260	0.7600	0.9800
	Medium Effect	0.5420	0.9360	1.0000	1.0000	1.0000
	Large Effect	0.9700	1.0000	1.0000	1.0000	1.0000
Product of Correlations Bobko & Rieck (1980)	Zero Effect	0.0000	0.0020	0.0000	0.0000	0.0020
	Small Effect	0.0140	0.0660	0.1920	0.7520	0.9840
	Medium Effect	0.5640	0.9520	1.0000	1.0000	1.0000
	Large Effect	0.9720	1.0000	1.0000	1.0000	1.0000

Note: For all analyses  $\rho = .5$  and  $\rho_{12} = .14$ . Small effect size = .14, medium effect size = .36, and large effect size = .51. Tests are 2-tailed, alpha = .05.

Table 4. Type 1 Error Rates and Statistical Power for Causal Step Methods

Method	Effect Size	Sample Size				
		50	100	200	500	1000
Judd & Kenny (1981)	Zero Effect	0.0000	0.0000	0.0000	0.0000	0.0000
	Small Effect	0.0080	0.0020	0.0000	0.0000	0.0000
	Medium Effect	0.1860	0.0480	0.0040	0.0000	0.0000
	Large Effect	0.3620	0.1080	0.0000	0.0000	0.0000
Baron & Kenny (1986)	Zero Effect	0.0000	0.0000	0.0020	0.0020	0.0020
	Small Effect	0.0140	0.0860	0.2360	0.7720	0.9820
	Medium Effect	0.5420	0.9240	1.0000	1.0000	1.0000
	Large Effect	0.9340	1.0000	1.0000	1.0000	1.0000
Joint Significance of " and \$	Zero Effect	0.0040	0.0020	0.0040	0.0020	0.0020
	Small Effect	0.0180	0.0860	0.2360	0.7720	0.9820
	Medium Effect	0.5660	0.9260	1.0000	1.0000	1.0000
	Large Effect	0.9440	1.0000	1.0000	1.0000	1.0000

Note: For all analyses " = \$ and  $JM = .39$ . Small effect size = .14, medium effect size = .36, and large effect size = .51. Tests are 2-tailed,  $\alpha = .05$

Table 5. Type 1 Error Rates and Statistical Power for Difference in Coefficients Methods

Method	Effect Size	Sample Size				
		50	100	200	500	1000
J - $JM$ Freedman & Shatzkin (1992)	Zero Effect	0.0320	0.0420	0.0380	0.0340	0.0480
	Small Effect	0.0960	0.2480	0.4580	0.8580	0.9900
	Medium Effect	0.7220	0.9640	1.0000	1.0000	1.0000
	Large Effect	0.9680	1.0000	1.0000	1.0000	1.0000
J - $JM$ McGuigan & Langholtz (1988)	Zero Effect	0.0020	0.0000	0.0000	0.0000	0.0000
	Small Effect	0.0020	0.0120	0.0800	0.5340	0.9580
	Medium Effect	0.3220	0.8640	1.0000	1.0000	1.0000
	Large Effect	0.8940	1.0000	1.0000	1.0000	1.0000
J - $JM$ Clogg et al. (1992)	Zero Effect	0.0480	0.0580	0.0480	0.0440	0.0540
	Small Effect	0.1680	0.2740	0.4740	0.8600	0.9900
	Medium Effect	0.7460	0.9640	1.0000	1.0000	1.0000
	Large Effect	0.9700	1.0000	1.0000	1.0000	1.0000
Simple vs. Partial Correlation Olkin & Finn (1995)	Zero Effect	0.0020	0.0000	0.0000	0.0000	0.0000
	Small Effect	0.0040	0.0240	0.1000	0.4880	0.9120
	Medium Effect	0.3440	0.8140	0.9880	1.0000	1.0000
	Large Effect	0.8220	0.9920	1.0000	1.0000	1.0000

Note: For all analyses " = \$ and  $JM = .39$ . Small effect size = .14, medium effect size = .36, and large effect size = .51. Tests are 2-tailed,  $\alpha = .05$

Table 6. Type 1 Error Rates and Statistical Power for Product of Coefficients Methods

Method	Effect Size	Sample Size				
		50	100	200	500	1000
First Order Test Sobel (1982)	Zero Effect	0.0020	0.0000	0.0000	0.0000	0.0000
	Small Effect	0.0020	0.0180	0.0860	0.5620	0.9640
	Medium Effect	0.3580	0.8760	1.0000	1.0000	1.0000
	Large Effect	0.9100	1.0000	1.0000	1.0000	1.0000
Second Order Test Aroian (1944)	Zero Effect	0.0000	0.0000	0.0000	0.0000	0.0000
	Small Effect	0.0020	0.0120	0.0800	0.5340	0.9580
	Medium Effect	0.3160	0.8640	1.0000	1.0000	1.0000
	Large Effect	0.8940	1.0000	1.0000	1.0000	1.0000
Unbiased Test Goodman (1960)	Zero Effect	0.0080	0.0120	0.0100	0.0120	0.0060
	Small Effect	0.0120	0.0200	0.1100	0.6020	0.9780
	Medium Effect	0.3960	0.8900	1.0000	1.0000	1.0000
	Large Effect	0.9160	1.0000	1.0000	1.0000	1.0000
Distribution of Products Test $P = Z_1 \cdot Z_2$ MacKinnon et al. (1998)	Zero Effect	0.0680	0.0340	0.0360	0.0420	0.0440
	Small Effect	0.2140	0.3980	0.6820	0.9900	1.0000
	Medium Effect	0.9400	1.0000	1.0000	1.0000	1.0000
	Large Effect	1.0000	1.0000	1.0000	1.0000	1.0000
Distribution of $\sqrt{F_1}$ MacKinnon et al. (1998)	Zero Effect	0.0620	0.0320	0.0400	0.0540	0.0480
	Small Effect	0.1940	0.3660	0.6420	0.9820	1.0000
	Medium Effect	0.9100	1.0000	1.0000	1.0000	1.0000
	Large Effect	0.9980	1.0000	1.0000	1.0000	1.0000
Asymmetric Distribution of Products Test $P = Z_1 \cdot Z_2$ MacKinnon & Lockwood (1999)	Zero Effect	0.0020	0.0000	0.0000	0.0020	0.0020
	Small Effect	0.0160	0.0720	0.2120	0.7600	0.9820
	Medium Effect	0.5580	0.9340	1.0000	1.0000	1.0000
	Large Effect	0.9520	1.0000	1.0000	1.0000	1.0000
Product of Correlations Bobko & Rieck (1980)	Zero Effect	0.0060	0.0000	0.0060	0.0060	0.0040
	Small Effect	0.0600	0.1920	0.4140	0.9080	0.9940
	Medium Effect	0.7820	0.9740	1.0000	1.0000	1.0000
	Large Effect	0.9860	1.0000	1.0000	1.0000	1.0000

Note: For all analyses  $\rho = .39$ . Small effect size = .14, medium effect size = .36, and large effect size = .51. Tests are 2-tailed, alpha = .05.

Table 7. Type 1 Error Rates and Statistical Power for Causal Step Methods

Method	Effect Size	Sample Size				
		50	100	200	500	1000
Judd & Kenny (1981)	Zero Effect	0.0000	0.0000	0.0000	0.0000	0.0000
	Small Effect	0.0000	0.0000	0.0000	0.0000	0.0000
	Medium Effect	0.0220	0.0000	0.0000	0.0000	0.0000
	Large Effect	0.0840	0.0040	0.0000	0.0000	0.0000
Baron & Kenny (1986)	Zero Effect	0.0000	0.0020	0.0000	0.0000	0.0000
	Small Effect	0.0180	0.0700	0.2400	0.7400	0.9880
	Medium Effect	0.5820	0.9340	0.9980	1.0000	1.0000
	Large Effect	0.9460	1.0000	1.0000	1.0000	1.0000
Joint Significance of " and \$	Zero Effect	0.0020	0.0020	0.0000	0.0000	0.0020
	Small Effect	0.0180	0.0700	0.2400	0.7400	0.9880
	Medium Effect	0.5820	0.9340	0.9980	1.0000	1.0000
	Large Effect	0.9460	1.0000	1.0000	1.0000	1.0000

Note: For all analyses " = \$ and  $JN = .59$ . Small effect size = .14, medium effect size = .36, and large effect size = .51. Tests are 2-tailed,  $\alpha = .05$ .

Table 8. Type 1 Error Rates and Statistical Power for Difference in Coefficients Methods

Method	Effect Size	Sample Size				
		50	100	200	500	1000
J - $JN$ Freedman & Shatzkin (1992)	Zero Effect	0.0380	0.0340	0.0260	0.0300	0.0460
	Small Effect	0.1460	0.2280	0.4900	0.8580	0.9920
	Medium Effect	0.7640	0.9720	1.0000	1.0000	1.0000
	Large Effect	0.9560	1.0000	1.0000	1.0000	1.0000
J - $JN$ McGuigan & Langholtz (1988)	Zero Effect	0.0000	0.0000	0.0000	0.0000	0.0000
	Small Effect	0.0000	0.0060	0.0760	0.5160	0.9720
	Medium Effect	0.3320	0.8440	0.9960	1.0000	1.0000
	Large Effect	0.9040	1.0000	1.0000	1.0000	1.0000
J - $JN$ Clogg et al. (1992)	Zero Effect	0.0500	0.0540	0.0460	0.0420	0.0540
	Small Effect	0.1880	0.2900	0.5000	0.8600	0.9920
	Medium Effect	0.7780	0.9720	1.0000	1.0000	1.0000
	Large Effect	0.9580	1.0000	1.0000	1.0000	1.0000
Simple vs. Partial Correlation Olkin & Finn (1995)	Zero Effect	0.0000	0.0000	0.0000	0.0000	0.0000
	Small Effect	0.0040	0.0160	0.0700	0.3920	0.7980
	Medium Effect	0.2900	0.6840	0.9560	1.0000	1.0000
	Large Effect	0.7540	0.9680	0.9980	1.0000	1.0000

Note: For all analyses " = \$ and  $JN = .59$ . Small effect size = .14, medium effect size = .36, and large effect size = .51. Tests are 2-tailed,  $\alpha = .05$ .

Table 9. Type 1 Error Rates and Statistical Power for Product of Coefficients Methods

Method	Effect Size	Sample Size				
		50	100	200	500	1000
First Order Test Sobel (1982)	Zero Effect	0.0000	0.0000	0.0000	0.0000	0.0000
	Small Effect	0.0000	0.0120	0.0900	0.5480	0.9800
	Medium Effect	0.3620	0.8640	0.9960	1.0000	1.0000
	Large Effect	0.9080	1.0000	1.0000	1.0000	1.0000
Second Order Test Aroian (1944)	Zero Effect	0.0000	0.0000	0.0000	0.0000	0.0000
	Small Effect	0.0000	0.0060	0.0760	0.5160	0.9720
	Medium Effect	0.3320	0.8440	0.9960	1.0000	1.0000
	Large Effect	0.9040	1.0000	1.0000	1.0000	1.0000
Unbiased Test Goodman (1960)	Zero Effect	0.0180	0.0120	0.0100	0.0200	0.0140
	Small Effect	0.0140	0.0200	0.1120	0.5980	0.9820
	Medium Effect	0.4080	0.8780	0.9960	1.0000	1.0000
	Large Effect	0.9220	1.0000	1.0000	1.0000	1.0000
Distribution of Products Test $P = Z_1 \cdot Z_2$ MacKinnon et al. (1998)	Zero Effect	0.0680	0.0500	0.0500	0.0480	0.0680
	Small Effect	0.2380	0.3840	0.6880	0.9800	1.0000
	Medium Effect	0.9560	0.9960	1.0000	1.0000	1.0000
	Large Effect	0.9980	1.0000	1.0000	1.0000	1.0000
Distribution of " $F_1$ " MacKinnon et al. (1998)	Zero Effect	0.0660	0.0440	0.0480	0.0520	0.0620
	Small Effect	0.2200	0.3540	0.6380	0.9600	1.0000
	Medium Effect	0.9280	0.9940	1.0000	1.0000	1.0000
	Large Effect	0.9960	1.0000	1.0000	1.0000	1.0000
Asymmetric Distribution of Products Test $P = Z_1 \cdot Z_2$ MacKinnon & Lockwood (1999)	Zero Effect	0.0020	0.0020	0.0000	0.0000	0.0000
	Small Effect	0.0140	0.0620	0.2200	0.7180	0.9880
	Medium Effect	0.5860	0.9340	0.9980	1.0000	1.0000
	Large Effect	0.9500	1.0000	1.0000	1.0000	1.0000
Product of Correlations Bobko & Rieck (1980)	Zero Effect	0.0240	0.0120	0.0180	0.0040	0.0220
	Small Effect	0.1300	0.2820	0.5600	0.9160	0.9940
	Medium Effect	0.8440	0.9780	0.9980	1.0000	1.0000
	Large Effect	0.9920	1.0000	1.0000	1.0000	1.0000

Note: For all analyses  $\rho = .5$  and  $\rho_{12} = .59$ . Small effect size = .14, medium effect size = .36, and large effect size = .51. Tests are 2-tailed, alpha = .05.