Deterministic and Small-World Network Models of College Drinking Patterns

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Department of Mathematics Technical Report

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August 2006

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Abstract

Drinking on college campuses, especially binge drinking, contributes to numerous unintentional injuries, sexual assaults, and poor performance in classes. We are interested in modeling college drinking in order to guide policymakers in the creation of laws that will help decrease college binge drinking and its effects. In order to model the manner in which college drinking spreads, we have created two different models. Our first model modifies a five equation, deterministic, homogeneous, compartmental model of college drinking developed by Scribner, et.al. (2006, manuscript). In order to consider the dynamics of college drinking, our model assumes that social interactions, social norms, and individual risks are most influential in students' decisions to consume alcohol. We focused our attention on binge drinkers by combining light and moderate drinkers into one class and by incorporating the social interactions between social drinkers and problem drinkers and between bingers and problem drinkers. As part of our investigation, we simulated different alcohol environments by varying the parameters. We analyze the implications of this model from both mathematical and sociological perspectives. However, we recognize that a homogeneous model of drinking does not accurately represent the individual-to-individual interaction, connection, and influence of members of the same group. Hence, we created a second model based on graph theory and small-world networks. This model considers a heterogeneously mixed population, where students are represented as unique individuals. In particular, the effects of clusters, or "clique" groups, are analyzed in relation to the dynamics of the system. Finally, we compare our deterministic model with our network model and give some recommendations.

1 Introduction

Alcohol consumption among college students is one of the most important public health problems in the United States. Countless studies have been conducted in order to emphasize the importance of the reduction of alcohol consumption amongst college students. In 1995, the National Institute of Alcohol Abuse and Alcoholism (NIAAA) reported that 88% of college students have consumed alcohol, including those under the legal drinking age [1]. Although this study was conducted over a decade ago, the rate of alcohol consumption amongst college students has not changed. Furthermore, a national survey indicates that 42% of college students admit that they have participated in binge drinking [2]. Approximately 25% of college students report that they have suffered academic consequences as a result of their drinking habits. These consequences include missing classes, falling behind in classes, doing poorly on exams and/or papers, and receiving lower grades overall [3]. Around 600,000 college students are unintentionally injured each year as a result of alcohol consumption [2]. Additionally, nearly 100,000 college students are sexually assaulted or raped because of alcohol misuse [3]. Sociologists, government officials, and parents across the country have developed programs such as Mothers Against Drunk Driving (MADD) and Alcoholics Anonymous (AA) in attempts to prevent or reduce this drinking both during and after college. While these programs have proven helpful for those with drinking problems, none of the programs have significantly reduced the number of students who begin to drink on college campuses across the United States.

The purpose of this paper is to understand the underlying dynamics behind college drinking from the perspective of group–to–group interactions and of the individual agents. To this end, we offer two models of college drinking patterns. The first is a deterministic, ordinary differential equations, compartmental model that assumes homogeneous mixing. It is based on a model developed by Dr. Richard Scribner, MD (Louisiana School of Public Health)and his research associates. The second model utilizes small-world networks to model students on an individual level and better understand the effect of the individual on the collective whole.

Our paper is divided into seven sections. In Sections 2 through 4, we offer an in-depth analysis of the dimensionalized deterministic model. We numerically analyze the stability of the alcohol-free equilibrium, the sensitivity of the system to parameter changes, and the long-term behavior of the solutions. While Sections 2 and 3 describe the dimensionalized model, Section 4 is specifically devoted to the explanation of the necessity of a non-dimensionalized model. In Section 5, we describe the main characteristics of our network model. We describe our original ring arrangement model and then explain how we determined the inefficiency of this model. We then discuss our reasons for using a random clustering model and explain the computer programs that we used to create simulations of this model. By creating these mathematical models of the drinking patterns of college students, we hope to better understand the manner in which college students become drinkers. Ideally, we would like to offer suggestions to decrease drinking on college campuses.

2 The Model of Scribner, et al.

We will first briefly describe the five compartment model developed by Dr. Richard Scribner, et al. [4]. In this model of alcohol consumption, drinkers are divided into five compartments (or classes): abstainers, light drinkers, moderate drinkers, problem drinkers, and bingers. Each compartment is defined by the amount of alcohol consumed in accord with the National Institute of Alcohol Abuse and Alcoholism drinker definitions [5]. The abstainer class is composed of individuals who consume less than one drink per month. The light drinker class consists of individuals who consume 1-13 drinks per month. The moderate drinker class is composed of individuals who consume 4-14 drinks per week. Members of the problem drinker class consume more than two drinks per day. Finally, individuals in the binge drinker class consume more than five drinks per sitting [5].

Students move in and out of each compartment as a result of dropout or graduation, individual risk factors, social interactions, and social norms. The dropout and graduation rates are modeled together in a single term as a continuous process by which students leave the school over the course of the year. Individual risk factors include all of the things that affect an individual's adverse risk level such as marketing, religious beliefs, values, and family background. These factors exclude peer pressure and the perception of other students' drinking habits (i.e. the college culture). The change in a student's drinking habits brought about by social interaction is due to direct peer pressure. Social norms affect how students move between drinking compartments depending on the student's perception of what is considered socially acceptable or popular in the college culture. All of the parameters that govern the movement of individuals from one compartment to another or out of the system depend on the wetness of the campus and the environment surrounding the campus. The number of total dropouts and graduates at the end of each year is replaced with the freshmen class and incoming transfer students at the beginning of the year. The "wetness" in our model represents the amount of physical and social support that alcohol is given at a particular college or university and its surrounding community. At the beginning of each year, the number of students who dropped out over the past year is calculated. This number of students is added back into the various compartments based on the proportion of the original population in each compartment at time $t = 0$. This aspect of the model developed by Scribner, et al. is described by a replacement equation. This will be explained in detail in Section 3.

2.1 Changes to the Model of Scribner, et al.

Statistics indicate that binge drinking is a more serious problem than either light drinking or moderate drinking [2]. Studies indicate that alcohol consumption by bingers must be reduced in order to control overall drinking patterns [1]. Thus, although the model of Scribner, et al. describes the dynamics of the college environment well, we believe that the difference between light drinkers and moderate drinkers plays an insignificant role in providing any insight to this end. Combining the light drinker compartment and the moderate drinker compartment into a "social drinker" compartment simplifies the model and directs our focus toward the resolution of the main problem—binge drinking. As a result of this modification we had to adjust the parameter values that influence the new compartment.

In addition to combining the light drinker and moderate drinker compartments, we also included two social interaction transition terms that were not present in the model of Scribner, et al.: s_{23} and s_{43}. These terms represent the social interaction between students in the social drinker compartment and the problem drinker compartment (s_{23}) and the social interaction between students in the binge drinker compartment and the problem drinker compartment (s_{43}). Simulations indicated that the exclusion of these social interaction transition terms could cause errors in our model because the impact of these social interactions on the general drinking population is significant. Small changes in these terms translate to large differences in the evolving dynamics of the system. Although it makes the system more complex, including these terms allows for a better understanding of the movement between compartments that are affected by these terms (i.e., the social drinker compartment, the problem drinker compartment, and the binge drinker compartment). See Figure 11. We also redefine the parameter s_{ij} and made it a rate. In the scriber et. al. model, the s_{ij} terms have the units of $\frac{1}{(time)(population)}$ This characteristic of Scribner et. al. makes it impossible to use the same model for various colleges or universities with different populations. In order to do so, one must adjust all the s_{ij} parameters accordingly without sufficient data on these parameter values might be impossible.

3 The Four Equation Model

Following the model of Scribner, et al., we denote the compartment i by N_i. N_1 is comprised of abstainers, N_2 of social drinkers, N_3 of problem drinkers, and N_4 of binge drinkers. The total number of students for a particular campus is denoted by N, where $N = N_1 + N_2 + N_3 + N_4$. See Figure 1 for a graphical representation.

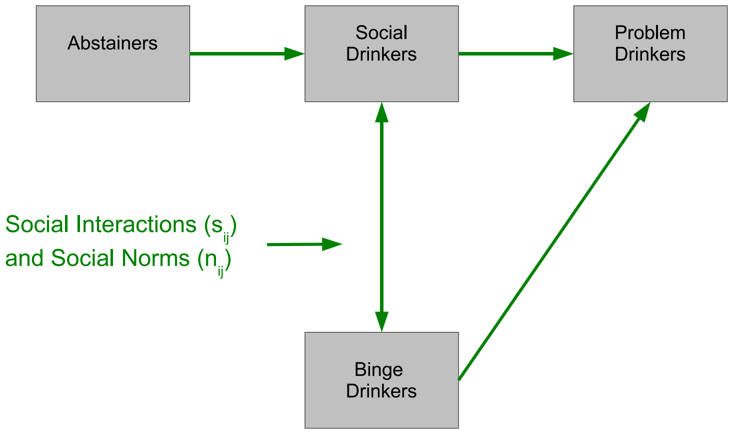

Figure 1: The four equation model in terms of compartments with different transitions

As mentioned earlier, students move in and out of each compartment due to graduation/dropout, individual risk factors, social interactions, and social norms. Each of the reasons that induce movement have associated rates.

The rate at which individuals continuously leave a compartment i due to graduation, transfer to another institution, or dropout is defined by d_i for $i = 1, 2, 3, 4$. The change in every compartment that results from the dropouts or graduation is proportional to the population in every compartment: d_1N_1, d_2N_2, d_3N_3, and d_4N_4. The transition rates associated with individual risk factors describe the rate at which an individual moves from compartment i to j due to individual choices and motivations induced by marketing, religious beliefs, family values, and personal values. This rate is denoted by r_{ij}. Social interactions have an associated transition rate, s_{ij}. This parameter describes the manner in which individuals move from compartment i to compartment j based on direct peer pressure from their friends or acquaintances in other compartments. The transition rate associated with social norms is defined as the rate at which individuals transition from compartment i to compartment j due to their perceptions about college drinking patterns that are considered "normal" or "cool". The change in each compartment brought about by social norms is jointly proportional to the current compartment population (in this case i) and the fraction of individuals in all other compartments, $s_{ij}[(\frac{N_i}{N}) + (\frac{N_k}{N}) + (\frac{N_s}{N})]$, that influence the populations of individuals in group i. Considering only the effect of social norms, $\frac{dN_i}{dt} \propto n_{ij}[(\frac{N_i}{N}) + (\frac{N_k}{N}) + (\frac{N_s}{N})]N_i$. On college campuses, it is not socially "normal" for students to become problem drinkers. For this reason, we did not assume that this compartment is affected by social norms in our model.

All of the rates are functions of the campus wetness, w, a parameter between 0 and 1 that represents the amount of physical and social support that alcohol is given at a particular college or university. For a given rate k_{ij}, this function of wetness is governed by the equation:

$$k_{ij}(w) = k_{ij}^{min} + (k_{ij}^{max} - k_{ij}^{min})w, \text{ for } w\epsilon\ [0, 1] \quad (1)$$

For example, if the campus and its surrounding community have lenient alcohol policies, this would result in a campus wetness near or equal to 1. When $w = 1$, the campus wetness causes the parameter to reach its maximum value, k_{ij}^{max}. Similarly, if a campus has very restrictive alcohol policies, the wetness would be near or equal to 0, and thus, $k_{ij}(w)$ will be close to its minimum value, k_{ij}^{min}.

3.0.1 The Resetting Equation

Following Scribner, et al., we accounted for the incoming freshmen classes and transfer students at the beginning of each year with a resetting/pulsed equation. While students are continuously leaving a certain compartment, students are added at the beginning of each year by the implementation of the resetting equation:

$$R_j(t_{k+1}) = c_j[N(0) - \sum_{i=1}^{4} N_i(t_k^-)] \quad (2)$$

.

The function R_j represents the number of students that will be added to each compartment j at the beginning of each academic year,t_{k+1}. To calculate this number for each compartment, determine the number of students that dropped out during the year by the subtracting the total number of students that remain at the end of year, t_k-, from the total number of students that the campus began with, $N(0)$. This total number of dropouts is then redistributed into each compartment j by the term c_j:

$$c_j = \frac{N_j(0)}{\sum_{p=1}^{4} N_p(0)}. \quad (3)$$

The quantity c_j is the initial percentage or fraction of the population that begins in compartment j, and we assume that every incoming class is divided into our four compartments in accord with these percentages.

For example, if we have a school with an initial total population of 1000 students, $N(0) = 1000$, where 200 students are considered abstainers, 300 students are social drinkers, 60 students are problem drinkers, and 440 students are binge drinkers, the c_j for each compartment are: $c_1 = 0.2$, $c_2 = 0.3$, $c_3 = 0.06$, and $c_4 = 0.44$. At the end of year 0, we find that 180 students have either graduated or dropped out. Using our replacement equation, and considering that we must round-off to the nearest integer to account for whole individuals, we calculate the number of students joining each compartment is as follows:

- 20% of the 180 new students will join the abstainer compartment, increasing this compartment's population by 36 new abstainers.
- 30% of the 180 new students will join the social drinker compartment, resulting in 54 more social drinkers.
- Approximately 6% of the 180 new students will join the problem drinker compartment, resulting in 11 new problem drinkers.
- 44% of the 180 students will join the binge drinker compartment, thus increasing the number of binge drinkers by 79.

3.1 A More Detailed Explanation of our Equations

$$\begin{aligned}
\frac{dN_1}{dt} &= -d_1N_1 + r_{21}N_2 + r_{31}N_3 - s_{12}\frac{N_1N_2}{N} - n_{12}\frac{N-N_1}{N}N_1 && (4)\\
\frac{dN_2}{dt} &= -d_2N_2 - r_{21}N_2 - r_{23}N_2 - r_{24}N_2 + r_{42}N_4 + s_{12}\frac{N_1N_2}{N} && (5)\\
&\quad -s_{23}\frac{N_2N_3}{N} + (s_{42}-s_{24})\frac{N_2N_4}{N} + n_{12}\frac{N-N_1}{N}N_1 - n_{24}\frac{N_4}{N}N_2 \\
\frac{dN_3}{dt} &= -d_3N_3 + r_{23}N_2 - r_{31}N_3 + r_{43}N_4 + s_{23}\frac{N_2N_3}{N} + s_{43}\frac{N_4N_3}{N} && (6)\\
\frac{dN_4}{dt} &= -d_4N_4 + r_{24}N_2 - r_{42}N_4 - r_{43}N_4 + (s_{24}-s_{42})\frac{N_2N_4}{N} && (7)\\
&\quad -s_{43}\frac{N_4N_3}{N} + n_{24}\frac{N_4}{N}N_2
\end{aligned}$$

$$\begin{aligned}
R_j(t_{k+1}) &= c_j[N(0) - \sum_{i=1}^{4} N_i(t_k^-)] && (8)\\
N(t) &= N_1(t) + N_2(t) + N_3(t) + N_4(t) && (9)
\end{aligned}$$

Where t_{k+1} and $t_k{}^-$ represent the beginning of year $k+1$ and the end of year k, respectively. It can also be noted that $N(0)$ is the total initial population, and

$$c_j = \frac{N_j(0)}{N(0)} \text{ for } j = 1, 2, 3, 4$$

is the proportion of the initial population in compartment j.

To better illustrate the impact of all alcohol inducing factors under consideration in this paper we will explain in detail one of the equations of our mathematical model. We take a closer look at the first equation of the system of equations (1) - (5),

$$\frac{dN_1}{dt} = -d_1N_1 + r_{21}N_2 + r_{31}N_3 - s_{12}\frac{N_1N_2}{N} - n_{12}\frac{N-N_1}{N}N_1,$$

and analyze it in detail to illustrate the role that each term plays in the change of the abstainer population over time.

The first term on the right hand side of the equation, $-d_1N_1$, represents the proportion of the abstainer population that leaves the abstainer compartment due to dropout or graduation. This term is negative since students are leaving the compartment. The second and third terms on the right hand side of the equation, $+r_{21}N_2 + r_{31}N_3$, are added to the equation to represent the movement of students into the abstainer compartment, N_1, from the social drinker compartment, N_2, and the problem drinker compartment, N_4, due to individual risk factors. The fourth term, $-s_{12}\frac{N_1N_2}{N}$, represents the movement of students from the abstainer compartment, N_1, to the social drinker compartment N_2 due to social interaction with individuals in compartment, N_2. This is modeled using the framework of epidemiology. Its representation utilizes the law of mass action to show that the interaction between two groups leads to the spread of alcohol consumption similar to the spread of a disease where the transmission depends on the proportion of infected individuals and the number of susceptible persons [6]. The final term, $-n_{12}\frac{N-N_1}{N}N_1$, represents the students that leave the abstainer compartment due to the their perceptions of the social norm of drinking at the college or university that they attend. Note that $\frac{N-N_1}{N} = \frac{N_2}{N} + \frac{N_3}{N} + \frac{N_4}{N}$. The remaining three equations can be described analogously.

3.2 Stability Analysis

3.2.1 Equilibrium with the Occurrence of Alcohol Consumption

The resetting equation component of the model leads to periodic long-term behavior, however, there is no closed form for the equilibrium. For this reason, we must resort to numerical means in order to find an approximation of the equilibrium for this system. We assume that an equilibrium is reached when the difference between two peaks or troughs is below a given tolerance level. In our case, we set the tolerance equal to 0.9. We utilized the *ode45* function in *Matlab* to verify that an equilibrium, as defined by our criteria, had been reached.

We integrated the system of equations for a fixed set of initial conditions and parameter values based on our understanding of the distribution of alcohol consumption on college campuses. These initial parameters are as follows:

$$r_{21} = 0.1, \quad r_{23} = 0.005, \quad r_{24} = 0.26, \quad r_{31} = 0.1, \quad r_{42} = 0.25, \quad r_{43} = 0.01,$$

$$s_{12} = 0.0014, \quad s_{23} = 0.0001, \quad s_{24} = 0.0016, \quad s_{42} = 0.0015, \quad s_{43} = 0.0002, \tag{10}$$

$$n_{12} = 0.09, \quad n_{24} = 0.1$$

We assumed an initial population distribution with 200 abstainers ($N_1 = 200$), 600 social drinkers ($N_2 = 600$), 25 problem drinkers ($N_3 = 25$), and 300 binge drinkers ($N_4 = 300$). First, we consider the peaks in each compartment for each year. These peaks occur at the beginning of the new academic year because the resetting equation replenishes the number of individuals that left each compartment due to dropout or graduation with the new incoming students. We look at the peak number of students in each compartment for every two consecutive years, i and $i+1$. We calculate the difference and consider only the largest difference. As long as this difference is above our tolerance, we continue to search for equilibrium. Once this equilibrium has been reached, all four compartments satisfy our tolerance level. Given the above initial conditions, parameter values, and a tolerance of 2, the equilibrium is reached at the end of the twentieth year and the beginning of the twenty-first year. This means that after twenty years, the net number of students moving between compartments is less than 2 (See Figure 2).

Similarly, for a tolerance of 0.9, equilibrium is reached at the end of the twenty-eighth year and the beginning of the twenty-ninth year. This means that the net number of students moving between compartments at the end of the twenty-eighth year is less than 0.9 (See Figure 2).

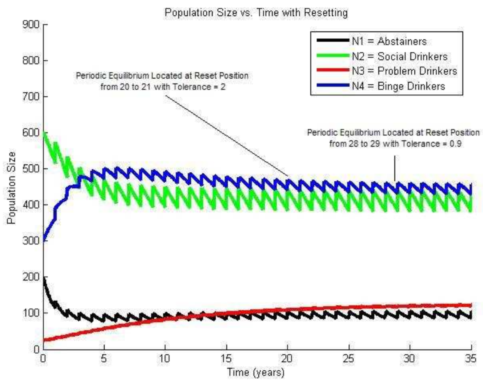

Figure 2: Equilibria for tolerances of 0.9 and 2 for the four equation model using the standard initial conditions and parameter values.

3.2.2 Alcohol–Free Equilibrium

In order to obtain more insight into the behavior of the four equation model, we remove the resetting equation and the dropout rates from all equations and analyze the point of alcohol–free equilibrium ($N_1 = N_1, N_2 = 0, N_3 = 0, N_4 = 0$). Using *Maple*, we calculated the Jacobian matrix and evaluated it at this equilibrium point:

$$J = \begin{pmatrix} 0 & -n_{12} - s_{12}N_1 + r_{21} & -n_{12} + r_{31} & -n_{12} \\ 0 & n_{12} + s_{12}N_1 - r_{23} - r_{24} - r_{21} & n_{12} & n_{12} + r_{42} \\ 0 & r_{23} & -r_{31} & r_{43} \\ 0 & r_{24} & 0 & -r_{43} - r_{42} \end{pmatrix}. \quad (11)$$

For realistic parameter values and population sizes, the stability of the system is almost entirely dependent on the $s_{12}N_1$ term since the other entries of the matrix are so small in comparison. For this reason, our numerical analysis of this matrix mainly focuses on the variation of the values of this term.

Using the same parameter values that are used in our *Matlab* simulations, we substitute values from the ranges of

$$\begin{array}{rclll}
\hat{s}_{12} = [.01, .8], & r_{21} & = & [.2, .09], & d_1 = 0.1, \quad n_{12} = [.08, 1], \\
\hat{s}_{24} = [.1,\ .7], & r_{42} & = & [.2, .08], & d_2 = 0.2, \quad n_{24} = [.08, .1], \\
\hat{s}_{42} = [0.4, .1], & r_{24} & = & [.01, .15], & d_3 = 0.2, \\
\hat{s}_{23} = [.01, .2], & r_{23} & = & [.05, .1], & d_4 = 0.2, \\
\hat{s}_{43} = [.01, .2], & r_{31} & = & [.8, .5], & \\
 & r_{43} & = & [.05, .08] &
\end{array} \tag{12}$$

into (1) to determine the eigenvalues. Using the minimum values of the ranges, for example, we obtain

$$J = \begin{pmatrix} 0 & -0.08 - s_{12}N_1 + 0.2 & -0.08 + 0.8 & -0.08 \\ 0 & 0.08 + s_{12}N_1 - 0.05 - 0.01 - 0.2 & 0.08 & 0.08 + 0.2 \\ 0 & 0.05 & -0.8 & 0.05 \\ 0 & 0.01 & 0 & -0.05 - 0.2 \end{pmatrix}. \tag{13}$$

A reasonable minimum for s_{12} is 0.4, which implies that there are only five abstainers within the entire school.

We vary the product $s_{12}N_1$ over a wide range of reasonable values while all other parameters are held constant within the ranges that are given above. If $s_{12}N_1$ is greater than its reasonable minimum, the alcohol–free equilibrium point is an unstable node. All other parameters are then varied individually to determine whether they affect the stability of the alcohol–free equilibrium point. For the above parameter ranges, it is not possible for this equilibrium point to be anything other than an unstable node. The instability of the alcohol–free equilibrium point for these parameter values indicates that is highly unlikely for a state to exist in which there are only abstainers.

4 Four Equation Non-Dimensionalized Model

4.1 Non-Dimensionalization

It is important to have a model that accounts for schools of any given size. While conducting trial simulations, we determined that the dimensional model gives drastically different answers for small versus large schools because of the dependence of every term of the equations on the population. In order to remove this dependence, we non-dimensionalized the model with respect to population.

To non-dimensionalize Equations (1)—(4), let $P_i = \frac{N_i}{N}$, where $N = N_1 + N_2 + N_3 + N_4$ represents the total population at time t, and N_i represents the number of students in compartment i. Thus, P_i represents the proportion of students in each compartment i. Since the s_{ij} terms are in units of $\frac{1}{\text{time} \cdot \text{population}}$, we set $\hat{s}_{ij} = \frac{s_{ij}}{N}$ to remove this parameter's dependence on population. Since $P_i = \frac{N_i}{N}$, we see that $N_i = N \cdot P_i$. Taking the derivatives of both sides of the equation gives the following:

$$N_i' = P_i' N + P_i N'. \tag{14}$$

We see that $N' = -\sum_{i=1}^{4}(d_i N_i)$. Through non-dimensionalization,

$$N' = -\sum_{i=1}^{4}(d_i N P_i) = -N\sum_{i=1}^{4}(d_i N_i)$$

.

Substituting for N_i and N, we obtain the following system of equations that is independent of population size:

$$\frac{dP_1}{dt} = -d_1P_1 + r_{21}P_2 + r_{31}P_3 - \hat{s}_{12}P_1P_2 - n_{12}(1-P_1)P_1 + P_1\sum_{i=1}^{4}(d_iP_i) \tag{15}$$

$$\begin{aligned}\frac{dP_2}{dt} &= -d_2P_2 - r_{21}P_2 - r_{23}P_2 - r_{24}P_2 + r_{42}P_4 + \hat{s}_{12}P_1P_2 - \hat{s}_{23}P_2P_3 \\ &\quad +(\hat{s}_{42} - \hat{s}_{24})P_2P_4 + n_{12}(1-P_1)P_1 - n_{24}P_4P_2 + P_2\sum_{i=1}^{4}(d_iP_i)\end{aligned} \tag{16}$$

$$\frac{dP_3}{dt} = -d_3P_3 + r_{23}P_2 - r_{31}P_3 + r_{43}P_4 + \hat{s}_{23}P_2P_3 + \hat{s}_{43}P_4P_3 + P_3\sum_{i=1}^{4}(d_iP_i) \tag{17}$$

$$\begin{aligned}\frac{dP_4}{dt} &= -d_4P_4 + r_{24}P_2 - r_{42}P_4 - r_{43}P_4 + (\hat{s}_{24} - \hat{s}_{42})P_2P_4 - \hat{s}_{43}P_4P_3 \\ &\quad +n_{24}P_4P_2 + P_4\sum_{i=1}^{4}(d_iP_i)\end{aligned} \tag{18}$$

Equations (3)—(6) now represent the rates of change of the proportions of students with respect to the total population within each compartment. Since all compartment populations are represented by proportions, schools of all sizes can be modeled without altering the parameters to account for changing population sizes.

Similarly, the resetting equations had to be non-dimensionlized with respect to population. As stated earlier, the purpose of the resetting equation is to replace the students that dropped out of the school during the year at the beginning of the next school year. Since this is completely dependent on the proportion of students who drop out, we must keep track of these students in order to accurately reset the model at the beginning of each year. For the purposes of resetting, we created an equation N_d that represents the total number of students who drop out from all four compartments. Note that

$$N_d' = -d_1N_1 - d_2N_2 - d_1N_1 - d_1N_1 - d_3N_3 - d_4N_4. \tag{19}$$

All five equations that are used can be represented in the following manner: $N_s = N_1 + N_2 + N_3 + N_4 + N_d$. Notice that N_s remains constant since there are no students leaving the system. Hence, $N_s = N(0)$. We can also non-dimensionalize N_d by letting $P_d = \frac{N_d}{N_s}$. Thus, the rate of change of the proportion of dropouts becomes

$$\frac{dP_d}{dt} = (1 - P_d)\sum_{i=1}^{4}(d_iP_i). \tag{20}$$

Replacing the proportion of the total population for each compartment at the beginning of year $k+1$ requires knowing the proportion of students who dropped out between year k and year $k+1$. At the beginning of year k, we have $P_d(t_k) = 0$, while at the end of year k, we have $P_d(t_k)$, and thus, $1 - P_d(t_k)$ is the proportion of the total population that remains in school. For year $k+1$, the initial proportion of students in each class is given by:

$$P_i(t_{k+1}) = [1 - P_d(t_k)]P_i(t_k) + c_iP_d(t_k), \text{ for each } i = 1, 2, 3, 4. \tag{21}$$

Multiplying the $1 - P_d(t_k)$ term by $P_i(t_k)$ scales the proportion of each compartment i, thus adjusting for the incoming class. The final term $c_iP_d(t_k)$ calculates the proportion of the incoming class that goes into each compartment i at the beginning of year $k+1$, where c_i is the initial ($t = 0$) proportion of students in each compartment. The replacing of each compartment is thus given by the change between $P_i(t_k)$ and $P_i(t_{k+1})$, as shown below:

$$\Delta P_i(t_k) = P_i(t_{k+1}) - P_i(t_k) = [c_i - P_i(t_k)]P_d(t_k). \tag{22}$$

4.2 Graphical Simulations

With the four equation model now non-dimensionalized, we determine appropriate ranges for all parameter values and test the validity of the overall system. Parameters for a five equation dimensional model were received courtesy of Dr. Benjamin Fitzpatrick. These values were determined by social scientists and medical doctors from Louisiana State University. Although these ranges were carefully selected, their effectiveness was limited to relatively small campus populations of approximately 1,000 students. Since our new non-dimensionalized model no longer depended on population, we adjusted the values Dr. Fitzpatrick provided to account for this. The final ranges that we determined are as follows:

$$\begin{array}{llll}
\hat{s}_{12} = [.01, .8], & r_{21} = [.2, .09], & d_1 = 0.1, & n_{12} = [.08, .1], \\
\hat{s}_{24} = [.1,\ .7], & r_{42} = [.2, .08], & d_2 = 0.2, & n_{24} = [.08, .1], \\
\hat{s}_{42} = [0.4, .1], & r_{24} = [.01, .15], & d_3 = 0.2, & \\
\hat{s}_{23} = [.01, .2], & r_{23} = [.05, .1], & d_4 = 0.2, & \\
\hat{s}_{43} = [.01, .2], & r_{31} = [.8, .5], & & \\
 & r_{43} = [.05, .08] & &
\end{array}$$

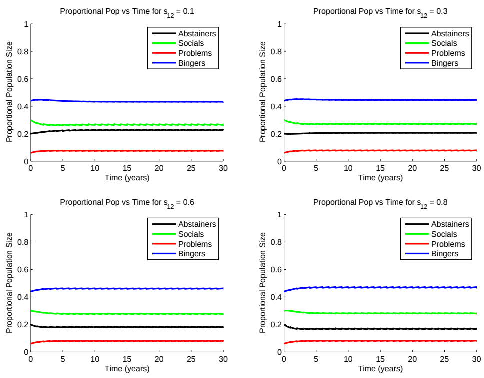

Figure 3: As s_{12} varies, abstainers become social drinkers due to social interaction.

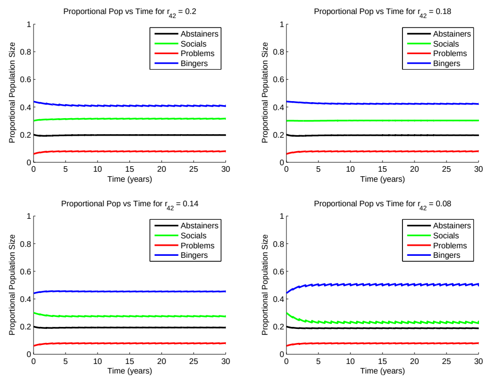

Figure 4: As r_{42} varies, bingers remain in the bingeing compartment and no longer become social drinkers due to individual risk.

4.2.1 Varying Parameter Values

To ensure that all parameters have a significant effect on the model and that the above ranges are accurate, every parameter range was varied over multiple values. In order to vary a parameter and observe its effects, all other values are held constant using a wetness of 0.5 (See Equation (6)) and initial conditions of $P_1 = 0.2, P_2 = 0.3, P_3 = 0.06, P_4 = 0.44$. For example, varying only the s_{12} (social interaction) parameter from its minimum to its maximum value causes the proportion of abstainers to visibly decrease as the proportion of social drinkers increases, see Figure 3. This particular type of variation informs campus policymakers that restricting social interactions with alcohol can help decrease the proportion of social drinkers attending the school.

Varying r_{42} shows the effects of individual risk involved with moving from binge drinking to social drinking. By varying this parameter value from its minimum to its maximum the graph illustrates that the more accessible alcohol is on campus, the more likely social drinkers will be to become binge drinkers due to their individual risk, see Figure 4.

Similarly, varying n_{24} from its minimum value to its maximum value illustrates the social drinkers becoming binge drinkers due to their perceptions of the percent of students in the school who are binge drinking, see Figure 5. As this perception increases, we see an increase in the proportion of students actually binge drinking. According to these results, if schools can decrease students' perceptions of binge drinking, the policymakers will be able to decrease binge drinking overall.

4.2.2 Varying Wetness

We first varied the wetness parameter for a school with proportions of student drinkers similar to the national average: $P_1 = 0.2, P_2 = 0.3, P_3 = 0.06, P_4 = 0.44$, see Figure 6. With a wetness parameter of 0.1, there is only an increase in the proportion of abstainers in the

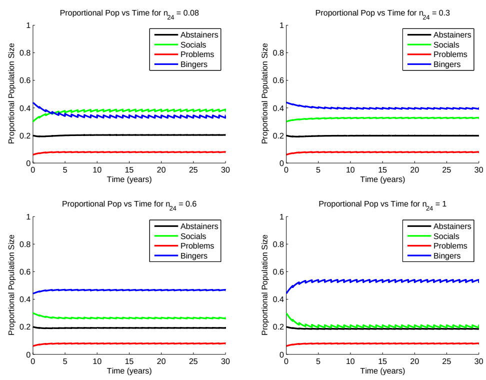

Figure 5: As n_{24} varies, social drinkers become bingers due to changing perceptions of social norms.

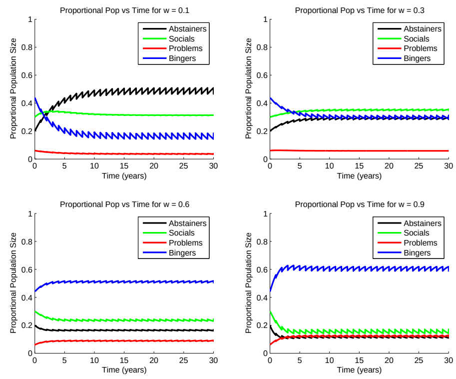

Figure 6: A typical proportion of student drinkers with varying wetness.

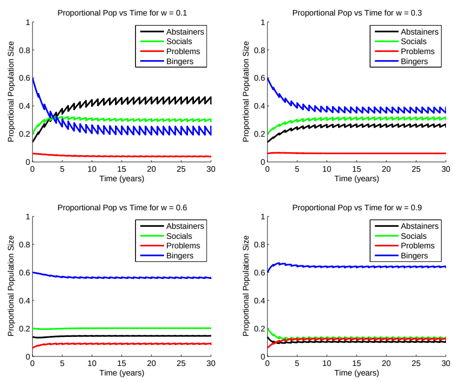

Figure 7: At 60% of the population, bingers remain the majority of student drinkers. $P_1(0) = 0.14$, $P_2(0) = 0.2$, $P_3(0) = 0.06$, $P_4(0) = 0.6$

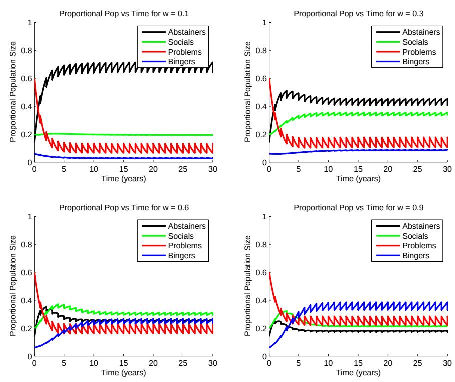

Figure 8: Even with 60% of the student body as problem drinkers, bingers still become the majority. $P_1(0) = 0.14$, $P_2(0) = 0.2$, $P_3(0) = 0.6$, $P_4(0) = 0.06$

school. This is reasonable since abstainers are dropping out at a slower rate than all other compartments and alcohol is not easily accessible in the college atmosphere to encourage drinking. As wetness increases, the proportion of abstainers and social drinkers begins to decrease while the proportion of problem drinkers and binge drinkers increases, as expected.

When looking at a school where abstainers begin as the majority, a wetness parameter of 0.1 causes abstainers and social drinkers to increase slightly, while problem drinkers and binge drinkers decrease, see Figure **??**. As the wetness increases, abstainers decrease, both problem drinkers and binge drinkers increase, and social drinkers increase initially, and then rapidly decrease before reaching periodic equilibrium.

With a school where the majority of the initial population is binge drinkers and the wetness is low, the abstainers and the social drinkers initially increase, whereas the problem drinkers and the binge drinkers initially decrease, see Figure 7. However, as the wetness increases, the binge drinkers decrease at a slower rate until the increase in wetness of the school causes the binge drinking population to increase in size. The increase in wetness also causes the social drinkers and abstainers to increase at a slower rate until it causes the social drinkers and abstainers to decrease.

Then, we simulated a school where the majority of the student body is problem drinkers, see Figure 8. At a school with a wetness parameter of 0.1, alcohol consumption is not supported, and problem drinkers are driven to become abstainers. When the wetness is at least 0.3, the difference between abstainers and problem drinkers is decreasing, which suggests that more students can remain alcoholics. With a wetness of 0.6, social drinkers become the majority, while abstainers and problem drinkers decrease. Since the model does not allow problem drinkers to directly become social drinkers, it seems that problem drinkers are moving to social drinkers after becoming members of the abstainer compartment as a result of attending programs such Alcoholics Anonymous. With a wetness of 0.9, bingers become the majority, while problem drinkers form the second largest compartment. It seems that problem drinkers are again moving through the drinking compartments through the abstainer compartments, this time to become bingers.

Next, we examined a school where the majority of the population is social drinkers, which is perhaps the most common case, see Figure 9. Under a wetness of 0.1, abstainers have a small majority over the social drinkers since alcohol consumption is not supported. As the wetness varies to 0.3, social drinkers have a significant majority over the abstainers. This suggests that even under mildly supportive conditions for alcohol, social drinking will prevail. As wetness increases to 0.6, social drinkers are converted to bingers since alcohol use is supported to a greater extent. This condition is even more extreme as wetness increases to 0.9.

It appears that regardless of the initial population, bingers will always become the majority. This supports our original hypothesis that binge drinking is the most influential of all drinking compartments.

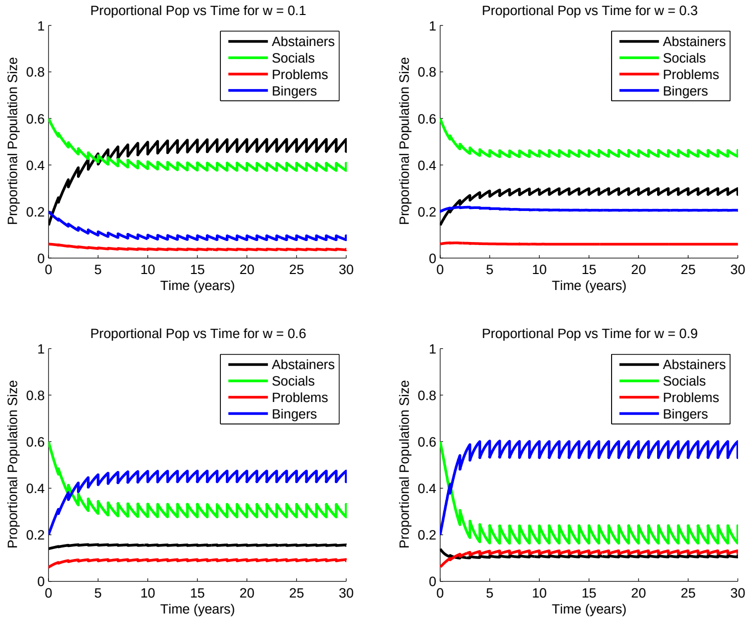

Figure 9: With an initial population of 60% social drinkers, bingers still become the majority. $P_1(0) = 0.14$, $P_2(0) = 0.6$, $P_3(0) = 0.06$, $P_4(0) = 0.2$

4.2.3 Equilibrium

In addition to varying the initial population proportions, of our simulations, we also calculated the stability of the simulations. To do this, we compared the peaks and troughs of each period of replacing of students with the respective peaks and troughs of the previous year in each compartment. When the compartment with the greatest peak or trough difference reached a given tolerance level, we concluded that equilibrium had been reached.

Using the same proportion of drinkers and abstainers that follows the national average, a tolerance level of 0.009, and wetness of 0, the school reaches equilibrium at the end of the tenth year and the beginning of the eleventh year, see Figure 10. This means that the proportion of students in each compartment does not fluctuate significantly after the tenth year.

If we change our wetness parameter to an average value of 0.5, the stability is reached within two to three years, see Figure 11.

Finally, we analyzed the periodic stability of the model with a wetness of 1. In contrast to a wetness of zero, equilibrium is reached in only four to five years. Since the difference between the initial and final conditions is much less then when wetness is zero, equilibrium is reached at a faster rate, see Figure 12.

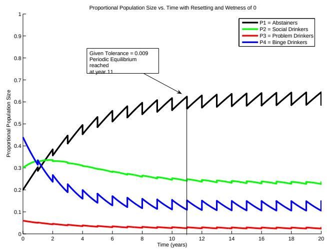

Figure 10: Equilibrium is reach at the beginning of the 11th year.

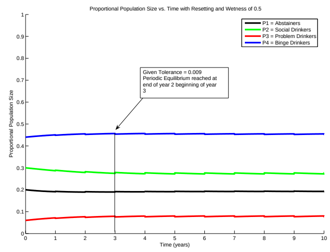

Figure 11: Equilibrium is reached in 2 to 3 years.

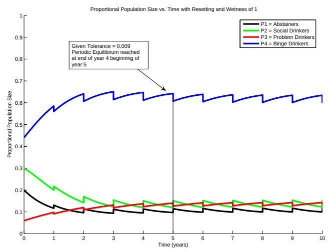

Figure 12: Equilibrium is reached in 4 to 5 years.

5 The Small-World Network Model

5.1 A Network Model

Now that we have looked at the four equation compartmental model and have gained a better understanding of the role of the rates and have identified the key parameters in a homogeneous population, we turn our attention toward a small-world network model in order to better understand a heterogeneous population. There are four different categories of small-world networks: social networks, information networks, technological networks, and biological networks. A social network is defined to be "a set of people or groups of people with some pattern of contacts or interactions between them" [7]. Since we are modeling the interactions of college students, we use the framework of social networks. This type of model builds and creates different individuals defined by a set of characteristics and connects them to each other in a way that defines our network. Since cliques are predominant in a college population we focus our efforts toward creating a model that includes these groups along with a few random connections. Such a network model would alow us to illustrate the individual-to-individual interactions and the effect of an individual on the dynamics of the entire population by means of the internal connections in each clique and the connections between cliques.

In order to understand small-world networks, we must first define a few key terms. *Graph theory* is the field of mathematics in which graphs and, by extension, small-world networks are studied. A *graph* is "a collection of points and lines connecting some (possibly empty) subset of them" [8], and a *small-world network* is a particular type of graph. Every network is composed of *vertices* and *edges*. A *vertex* is the rudimentary unit of a network and is represented by a dot. *Edges* are the lines that connect every two vertices. If a graph is representing a network, *undirected* edges are used to define connections between vertices. When things are flowing to a particular vertex,*directed* edges are used. A network is *complete* or *fully connected* if every vertex is connected to every other vertex. Finally, a *cluster* is subset of vertices within a graph that are complete or nearly complete [7].

In our small-world network model, the graph or network represents the entire student population at a college or university. Each vertex symbolizes an individual student, and an edge between two vertices represents an acquaintance relationship between the two students. To model the various social cliques and groups of students at a college campus, we want random-clusters.A cluster in our model represents a group of students who are very tightly connected. As a result of their tight connection, every group or clique member will have a greater influence on the other members in the clique.

5.2 Individualizing the Students

To individualize the students, each individual needed to have specific attributes. To accomplish this, we assigned a compartment number, m, and a change variable, c, to each student. The compartment number, m, is either 1, 2, 3, or 4 depending on whether the student is an abstainer, a social drinker, a problem drinker, or a binger, respectively. The change variable, c defines the unlikeliness for an individual to change or move to another compartment due to peer pressure or his/her perspective of what is popular or accepted by his/her circle of friends. For the remainder of this paper we will assume that an individual's vulnerability to change is inversely related to his/her ability to change others. Thus, the lower the change variable, c, the more likely he/she is to change compartments and the more unable he/she is to influence others to change. This attribute allows for individuals within a drinking compartment to be differentiated from each other. The change variable,c, is a random number that is chosen from a beta distribution, whose probability density function is:
The probability density function of the beta distribution is defined as:

$$F(x) = \frac{1}{B(\alpha,\beta)}(x^{\alpha-1})(1-x)^{\beta-1}$$

with $0 < x < 1$ and where

$$B(\alpha,\beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1}dt,$$

for α and $\beta > 0$. [9]. For a given α and β we obtain a particular probability distribution and from there randomly select a change variable, c, for each student. In this paper, we will use a beta distribution with $\alpha = .41$ and $\beta = 1.67$, see Figure 13. These constants were chosen particularly because this allows us to randomly assign lower c values to a greater number of students. Recall we assume that people are typically very easily influenced by their peers and the social norms of their school, therefore the tendency to change will be more likely for most individuals. The change variable, c as chosen from the beta distribution with $B(.41, 1.67)$ mathematically models this assumption. Because of the inverse relationship between vulnerability to change and ability to induce change, the graph demonstrates that a student who is easily influenced to change is also not able to easily influence others to change and vice versa.

Along with the change variable, a student is given a position in the unit square (i.e., $[0, 1] \times [0, 1]$). The x-coordinates and y-coordinates that determine students' positions are determined using uniform distributions in the interval $[0, 1]$ and allow each student to be placed in an exact location within the unit square. This placement is important in order to calculate the distance between two students. In order to form non-random clusters, we will need to specify different circles of a certain radius and connect all the vertices (in this case students) within the circle. Each student is assigned to one of the four compartment numbers similar to those used in the deterministic model: $m = 1$ if the student is an abstainer, $m = 2$ if the students is a social drinker, $m = 3$ if the student is a problem drinkers, and $m = 4$ if the student is a binger. Students are placed in a compartment by choosing a radius from a uniform distribution. Since initially, students have to randomly be placed in one drinking compartment, these compartments are defined by intervals in the [0,1] range. The initial compartment placements are based on percentages from national drinking averages on college campuses [2]. Since these averages at colleges and universities are approximately 20% abstainers, 30% social drinkers, 44% binge drinkers, and 6% problem drinkers, we use the following ranges: Abstainers = $[0, .2)$, Social Drinkers = $[.2, .5)$,Binge Drinkers = $[.5, .94)$, Problem Drinkers = $[.94, 1]$ Students are placed into these compartments by first calculating n_i, the number of students that are needed in compartment i where i = abstainers, social drinkers, problem drinkers, and binge drinkers:

$$n = (\text{total number of students})(\text{ proportion of students in compartment } i).$$

We then randomly choose n_i numbers between [0,1]. These random numbers are then scaled to the appropriate ranges for the compartment, i, that we are building. After giving students decimal representations, students are given whole number representations for their respective compartments in an effort to keep the programming as simple as possible. As mentioned earlier, these compartments are numbered $m = 1, 2, 3, 4.$ are numbered as follows: Abstainers = 1, Social Drinkers = 2, Problem Drinkers = 3, Binge Drinkers = 4.

5.3 Developing Connections Between Students

To build the small-world network, we build both random and clustered connections. When only random connections exist in a system, it is expected that the distribution of connections follow a power law.

If the students are only connected through clusters, the model does not accurately represent the college atmosphere because it ignores the unexpected connections that exist between students from different cliques but that know each other and share common interests. Many college students interact with multiple "social circles" and are, therefore, included in and connected to more than one cluster. A small number of random connections allows for the interactions between multiple social circles.

In order to create the random-clustering model, we first assume a particular population of students. In order to create connections between these students, we assume that an imaginary circle with a particular radius exists around each vertex (i.e. student). If another vertex exists within the "circle", then the two students form a connection. Since we recognize that students have multiple "circles" of friends, we create $\ln(N)$, where $N =$ the total population, random connections between the students.

After the two types of connections are formed, we have created a small-world network in which students are connected across the graph in addition to the connections with their neighbors. Next, we consider all of the students as individuals and count the number of connections for each student. This number of connections then becomes another defining parameter, f for each student. Due to the clustering connections, the distribution no longer follows a power law.

5.4 Changing the Network

In the process of building this network, we have made two crucial assumptions:

1. Any student in any compartment has the ability to move to any other compartment.
2. A student's ability to change another student is affected by the compartment that the influential student occupies.

Although these assumptions do not completely emulate the actual population of college and university students, these assumptions were allowed for the sake of simplicity.

To show the students' progressions of movement between compartments, we need to determine the manner in which students interact and the way that they are influenced to change by the students who surround them.

First, we consider one student, w_i, and calculate the influences from the peers to whom he is connected, say w_j for $j = 1, \ldots, k$. This calculation considers the change variable, c_i, for the student and the change variables for the student's peers, c_j, where $j = 1, \ldots, k$ is a student near the student, a number between 1 and k. This is done by comparing the averages of the change variables of student's peers, $\frac{\sum_{j=1}^{k} c_j}{k} > c_i$, to the student's own change variable c_j.
If $\frac{\sum_{j=1}^{k} c_j}{k} < c_i$, we move onto the next student. However, if $\frac{\sum_{j=1}^{k} c_j}{k} \geq c_i$, a change will occur in student w_i. We then calculate the compartment to which student w_i will move by considering the compartments of his k peers, m_j for $j = 1, \ldots, k$ and his own m_i. The new compartment of student w_i is defined in decimal form by:

$$\frac{\sum_{j=1}^{k} m_j + m_i}{k+1} \quad = \quad \text{new compartment for } w_i. \tag{23}$$

We then change w_i's new compartment given by (22) to its integer value as explained in Section 6.2.

6 Validation of the Network Model

6.1 Network Simulations without Restrictions

In order to obtain preliminary results for a network model that semi-simulates the effect of dropouts and individual risk factors, we allow a student drinker in any drinking compartment to move to any other compartment through social interactions. We understand that dropouts and individual risk factores are intrinsic movements and not cause by social factors, but hope to gain some insight to the evolution of the process by making this assumption. Also, a student's ability to change another student is influenced by the compartment that the influential student occupies. We allow binge drinkers to be most influential of all drinkers, followed by social drinkers, abstainers, and problem drinkers. This assumption is made because binge and social drinkers typically drink with others which allows them to be more easily influenced. Problem drinkers typically drink alone and are, therefore, least influenced by others.

For a standard population of 20% abstainers, 30% social drinkers, 6% problem drinkers, and 44% binge drinkers, simulations predict that nearly the entire student body will become social and binge drinkers, see figures 17, 18 and 19). Whether the social or binge drinkers become the majority is dependent on the the clustering of the student drinkers. If clusters of binge drinkers form from super-connected bingers, the binge drinkers will ultimately become the majority population. The case is similar for the social drinkers. We see these dynamics because if anyone is super-connected, they will have a much greater influence on the other members of their cliques.

Since the final outcome of the drinking population seems to depend on clustering, we varied the connection radius and noted its changes on the dynamics of the system. Increasing the radius increases the level of connectivity and creates larger clusters of drinkers. Eventually, these increasingly large clusters begin to merge into a single cluster that behaves more as a homogeneous system. Extreme interconnectedness also causes the entire student population to become the same type of student drinkers. The specific compartment that becomes the majority varies from simulation to simulation since a single drinker can change the status of the remaining population. This ability of a single drinker to convince others also causes more rapid changes in drinking status among students. Of the four types of student drinkers, it seems that social drinkers and binge drinkers are the most influential since they form groups of convincing students most often.

From the model, we can see that extreme interconnectivity causes dramatic changes in the dynamics of the system as well as the time it takes for a steady-state to be reached. Various recommendations can be made to colleges and universities based on our model of student drinking. First of all, it seems that clusters of student drinkers are able to have the most influence on the entire campus populations. To reduce drinking levels, administrators can use this information to construct clusters of abstainers that should encourage other students to become abstainers. They can also isolate clusters of drinkers, such as fraternities, from the remainder of the population and prevent their drinking habits from spreading to other students. Since social interaction can control drinking patterns, administrators should organize social activities that promote interaction using alcohol-free environments such as intermural sports, theater, concerts, etc.

6.2 Changes to the Models

In order to compare the deterministic and small-world network models, we must adjust each model to accommodate for the differences and similarities between the two.

Movement due to individual risk factors is not considered in the network model because these are intrinsic, not caused by social factors. The movement in the network model is based on the students who are connected to a particular student. Thus, we can only consider the changes that occur because of the social interaction parameters and the social norm parameters in the deterministic model. Furthermore, the dropout/graduation rate cannot be considered in the network model because we are not creating or destroying students (represented by vertices) or the acquaintance relationships between the students (represented by edges) once the process has begun. These restrictions force the r_{ij} and the d_i terms to be set to zero in the deterministic model. Since the d_i terms are no longer included, the resetting equation has no effect because there is no change in the total population.

We must restrict the movement between compartments in the small-world network model so that it reflects the restricted movements in the deterministic model. Without compartmental movement restrictions in the small-world network model, the students are free to move from any compartment to any other compartment. For example, a student who is a binge drinker could become an abstainer in a single time step due to social interaction. In this model, a time step has the units of 1 - 2 years. The deterministic model has unit steps much smaller and therefore does not allow for a social interaction change of this magnitude, so we must remove this option in the small-world network model in order to compare the two models. We calculate the possibility of change and the compartment to which the student would move, but if the calculations indicate that the student should move to the abstainer compartment, he or she is forced to move to the social drinker compartment instead. Another change to the small-world network model is the restriction of movement once a student has reached the problem drinker compartment. Since our deterministic model only allows movement to occur between the problem drinker compartment and the abstainer compartment as a result of no individual risk factors, we must reflect this in the small-world network model. In order to do this, we skip the problem drinkers in our calculations. Since the problem drinkers are not moving out of their compartment due to social interactions or social norms, there is no reason to consider the possibility of change or movement, their change calculations are skipped. However, the problem drinkers still have the abilities to influence other students in the model, so they are still considered in the calculations of the influences they have on other students who are connected to them. Another important change is the removal of the change variable. The change variable gives our model heterogeneity and defines the vulnerability to change. Without this change variable, the students are only defined by their compartments, and thus, the small-world network model behaves in a manner that is more like the deterministic model. To accommodate for the homogeneity, we must also connect every student to every other student in the graph. This gives us a *complete* graph for the network.

6.3 Model Comparison

After completing the adjustments to the deterministic model as described above, we compared the final results of both models to ensure that the network model accurately describes college drinking patterns using homogeneous mixing. (See Figures 20,21, 22, 23, and 24).
Our first comparison examined the similarities and differences between the deterministic model and a network model of complete connectedness and homogeneity. In its final state, the modified deterministic model predicts that the entire student population will eventually become problem drinkers. As this model approaches its final state, binge drinkers remain a significant proportion of the population while abstainers and social drinkers convert to problem drinkers much more rapidly. In the final state of the network model, the entire student population will again become problem drinkers. However, the transition to this state is somewhat different from the deterministic model. As time progresses the social drinkers unrealistically become 75% of the student body population. In the deterministic model, the maximum proportion of social drinkers is a mere 35%. As it approaches its final state, the network model also predicts that the only drinkers remaining will be problem drinkers and binge drinkers. Overall, both models conclude that when drinking patterns are controlled only by social parameters, the entire student population will consist of problem drinkers.

Our second comparison examined the same deterministic model as before, but used a network model of nearly complete connectedness and homogeneity. As the nearly complete network model progresses, most students still become problem drinkers but do so over a larger number of time steps (See figures 25, 26 and 27). At its final state, a small percentage of students still remain as binge drinkers while all other students have become problem drinkers (See figure 28). Since the model no longer accounts for homogenous interaction, a small number of students lack complete connectedness to the remaining student body. It is their lack of connections that allow them to remain as binge drinkers and refrain from becoming problem drinkers.

Based on these comparisons, a network model of college drinking patterns provides information that cannot be accounted for in the deterministic model. For instance, the network model reveals that a student's drinking classification depends on their connections with other students. Students that are completely connected are more likely to convert to the same drinking compartment as the students they are connected with, while students without any connections are unable to change.

7 Conclusions

From the analysis developed in the first half of this paper, many things can be concluded about the four equation, deterministic model. First, the social influence and social norm parameters have the greatest influence on the way students move throughout the compartments. Along with these influences, the wetness of a school and its surrounding community significantly changes how quickly equilibrium is reached. With a higher wetness value we can also see that there are typically more binger students and this compartment always has the most students no matter what the initial conditions. After stability analysis, it can also be seen that an alcohol-free equilibrium is unstable and a student population will not typically stay alcohol-free.

In the analysis of the small-world network model we found that the connectivity of a network has the greatest influence on the tendencies of the system. For the networks that have larger "cliques" or a greater radius of connection, an alcoholic stability is reached much sooner.

In comparing the two models we verified the individual conclusions from each and found that eventually both models show the same results, though not necessarily in the same amount of time because we could not specify an exact time step for the network model. We also concluded that the small-world network model more accurately represents a college population because of its heterogeneity versus the homogeneity of the deterministic model.

8 Future Work

In the future, work could be done to compare the results from the deterministic model to data collected from surveys of college students. Comparing this data can give more insight into the parameter values and long-term effects of these changes. For the small-world network model, it is suggested to add the change due to individual risk in the influences on individual students. This could be done by considering a probability distribution for each individual student. This probability distribution could give each student a probability of movement to any particular compartment. To evaluate this, a certain percentage of students would change at each time step to their highest probability. The graduation or dropout rates from the deterministic model should be included as well. Since the college student population is regularly changing, it is necessary to add this to properly model the college atmosphere. Along with the change in the number of students, a creation and destruction of edges or acquaintances would have to occur. Students in the college environment do not typically have the same friends throughout their entire college careers and these bonds change significantly after graduation. While the deterministic model gives ranges for parameters based on the "wetness" of a campus and surrounding community, the network model does not include this. Inclusion of a wetness term or equation would give the ability to model various types of schools and communities more accurately.

9 Acknowledgements

This research was conducted at the Applied Mathematical Sciences Summer Institute (AMSSI) and has been partially supported by grants given by the Department of Defense (through its

ASSURE program), the National Science Foundation (DMS-0453602), the National Security Agency (MSPF-06IC-022). Substantial financial and moral support was also provided by Don Straney, Dean of the College of Science at California State Polytechnic University, Pomona. Additional financial and moral support was provided by the Department of Mathematics at Loyola Marymount University and the Department of Mathematics & Statistics at California State Polytechnic University, Pomona. We would like to thank Dr. Benjamin Fitzpatrick, Dr. Stephen Wirkus, and Dr. Zoltan Toroczkai for their helpful conversations. We greatly appreciate the assistance of Mr. Adam Schneider, a computer science undergraduate at the University of Missouri-Rolla, for helping us write our small-world network program in *C++*. Furthermore, we would like to express our sincerest gratitude to our research advisor, Dr. Erika Camacho, for her direction, devotion, and support for the duration of this project. We would also like to thank Ms. Laura Smith for her assistance with research, computer programming knowledge, and overall contribution to our project. Without the help of any of these people, our project would not have been realized. The authors are solely responsible for the views and opinions expressed in this research; it does not necessarily reflect the ideas and/or opinions of the funding agencies and/or Loyola Marymount University or California State Polytechnic University, Pomona.

References

[1] Donna E. Shalala. (29), July 1995. http://pubs.niaaa.nih.gov/publications/aa29.htm.

[2] Statistics, 2001. http://www.intheknowzone.com/binge/stats.htm.

[3] A snapshot of annual high-risk college drinking consequences, 2005. http://www.collegedrinkingprevention.gov/StatsSummaries/snapshot.aspx.

[4] Ben Fitzpatrick Geoffrey Jacquez Jeremy Thibodeaux Robert Rommel Neal Simonsen Richard Scribner Azmy Ackleh ˙Ecosystem modeling of college drinking: Development of a deterministic-compartmental model. 2006.

[5] Mary C. Dufour. What is moderate drinking? *Alcohol Research and Health*, 1, 1999. http://pubs.niaaa.nih.gov/publications/arh23-1/05-14.pdf.

[6] Herbert W. Hethcote. The mathematics of infectious diseases. *SIAM (Society for Industrial and Applied Mathematics) Review*, 42(4):599–653, 2000.

[7] M.E.J. Newman. The structure and function of complex networks. *SIAM (Society for Industrial and Applied Mathematics) Review*, 45(2):167–256. Department of Physics, University of Michigan, Ann Arbor, MI and Santa Fe Institute, Santa Fe, NM.

[8] Eric W. Weisstein. Graph. From Mathworld - A Wolfram Web Resource, http://mathworld.wolfram.com/Graph.html.

[9] Lee Bain and Max Engelhardt. *Introduction to Probability and Mathematical Statistics*. Brooks/Cole, 1992.

[10] C.S. Holling D. Ludwig D.D. Jones ˙ Qualitative analysis of insect outbreak systems: The spruce budworm and forest. *Journal of Animal Ecology*, 47, 1978.

[11] Duncan J. Watts and Steven H. Strogatz. Collective dynamics of 'small-world' networks. *Nature*, 393:440–442, 1998.

[12] Larry Samuelson Emilia Sansone Avner Shaked Ilan Eshel Dorothea Herreiner ˙ *Sociological Methods and Research*, July.

[13] Jesper Dall and Michael Christensen. Random geometric graphs. *Physical Review E*, 66(1):016121, July 2002.

[14] Cristopher Moore and M.E.J. Newman. Epidemics and percolation in small-world networks. *Physical Review E*, 61(5):5678 – 5682, May 2000.

[15] Matthew Mulford John Orbell Langche Zeng ˙ *American Sociological Review*, 61(6):1018–1032, December 1996. Accessed through JSTOR 2006 July 13.

[16] Hyojoung Kim and Peter S. Bearman. The structure and dynamics of movement participation. *American Sociological Review*, 62(1):70–93, February 1997.

[17] Noah Mark. Beyond individual differences: Social differentiation from frist principles. *American Sociological Review*, 63, June 1998.

[18] Andreas Flache James A. Kitts Michael W. Macy ˙ *Computational & Mathematical Organization Theory*, 5(2):129–145, July 1999.

[19] Michael W. Macy and Robert Willer. From factors to actors: Computational sociology and agent-based modeling.

[20] Steven H. Strogatz. *Nonlinear Dynamics and Chaos*. Westview Press, 1994.

[21] D.J. Daley and J. Gani. *Epidemic Modeling: An Introduction*. Cambridge University Press, 1999.

[22] G. Martinez C. Nesmith G. Chowell J. Bracamonte M. Gorritz ˙ Scaling laws and dynamics of sexual activity with interracial and multi-ethnic mixing. Mathematical and Theoretical Biology Institute, Los Alamos National Laboratory, Center for Nonlinear Studies, August 2003.

[23] The Mathworks, Inc., August 2005. Users Guide Version 5 for use with MatLab, www.mathworks.com.

[24] Duane Hanselman and Bruce Littlefield. Prentice Hall, 2005. Department of Electrical & Computer Engineering, University of Maine.

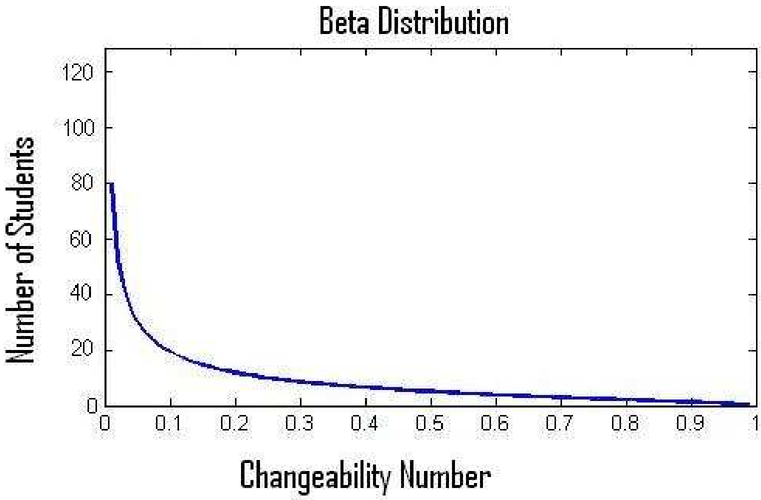

Figure 13: The Beta Distribution with $\alpha = 0.41$ and $\beta = 1.67$.

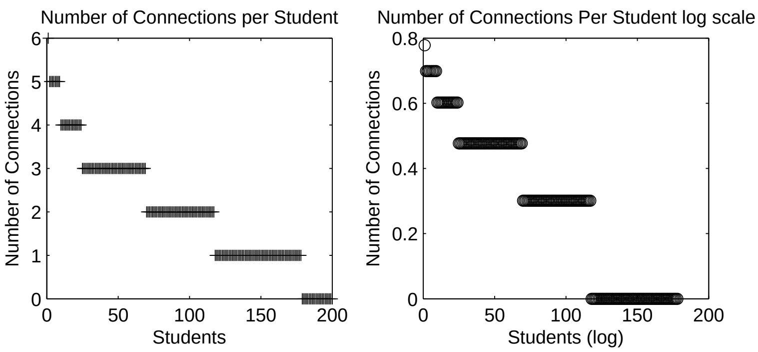

Figure 14: With an initial population of 200 students and a completely random graph, the distribution of connections follows a power law.

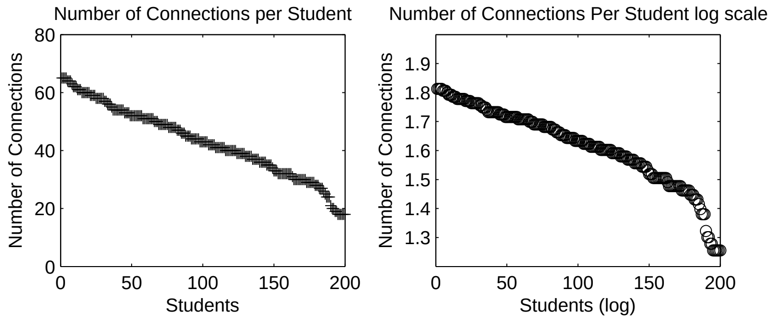

Figure 15: With an initial population of 200 students and a graph with cluster connections and few random connections, the distribution of connections no longer follows a power law.

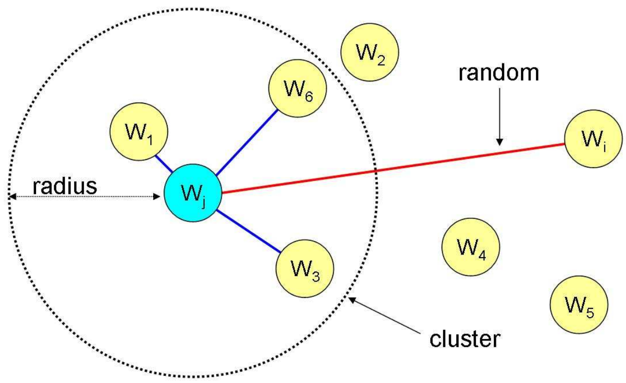

Figure 16: Illustration of random and nearest neighbor connections.

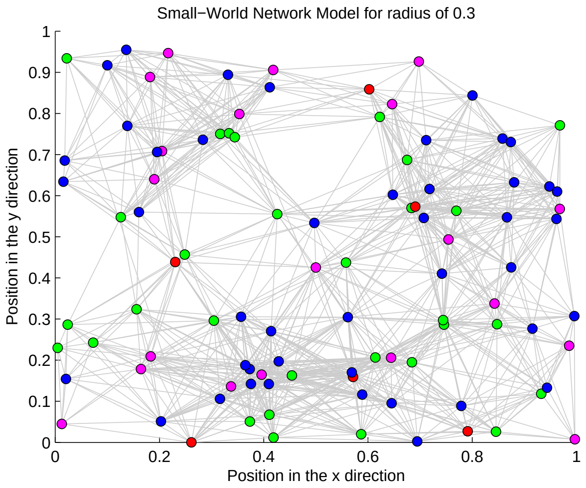

Figure 17: The standard network model at a time step of zero.

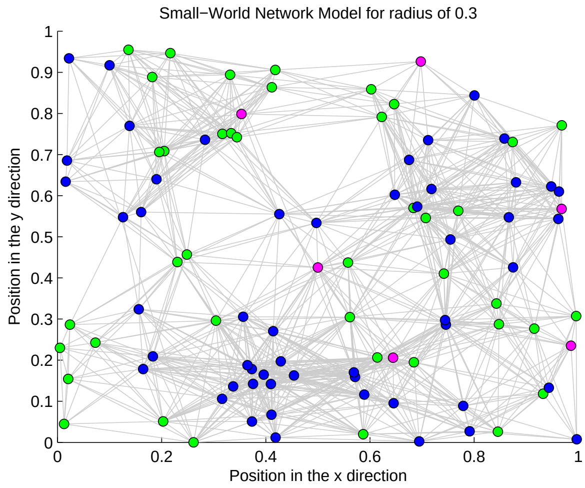

Figure 18: The standard network model at its mid-range time step.

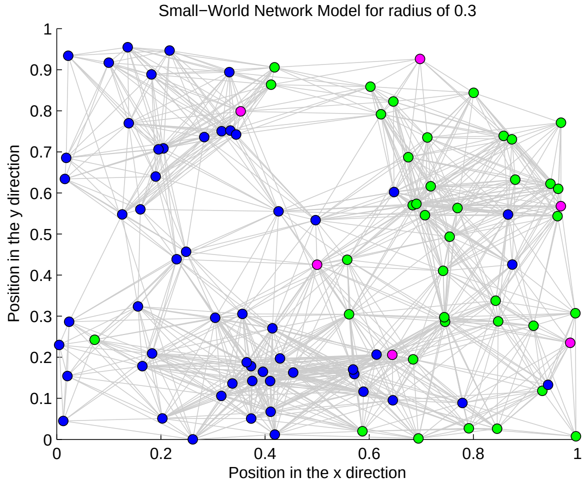

Figure 19: The network model at its final time step.

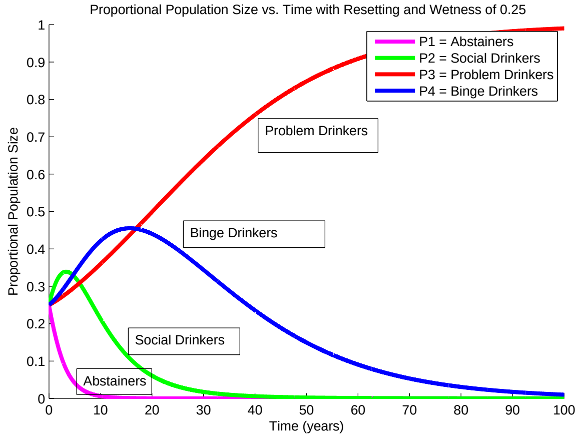

Figure 20: The deterministic model adjusted for social interactions.

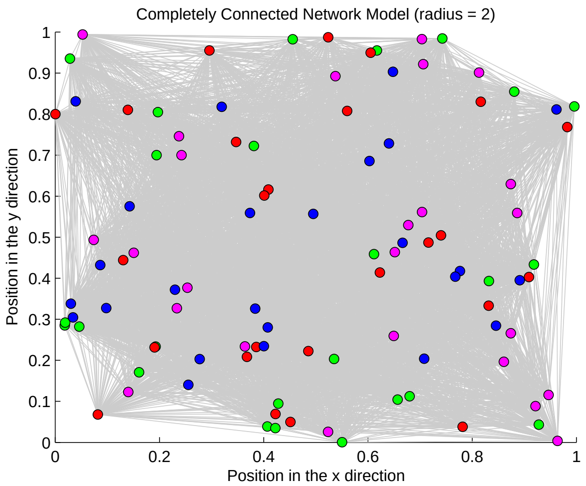

Figure 21: The network model at a time step of zero.

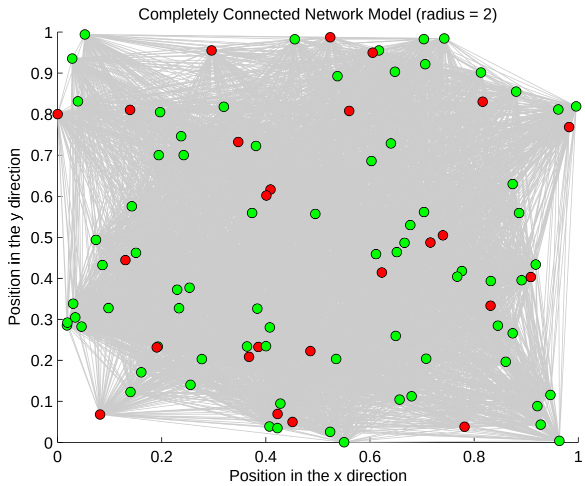

Figure 22: The network model at a time step of one.

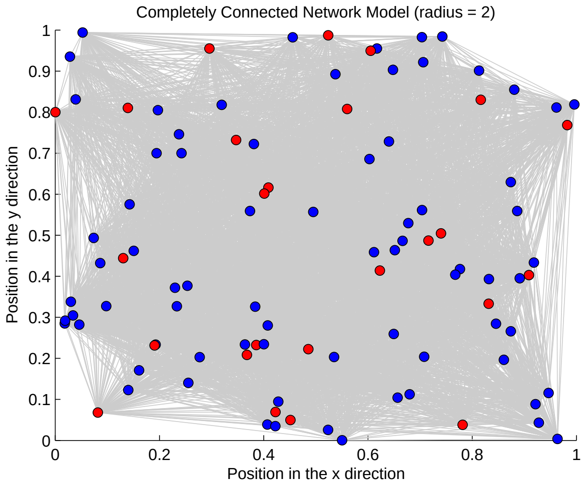

Figure 23: The network model after a time step of two.

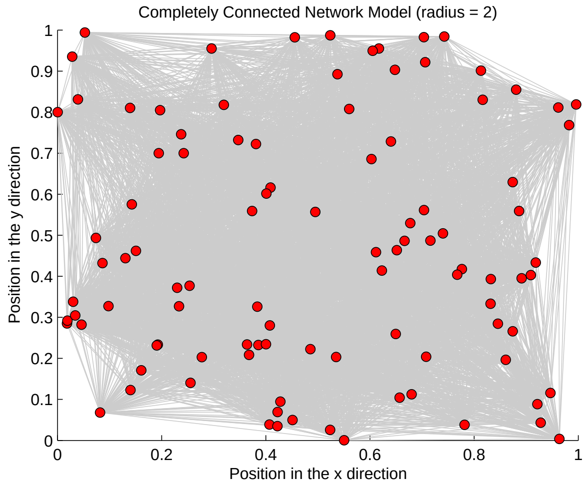

Figure 24: The network model at its final time step.

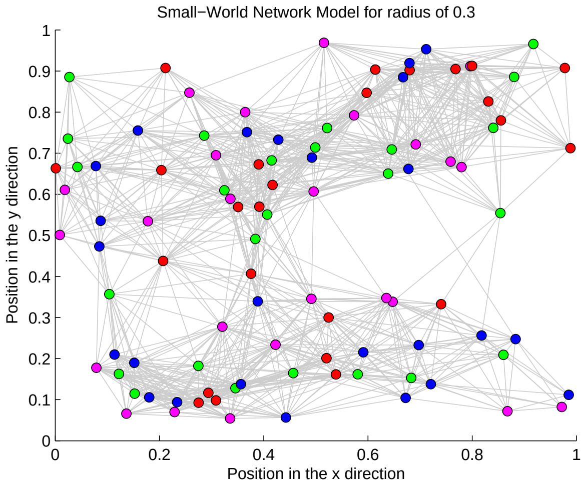

Figure 25: The nearly complete network model at a time step of zero.

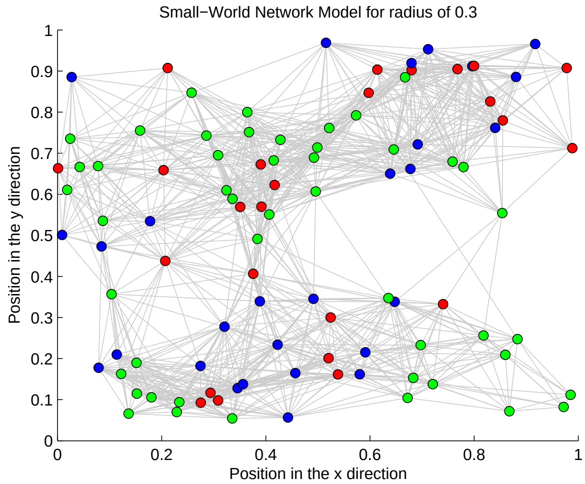

Figure 26: The nearly complete network model at a time step of one.

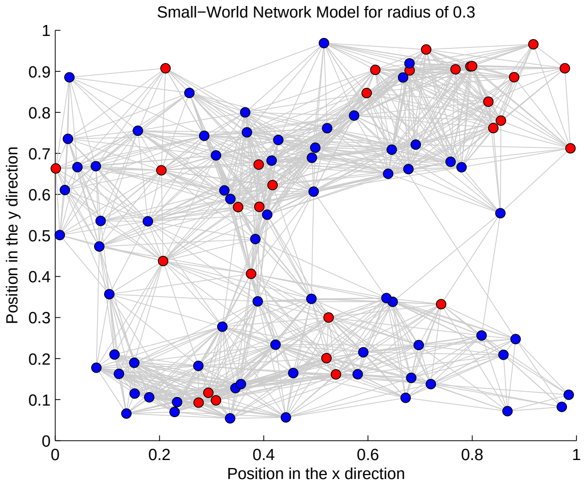

Figure 27: The nearly complete network model after a time step of two.

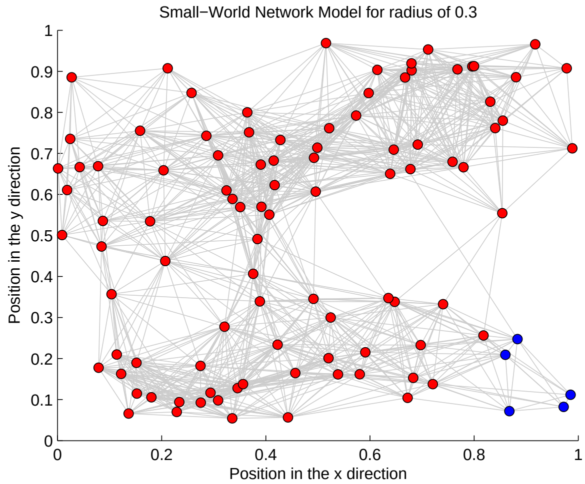

Figure 28: The nearly complete network model at its final time step.