Abstract—Network lifetime is a critical issue in wireless sensor networks. In the coverage problem, sensors can be partitioned into many subsets to prolong network lifetime. These subsets are activated successively and each of them completely covers an interest region. Many centralized algorithms have been proposed to solve this problem. A very few distributed versions have also been presented but none of them obtains a good approximation ratio. In this paper, we first reduce the coverage problem to a related domatic number problem in directed graphs. We next prove the lower and upper bounds of this domatic number. Based on this proof, we present two localized and distributed algorithms to solve the coverage problem with an objective of maximizing network lifetime. The approximation ratio of these distributed algorithms is within a factor of $O(\log n)$. Theoretical analysis and performance evaluation results are also presented to verify our approach.

Keywords: energy efficiency, coverage problem, dominating set partition, algorithms

I. INTRODUCTION

Recent advances in embedded processing and wireless networking have made possible creation of wireless sensor networks. Wireless sensor networks can be used in a wide-range of potential applications such as national security, surveillance, health care, biological detection, environmental monitoring, and many other applications [8], [9], [10]. A wireless sensor network consists of a large number of tiny sensor nodes to accomplish a larger sensing task. Sensor nodes are small devices equipped with one or more sensors, one or more transceivers, processing, storage resources and possible actuators [13]. Sensors in a network can cooperatively gather information from an interest region of observation and transmit this collected information to base stations.

Energy efficiency is one of the most critical issues in wireless sensor networks. With the current available technology, sensors are battery powered and have a limited weight. These characteristics globally affect the application lifetime. Thus prolonging network lifetime is an important part of the system design in wireless sensor networks. One of the most prominent methods is based on the schedule sensor activity so that a set of active sensors can handle a required task while the rest of the redundant sensors can enter into sleep mode. In [15], the authors pointed out that the energy consumption in the sleep state is about 15 times less than that in the active state. In addition, the lifetime of a battery discharging in short bursts with significant off-time is approximately twice as much as in a continuous mode of operation [14].

An important problem addressed in wireless sensor networks is the coverage problem. The goal is to have each location in the physical space of interest within the sensing range of at least one sensor. In wireless sensor networks, sensors are densely deployed. The number of sensors deployed is usually higher than needed due to the lack of precise sensor placement. Thus it is possible to turn some sensors off while guaranteeing the complete coverage of the interest region.

In this paper, we study the target coverage problem with an objective of maximizing network lifetime, called the maximum lifetime target coverage problem. Besides satisfying the coverage requirement, the goal of this problem is also to maximize network lifetime. Network lifetime is defined as the total lifetime of a network that all the targets are completely covered at any time instant. To study this problem, we use a discrete target model, in which the purpose is to cover a set of fixed targets. Each target is monitored by a set of sensors. Indeed, we can transform the area coverage into the discrete target model by dividing the area into a number of fields where each field is monitored by the same set of sensors [5], [4]. Here, we can treat each field as a target.

Many works in literature have been proposed to solve
the above problem. The major approach of these proposals is to organize sensors into a number of cover sets such that each set completely covers all the targets [1], [2], [5], [4]. These cover sets are activated successively, so that at any time instant, only one set is active. The sensors from the active set are in the active state (e.g., transmit, received or idle) and all the other sensors are in the sleep state. Note that the cover sets can be disjoint [2], [5] or non-disjoint [1], [4]. Unfortunately, all these algorithms are centralized versions, which is impractical. A very few distributed algorithms [16], [17] were proposed, but the authors did not provide any worst-case analysis and performance ratio.

Focused on distributed and localized solutions, we present two \(O(\log n)\)-approximation distributed algorithms for the maximum lifetime target coverage problem. In particular, we first reduce this problem to the domatic number problem in directed graphs. The domatic number is defined as the maximum number of disjoint dominating sets in a given graph. For undirected graphs, the authors in [11] recently have just analyzed the lower bound of this number. Using a similar approach, we obtain the upper bound and lower bound of the domatic number in a directed graph.

Motivated by the above relationship, we continue showing how to reduce an original graph \(G\) describing the coverage relationship between the sensors and the targets to a directed graph \(G'\) whose vertices only represent the sensors, i.e., no targets involved. Any dominating set (see Section III for the definition of dominating set) of this directed graph \(G'\) guarantees a complete coverage of all the targets in the original graph \(G\). This transformation can be constructed locally and distributively, requiring only a constant number of communication rounds.

We next present two distributed and localized algorithms to find the maximum disjoint dominating sets of \(G'\). In [12], the authors proposed the distributed algorithms to solve this problem in undirected graphs using the color class method and probability arguments. We adopt this scheme to our directed graphs. We also show that these algorithms can obtain an \(O(\log n)\) approximation ratio with a high probability. Note that our distributed algorithms are completely localized, from the graph construction to the solution of maximum number of disjoint dominating sets.

The contributions of this paper are the following:

1) Introduce a new model of maximizing network lifetime of the target coverage problem by reducing it to the domatic number problem in directed graphs

2) Show the bounds of this domatic number and the relationship of the coverage problem and the domatic number problem

3) Present the solutions and their performance analysis

4) Analyze the performance of our approach through simulations.

The rest of this paper is structured as follows. In section II, we present energy efficient coverage and dominating sets related works. Section III describes the problem formulation as well as shows the lower and upper bounds of the domatic number in the directed graphs. We introduce the problem transformation from the maximum lifetime coverage to the related domatic number in section IV. Two \(O(\log n)\) distributed algorithms and their theoretical analysis are presented in section IV. Section V reveals the performance of these algorithms through the simulation results and section VI concludes our paper.

II. Related Work

A. Coverage Problem

The maximum lifetime coverage problem in wireless sensor networks have been studied extensively. This problem is NP-complete. The most relevant work to our approach is [3]. The authors in [3] modeled the coverage problem as the disjoint dominating sets in an undirected graph. A graph-coloring mechanism was proposed for computing the disjoint dominating sets. However, this dominating set does not guarantee complete coverage. Also, this algorithm is a centralized one, which is impractical.

Other related works are [1], [2], [5], [4]. Both [5] and [2] proposed energy efficient centralized mechanisms by dividing the sensor nodes into disjoint sets and each set completely covers the monitored region. The goal of this approach is to determine a maximum number of disjoint sets, as this has a direct impact on conserving energy resources as well as on prolonging network lifetime. Specifically, the disjoint set cover problem in [2] was reduced to a maximum flow problem, which is then modeled as a Mixed Integer Programming. In [5], a polynomial time heuristic called Most Constrained-Least Constraining Covering Heuristic was proposed to compute the disjoint covers successively.

In [4], Berman et al. introduced another approach for the maximum network lifetime problem using a packing Linear Programming technique. In this approach, sensors are divided into non-disjoint sets. They proposed an
approximation algorithm with the performance ratio of \((1 + \varepsilon)(1 + 2 \ln n)\) for any \(\varepsilon > 0\). The running time of this polynomial-time approximation scheme is quite high. It is necessary to study faster heuristics with a good performance. In our previous work [1], we proposed another model called Maximal Set Covers (MSC) in which we can obtain a \((1 + \varepsilon)^2\)-approximation algorithm. We also showed that organizing sensors into non-disjoint sets achieves a better result in terms of network lifetime than organizing sensors into disjoint sets. All of these four algorithms are the centralized versions, which is impossible from the practical point of view.

There are also a few distributed algorithms about the coverage problem [16], [17]. In [16], power conservation was obtained through coverage-based off-duty eligibility rule and back-off-based node-scheduling scheme. The off-duty eligibility rule determines whether a node’s sensing area is included in its neighbors’ sensing area. In [17], Zhang and Hou addressed an important observation which is that an area is completely covered if there are at least two disks that intersect and all crossings are covered. Based on this observation, the authors proposed a distributed algorithm called optimal geographical density control (OGDC). At any time, a node can be in one of the three states: UNDECIDED, ON, and OFF. The algorithm runs in rounds and using the backoff mechanism to decide the status of each node. Unfortunately, none of these works analyzes the performance ratio or presents any worst-case analysis.

B. Dominating Sets Problem

Because the concept of dominating sets is highly related to the clustering in wireless networks such as routing and virtual backbone, many works have been proposed in literature [6], [7], [11], [12]. One of the standard problems is to find the minimum dominating set of a given graph. This problem is NP-hard. Most of the works in this area just focused on finding only one minimum dominating set. The only two works highly related to ours are [11], [12].

In [11], Feige et al. studied the domatic number problem. The authors proved that any undirected graph \(G\) has at least \(\frac{(1-o(1))(\delta+1)}{\ln n}\) disjoint dominating sets where \(\delta\) and \(n\) are the minimum degree and number of vertices of \(G\) respectively. The authors later used the Lovasz Local Lemma [18] to give a tighter bound of the domatic number. A nonconstructive argument showed that the domatic number is at least \(\frac{(1-o(1))(\delta+1)}{\ln \Delta}\) where \(\Delta\) is the maximum degree of \(G\). An important result was also presented in this paper. Specially, the authors showed that for every \(\varepsilon > 0\), approximating the maximum disjoint dominating sets problem within a factor of \((1 - \varepsilon) \ln n\) is impossible, unless \(NP \subseteq \text{DTIME}(n^{1+\varepsilon})\). It also yields a \((1 - o(1))\ln \Delta\)-hardness. They also proposed a centralized algorithm that produces a domatic partition of \(\omega(\delta/\ln \Delta)\) sets in undirected graphs. Their constructive proof can be turned into an efficient distributed algorithm for the domatic number problem. In this paper, we use a similar method to prove the bounds of the domatic number for directed graphs.

In [12], Moscibroda and Wattenhofer also studied the maximum number of disjoint dominating sets in undirected graphs. They used the similar probability arguments in [11] to build three distributed algorithms of which the approximation ratio is within a factor of \(O(\log n)\). In addition, the authors studied the Maximum k-tolerant Clustering Lifetime problem. This problem is to find the maximum number of disjoint dominating sets such that any node in a given graph has at least \(k\) dominators within its neighbors. All of these algorithms target to the network lifetime. In our paper, we adopt this idea to directed graphs. We prove that in directed graphs, the approximation ratio \(O(\log n)\) still holds.

III. Problem Formulation

In this section, we give the formal definitions of the maximum lifetime target coverage problem, the domatic number in directed graphs, and the maximum lifetime dominating sets problem. Next, we present the upper and lower bounds of this domatic number. In this section, we also introduce our notations that are being used throughout the paper unless stated otherwise.

A. Maximum Lifetime Target Coverage

Let us assume that \(n\) sensors \(s_i\), \(i = 1...n\), are randomly deployed to cover \(m\) targets \(r_j\), \(j = 1...m\). Sensor \(s_i\) covers target \(r_j\) if and only if target \(r_j\) is in the transmission range of sensor \(s_i\). The coverage relationship of sensors and targets can be described by a bipartite graph \(G = (X, Y; E)\) where \(X\) is the set of all the sensors, \(Y\) is the set of all the targets, and \(xy \in E\) if and only if sensor \(x\) covers target \(y\). Here, we have \(|X| = n\) and \(|Y| = m\). Figure 1(a) shows an example with three sensors and three targets. In this example, target \(r_1\) is covered by either sensor \(s_1\) or \(s_2\). Let \(R_j\), \(j = 1...m\) be the set of all the sensors that cover target \(r_j\). As can be seen in Figure 1(a), \(R_1 = \{s_1, s_2\}\), \(R_2 = \{s_1, s_2, s_3\}\), and \(R_3 = \{s_2, s_3\}\).

We also assume that the maximum lifetime that each sensor \(s_i\) can be active is \(b_i\) unit time. Note that \(b_i\) can
be different for each sensor, which is more practical than most of the previous models, where all sensors have the same maximum energy. Without loss of generality, we assume that \( b_i \in \mathbb{N} \). The Maximum Lifetime Target Coverage problem (MTC) can be formally defined as follows:

**Definition 1: Maximum Lifetime Target Coverage Problem (MTC):** Given a wireless sensor network with \( n \) sensors \( s_i, i = 1...n \) and \( m \) targets \( r_j, j = 1...m \), find a family of ordered pairs \( (S_k, t_k), k = 1...p \), such that \( \sum_{k=1}^{p} t_k \) is maximized. For each sensor \( s_i \), \( s_i \) appears in \( S_1, ..., S_p \) with total time of at most \( b_i \) and each \( S_k \) completely covers all the targets.

For the example in Figure 1(a), the maximum lifetime of sensors \( s_1, s_2, s_3 \) are 2, 3, and 2 respectively. By the above definition, we can find this following family of ordered pairs \( \{(s_1, s_3), 2\}, \{(s_2), 3\} \). Thus the total network lifetime is 5 which is optimal.

**B. Domatic Number in Directed Graphs**

Given a directed graph \( G' = (V', E') \), let us first introduce some notations that are being used throughout the paper. For an arbitrary vertex \( v \in V' \), let \( N^-(v) \) be the set of its incoming neighbors, i.e., \( N^-(v) = \{u \mid uv \in E'\} \). The in-degree \( \delta^-(v) \) is the number of its in-coming neighbors, thus \( \delta^-(v) = |N^-(v)| \). Let \( N^-(v) = N^-(v) \cup \{v\} \). The maximum and minimum in-degree of a graph \( G' \) can be written as \( \Delta^- \) and \( \delta^- \) respectively where \( \Delta^- = \max_{v \in V'} \delta^-(v) \) and \( \delta^- = \min_{v \in V'} \delta^-(v) \). We are now ready to introduce the following definitions:

**Definition 2: Dominating Set:** A dominating set of a directed graph \( G' \) is a subset \( D \subseteq V' \) such that \( \forall v \in V' \), either \( v \in D \) or there exists a node \( u \in V' \) such that \( u \in D \) and \( uv \in E' \).

**Definition 3: Domatic Number:** The domatic number \( D(G') \) of a directed graph \( G' \) is the maximum number of disjoint dominating sets of graph \( G' \).

Informally, given a directed graph \( G' = (V', E') \), we need to partition the vertices into a number of disjoint sets such that each set is a dominating set. The domatic number problem asks us to find a maximum number of disjoint dominating sets. This problem is well-known to be NP-hard.
Theorem 1: Given a directed graph \( G' = (V', E') \), the domatic number \( D(G') \) is bounded as:
\[
\frac{(\delta^- + 1)(1 - o(1))}{\ln n'} \leq D(G') \leq (\delta^- + 1)
\]
where \( n' = |V'| \) and the term ”\( o(1) \)” goes to zero as \( n \) goes to \( \infty \).

Proof: For the upper bound, it is easy to see that \( D(G') \leq (\delta^- + 1) \). Let \( v \) be a vertex with \( \delta^-(v) = \delta^- \), then \( v \) is dominated at most by its \( \delta^- \) in-coming neighbors and itself. Thus \( D(G') \leq (\delta^- + 1) \).

For the lower bound, we use the similar approach as in [11]. For each vertex in the graph, randomly assign a color \( c \) in the range \([1, ..., (\delta^- + 1)/\ln(n' \ln n')]\). Let \( A_{v,c} \) be the event that there is no vertex of color \( c \) in \( N^-_c(v) \). If this event is true for some color \( c \) and an arbitrary vertex \( v \), it means that the color class \( C_c \) (\( C_c \) is the set of all vertices that have the same color \( c \)) does not form the dominating set. Let \( P[A_{v,c}] \) denote the probability of the event \( A_{v,c} \). We have:
\[
P[A_{v,c}] = \prod_{u \in N^-_c(v)} \left( 1 - \frac{\ln(n' \ln n')}{(\delta^- + 1)} \right)
\]
\[
= \left( 1 - \frac{\ln(n' \ln n')}{(\delta^- + 1)} \right)^{\delta^-(v)+1}
\]
\[
\leq e^{-\ln(n' \ln n')/\ln n'} = \frac{1}{n' \ln n'}
\]

Next, let \( B_v \) be the event that vertex \( v \) does not have all colors \( c \in [1, ..., (\delta^- + 1)/\ln(n' \ln n')] \) in \( N^-_c(v) \). We have:
\[
P[B_v] = P\left( \bigcup_{c=1}^{\delta^- + 1} A_{v,c} \right)
\]
\[
\leq \frac{(\delta^- + 1)}{\ln(n' \ln n')} \cdot \frac{1}{n' \ln n'}
\]

Then the expected number of bad event \( A_{v,c} \) is:
\[
E[A_{v,c}] \leq \frac{(\delta^- + 1)}{\ln(n' \ln n')} \cdot \frac{1}{\ln n'}
\]

And the expected number of colors that forms dominating sets is at least:
\[
\frac{(\delta^- + 1)}{\ln(n' \ln n')} \left( 1 - \frac{1}{\ln n'} \right) = \frac{(\delta^- + 1)}{\ln n'} \left( 1 - \frac{\ln \ln n' + 1}{\ln(n' \ln n')} \right)
\]

Hence \( D(G') \geq \frac{(\delta^- + 1)(1 - O(\log \log n'/\log n'))}{\ln n'} \).

When \( n' \to \infty \), \( D(G') \geq \frac{(\delta^- + 1)(1 - o(1))}{\ln n'} \).

Based on the above lower bound of the domatic number as well as the probability arguments, one can see that there exists an \( O(\log n') \)-approximation distributed algorithm for the maximum number of disjoint dominating sets. With this motivation, we reduce the maximum lifetime coverage problem to the maximum lifetime dominating sets problem, which is defined as:

Definition 4: Maximum Lifetime Dominating Sets Problem (MDS): Given the directed graph \( G' = (V', E') \) where each node \( v \in V' \) has a maximum lifetime \( b'_v \), find a family of ordered pairs \( (D_k, t_k) \), \( k = 1, ..., p' \), such that each set \( D_k \) is a dominating set and \( \sum_{k=1}^{p'} t_k \) is maximized.

IV. COVERAGE PROBLEM TRANSFORMATION AND SOLUTIONS

In this section, we show how to transform the maximum lifetime target coverage problem (MTC) to the maximum lifetime dominating sets problem (MDS). We also present the relationship between these two problems. Next, two algorithms are discussed to solve the problem.

A. Problem Transformation

Given a bipartite graph \( G = (X, Y; E) \) as described in section III-A, first we transform this graph \( G \) to the directed graph \( G' = (V', E') \) as follows:

Step 1: Make a directed clique \( R_i \) for each set \( R_j \). Recall that for a given graph \( G \), \( R_j \) is the set of all the sensors that cover target \( r_j, j = 1, ..., m \), i.e., \( R_j = \{ s_i \mid s_i \text{ covers } r_j \} \). The vertices in the clique \( R_j \) are the sensors in the set \( R_j \). For each clique, relabel each vertex to \( s_{ij} \). For example, Figure 1(c) shows three cliques for the graph \( G \) in Figure 1(a).

Step 2: Connect the above cliques as follows:

- Given two vertices \( s_{ij} \) and \( s_{i'j'} \), if \( i = i' \), add an directed edge from \( s_{ij} \) to \( s_{i'j'} \).
- Next, add a directed edge from \( s_{ij} \) to each of the neighbors of \( s_{i'j'} \) in the clique \( R_{i'} \).

For example, the directed graph \( G' \) in Figure 1(b) represents the graph \( G \) in Figure 1(a) after the transformation. At this step, we successfully transform the given bipartite graph \( G \) to the directed graph \( G' \) where \( V' \) consists only the sensors. Next, we need to assign the maximum lifetime of each vertex in graph \( G' \). Recall that each sensor \( s_i \) in the graph \( G' \) has the maximum lifetime \( b_i \). Let \( d_i \) be the degree of sensor \( s_i \) in \( G \).
Step 3: Assign the maximum lifetime $b_i' = b_i/d_i$ for each vertex $s_{ij}$. The natural number next to each vertex in Figure 1(b) indicates its maximum lifetime.

In Figure 1(b), we can find the following dominating sets and their associate active time: $\{(s_{11}, s_{33}), 1\}, \{(s_{21}), 1\}, \{(s_{2}, s_{12}), 1\}, \{(s_{22}), 1\}, \{(s_{23}), 1\}$). Note that each dominating set completely covers all the targets in $G$. In addition, these dominating sets correspond to these cover sets $\{(s_1, s_3), 1\}, \{(s_2), 1\}, \{(s_3, s_1), 1\}, \{(s_2), 1\}, \{(s_2), 1\}$, which is also the optimal solution for the original problem in the graph $G$.

This directed graph can be constructed distributively and locally. We assume that the communicating range of a sensor is at least twice its sensing range. With this assumption, any two sensors that cover the same target can communicate with each other [17]. The localized and distributed algorithm of this transformation is shown in Algorithm 1. Let $T_i$ denote a set of the targets that sensor $s_i$ can cover, i.e., $T_i = \{ j \mid s_i \text{ covers } r_j \}$. In this algorithm, each sensor $s_i$ sends its id number and the set $T_i$ to all its neighbors. Let $N(i)$ denote the neighbors of sensor $s_i$ in the network. Based on all these information from its neighbors, the clique $R'_i$ is created (from line 9 to line 15.) The rest of the algorithm is to connect these cliques.

Lemma 1: For any bipartite graph $G$, $n' = \bar{d} \cdot n$ where $\bar{d}$ is the average degree of all the sensors.

Proof: This is obvious. Recall that $n$ is the number of sensors in the bipartite graph $G = (X, Y; E)$, i.e., $n = |X|$, and $n' = |V'|$ where $V'$ is the set of vertices in the transformed directed graph $G'$. We have:

$$n' = \sum_{i=1}^{n} d_i = n \cdot \bar{d}$$

Note that $\bar{d}$ can represent the coverage probability of the network, that is, how many targets that one sensor can cover.

Lemma 2: A dominating set $D$ of $G'$ is a cover set of $G$.

Proof: Given a dominating set $D$ of $G'$, we need to prove that this set completely covers all the targets in $G$. By contradiction method, assume that there exists one target $r_j$ that is not covered. Let $R'_j$ be the clique of the set $R_j$. Then there exists one vertex $v \in R_j$ such that $v$ is not in $D$ and not dominated by $D$.

Algorithm 1 Transformation

1: Send $(i, T_i)$ to all neighbors
2: Receive $(k, T_k)$ from all $k \in N(i)$
3: for $j \in T_i$ do
4: Split $s_i$ to $s_{ij}$
5: $b'_{s_{ij}} = b_i/|T_i|$
6: $N^-(s_{ij}) = \emptyset$
7: $\delta^-(s_{ij}) = 0$
8: end for
9: for $k \in N(i)$ do
10: for $z \in T_k$ do
11: if $z == j$ then
12: $N^-(s_{ij}) = N^-(s_{ij}) \cup \{s_{kz}\}$
13: end if
14: end for
15: end for
16: Let $S'(i)$ denote the set of all $s_{ij}$ that are splitted from $s_i$
17: for each $s_{ij} \in S'(i)$ do
18: $N^-(s_{ij}) = N^-(s_{ij}) \cup (S'(i) - \{s_{ij}\})$
19: $N^-(s_{ij}) = N^-(s_{ij}) \cup N^- (S'(i) - \{s_{ij}\})$
20: end for
21: $\delta^-(s_{ij}) = |N^-(s_{ij})|$
22: Return $\{b'_{s_{ij}}, \delta^-(s_{ij}), N^-(s_{ij})\}$

Lemma 3: A cover set $S$ in $G$ can be transformed to a dominating set $D$ of $G'$.

Proof: Let sensor $s_i \in S$ be the sensor that covers the least number of targets. Without loss of generality, assume that $s_i$ covers a target $r_j$. Then select vertex $s_{ij}$ as the dominating node, i.e, $D = \{s_{ij}\}$. The rest of the vertices in clique $R'_j$ become the dominatee nodes. Also, the out-going neighbors of $s_{ij}$ become the dominatee nodes. Repeat this process for the other sensors in $S$, we have a dominating set $D$.

Theorem 2: A solution of the MDS problem is a solution of the MTC problem.

Proof: Assume that we can find $k$ dominating sets such that the total lifetime is maximum. From Lemma 2, each dominating set is also a cover set of $G$. Thus we have $k$ cover sets of $G$ so that each set completely covers all the targets. Also, by the construction of the graph $G'$, the maximum lifetime of each vertex in all $D$ will not exceed the maximum lifetime of each sensor in $G$. 

\[\]
**B. The Solutions**

In this section, we present two distributed algorithms and their theoretical analysis. As mentioned in section III, we can use the idea of the domatic number problem to solve the maximum lifetime dominating sets. In particular, we can find the maximum number of disjoint dominating sets of $G'$. Note that although the solution of the MDS problem of $G'$ is disjoint, the corresponding cover sets of $G$ are not necessary to be disjoint.

Recall that $b'_c$ is the maximum lifetime of each vertex $v$ in $G'$. For the sake of simplicity, let us first look at this special scenario. Assume that after transforming the non-uniform maximum lifetime of each sensor in $G$ to $b'_c$ in $G'$, all $v \in V'$ have the same maximum lifetime, i.e., $b'_v = b' \forall v' \in V'$. From the probability arguments in section III-B, one can see that if we randomly and independently assign a color in a certain range, we can obtain a certain number of dominating sets within a factor of $O(\log n)$. Similarly to the argument in section III-B, we choose $[1, \ldots, \frac{\delta^-(v)+1}{3\ln n}]$ as the color range for each vertex $v$ where $\delta^-(v) = \min_{u \in N^-(v)} \delta^-(v)$. After randomly choosing their own colors, all the vertices that have the same color, i.e., in the same color class, are active for $b'$ unit time. The active time can be decided locally as follows. Assume that the vertex $v$ chooses the color $c_v$, then vertex $v$ can be active from $b' \cdot (c_v - 1)$ to $b' \cdot c_v$. The detail of this algorithm is shown in Algorithm 2.

**Algorithm 2** Special Case

1. $\{b'_v, \delta^-(v), N^-(v)\} = \text{Transformation}(s_i)$
2. Send $\delta^-(v)$ to all the neighbors
3. Receive $\delta^-(u)$ from all $u \in N^-(u)$
4. $\delta^-(v) = \min_{u \in N^-(v)} \delta^-(v)$
5. Randomly choose a color $c_v \in [1, \ldots, \frac{\delta^-(v)+1}{3\ln n}]$
6. Sensor $v$ is active from $b'_v \cdot (c_v - 1)$ to $b'_v \cdot c_v$

**Lemma 4:** The above algorithm can form at least $\frac{(\delta^--1)}{3\ln n}$ dominating sets with a probability of $1 - o(n^{-1})$.

**Proof:** To prove this, we use the same approach as in section III-B. Let $C_v^c$ denote a set of all vertices that choose a color $c$ for $c \in [1, (\delta^-(v)+1)/(3\ln n)]$. Again, let $A_{v,c}$ be the event that there is no vertex of color $c$ in $N^-(v)$. Note that vertex $v$ can randomly and independently choose a color $c \in [1, \ldots, \frac{\delta^-(v)+1}{3\ln n}]$ (line 5 in Algorithm 2). If this event is true for some color $c$ and an arbitrary vertex $v$, it means that the color class $C_v$ does not form a dominating set. Let $P[A_{v,c}]$ denote the probability of the event $A_{v,c}$. We have:

$$P[A_{v,c}] = \prod_{u \in N^-(v)} \left(1 - \frac{3\ln n}{\delta^-(u)+1}\right) \leq \left(1 - \frac{3\ln n}{\delta^-(v)+1}\right)^{\delta^-(v)+1} \leq e^{-(\delta^-(v)+1)(3\ln n)/\delta^-(v)+1} < e^{-3\ln n} = \frac{1}{n^3}$$

Next, let $B_v$ be the event that vertex $v$ does not have all colors $c \in [1, \ldots, (\delta^-(v)+1)/(3\ln n)]$ in $N^-(v)$. We have:

$$P[B_v] = P \left[ \bigcup_{c=1}^{\delta^-(v)+1} A_{v,c} \right] \leq \frac{\delta^-(v)+1}{3n^3\ln n}$$

Thus the probability $P$ of an event that there exists at least one vertex $v \in V'$ such that $v$ does not have all colors $c \in [1, \ldots, (\delta^--1)/(3\ln n)]$ is bounded by:

$$P \leq P \left[ \bigcup_{v \in V'} B_v \right] \leq \sum_{v \in V'} \frac{\delta^-(v)+1}{3n^3\ln n} \leq \frac{\delta^-(v)+1}{3n^3\ln n} \cdot \frac{d}{n^2}$$

The expected number of colors that forms dominating sets is at least:

$$\frac{\delta^-(v)+1}{3\ln n} \left(1 - \frac{d}{n^2}\right) \geq \frac{\delta^-+1}{3\ln n} \left(1 - o(1)\right)$$

Hence Algorithm 2 can form at least $\frac{\delta^-+1}{3\ln n}$ dominating sets with a probability of $1 - o(n^{-1})$.

**Theorem 3:** The approximation ratio of Algorithm 2 is within a factor of $O(\log n)$.

**Proof:** Let $L$ be the lifetime of all the dominating sets obtained from Algorithm 2 and let $L^*$ be the optimal solution of the coverage problem. We first prove the upper bound of $L^*$. From Theorem 2, $L^*$ is also the optimal solution of the maximum lifetime dominating sets of $G'$. From the upper bound of the domatic number in Theorem 1, it is easy to see that $L^* \leq b' \cdot (\delta^-+1)$. Next, we prove the lower bound of $L$. From Lemma 4, we have:

$$L \geq b' \cdot \frac{\delta^-+1}{3\ln n}$$
Thus the approximation ratio is within a factor of $O(\log n)$.

We are now ready to consider a more general case which is that all $b'_v$ of all $v \in V'$ are not the same. We can use Algorithm 2 of the above special case for this general problem case. Specially, each vertex $v$ randomly chooses a color $c$ in the range $[1, ..., \delta^-(v) + 1/(3 \ln n)]$. Each color class $C_c$ is active for $b'_v = \min_{u \in V'} b'_u$ unit time. However, this idea does not take an advantage of the "remaining" energy of each sensor. A better way to solve this problem is to divide $b'_v$ to $b'_v$ time-unit slots [12]. At each time-unit slot, a vertex can randomly and independently choose its color. In other words, each vertex can choose $b'_v$ many colors in a certain range. After choosing its colors, all these color classes are activated successively. Each color class stays up for one unit time. The details of this algorithm is shown in Algorithm 3.

**Algorithm 3 General Algorithm**

1. $\{b'_v, \delta^-(v), N^-(v)\} = \text{Transformation}(s_i)$
2. Send $b'_v$ to all neighbors
3. Receive $b'_u$ from all $u \in N^-(v)$
4. $b''_v = \max_{u \in N^-(v)} b'_u$
5. $\tau_v = \sum_{u \in N^-(v)} b'_u$
6. Send $(b''_v, \tau_v)$ to the all neighbors
7. Receive $(b''_u, \tau_u)$ from all $u \in N^-(v)$
8. $b_v = \max_{u \in N^-(v)} b'_u$
9. $\hat{\tau}_v = \min_{u \in N^-(v)} \tau_u$
10. $C = \emptyset$ /* To hold $b'_v$ colors */
11. for $j$ from 0 to $b'_v$ do
12. Randomly choose a color $c_j \in [0, ..., (3 \ln (b''_v n))]
13. $C = C \cup \{c_j\}$
14. end for
15. At each time slot $t$, sensor $v$ is active if $c_t \in C$

Similar to the special case, we prove that Algorithm 3 can obtain a solution with an approximation ratio of $O(\log n)$.

**Lemma 5:** Algorithm 3 can form at least $(\frac{\tau}{3 \ln (b''_v n)} + 1)$ dominating sets with a probability of $1 - o(n^{-1})$ where $\tau = \min_{u \in V'} \sum_{v \in N^-(u)} b'_v$ and $b_{\max} = \max_{v \in V'} b'_v$.

**Proof:** Let $C_c$ denote a set of all vertices that choose a color $c$ for $c \in [0, ..., (3 \ln (b''_v n))]$. Again, let $A_{v,c}$ be the event that there is no vertex of color $c$ in $N^-(v)$. If this event is true for some color $c$ and an arbitrary vertex $v$, it means that the color class $C_c$ does not form a dominating set. Let $P[A_{v,c}]$ denote the probability of the event $A_{v,c}$. We have:

$$P[A_{v,c}] = \prod_{u \in N^-(v)} \prod_{i=1}^{b'_u} \left(1 - \frac{3 \ln (b''_v n)}{\tau_v}\right)$$

$$\leq \prod_{u \in N^-(v)} \left(1 - \frac{3 \ln (b''_v n)}{\tau_v}\right)^{b'_u}$$

$$= \left(1 - \frac{3 \ln (b''_v n)}{\tau_v}\right)^{\sum_{u \in N^-(v)} b'_u}$$

$$= \left(1 - \frac{3 \ln (b''_v n)}{\sum_{u \in N^-(v)} b'_u}\right)^{\sum_{u \in N^-(v)} b'_u}$$

$$\leq e^{-3\ln (b''_v n)} = \frac{1}{(b''_v n)^3}$$

Next, let $B_v$ again be the event that vertex $v$ does not have all colors $c \in [0, ..., (3 \ln (b''_v n))]$ in $N^-(v)$. We have:

$$P[B_v] = P \left[ \bigcup_{c=0}^{b''_v n} A_{v,c} \right]$$

$$\leq \frac{\hat{\tau}_v}{3 \ln (b''_v n)} \cdot \frac{1}{(b''_v n)^3}$$

$$\leq \frac{\delta^-(v) + 1}{3 \ln (b''_v n)} \cdot \frac{n(b''_v n)^2}{(b''_v n)^3}$$

Thus the probability $P$ of an event that there exists at least one vertex $v \in V'$ such that $v$ does not have all colors $c \in [0, ..., (3 \ln (b''_v n))]$ is bounded by:

$$P \leq P \left[ \bigcup_{v \in V'} B_v \right] \leq \frac{\delta^-(v) + 1}{3 \ln (b''_v n)} \cdot \frac{\hat{\tau}_v}{(b''_v n)^2}$$

Hence Algorithm 3 can form at least $\left(\frac{\tau}{3 \ln (b''_v n)} + 1\right)$ dominating sets with a high probability.

**Theorem 4:** The approximation ratio of Algorithm 3 is within a factor of $O(\log n)$.

**Proof:** Let $L$ be the lifetime of all the dominating sets obtained from Algorithm 2 and let $L^*$ be the optimal solution of the coverage problem. We first prove the upper bound of $L^*$. It is easy to see that $L^* \leq \tau$. Next, we prove the lower bound of $L$. From Lemma 5, we have:

$$L \geq \frac{\tau}{3 \ln (b_{\max} n)} + 1 = \frac{L^*}{3 \ln (b_{\max} n)} + 1$$

Thus the approximation ratio is within a factor of $O(\log (b_{\max} n))$ which can be reduced to $O(\log n)$.
V. Simulation Results

In the previous section, we evaluate our algorithms through theoretical analysis. In this section, we conducted some simulation experiments to measure the performance (in terms of network lifetime) of Algorithm 3. We study three network parameters that may affect network lifetime: 1) \( m \), which is the number of targets that a sensor network need to monitor. 2) \( d \), which is the percentage of the total targets that a sensor can monitor. 3) \( \text{gap} \), which is the maximum variance of the maximum lifetime \( b_i \) of all the sensors, i.e., \( \text{gap} = b_{\text{max}} - b_{\text{min}} \) where \( b_{\text{max}} = \max_{s_i \in X} b_i \) and \( b_{\text{min}} = \min_{s_i \in X} b_i \). Note that sometimes we refer \( b_i \) as the sensor’s energy.

For each simulation, we conducted the experiments for the networks of sizes from 10 to 100 sensor nodes. The coverage relationship was randomly constructed according to \( m \) and \( d \). The maximum lifetime \( b_i \) of each sensor node was also randomly assigned considering \( \text{gap} \). For each network size, 100 network instances were investigated and the results averaged.

A. Effects of the Number of Targets

The purpose of this simulation is to study how \( m \) affects network lifetime. \( m \) indicates the number of targets that a sensor network needs to monitor, and it is the primary factor for people to decide how many sensors to deploy to fully cover all the targets. Three cases were studied for this simulation, where \( m \) was set to 2, 5 and 15 respectively. In this simulation, \( d \) was set to 50% which means each sensor node monitors 50% of the total targets, and \( \text{gap} \) was set to 5 unit energy. We compare network lifetime of the three cases. The results are shown in Figure 2.

Figure 2 compares the network lifetime of the three cases where \( m \) is set to 2, 5 and 15 respectively. It is shown that for all the networks, the networks where \( m = 2 \) have the longest network lifetime. On average, the network lifetime of the networks where \( m = 5 \) are 61.17% of that of the ones where \( m = 2 \), and the network lifetimes of the networks where \( m = 15 \) are 53.57% of that of the ones where \( m = 2 \). This simulation complies with an obvious fact that the more the targets needed to be monitored, the less the network lifetime. The network lifetime increases as the network size becomes larger, since more possible dominating sets can be constructed to extend network lifetime.

B. Effects of Sensing Range

The purpose of this simulation is to evaluate how \( d \) affects network lifetime. Recall that \( d \) is the percentage of the total targets that a sensor can monitor. Indeed, \( d \) reflects the sensing range of each sensor. In particular, the larger the sensing range, the more the targets a sensor can cover. In parallel, \( d \) increases as sensing range increases. To increase the number of targets that a sensor can monitor is a way to help with increasing the probability of target coverage. Three cases were studied for this simulation, where \( d \) was set to 80%, 50% and 20% respectively. In this simulation, 10 targets were deployed, and \( \text{gap} \) was set to 5 unit energy. We compared network lifetime of the three cases. The results are shown in Figure 3.

From Figure 3, it can be seen that the larger the \( d \),
the longer network lifetime. For all the networks, the networks where \( d = 80\% \) have the longest network lifetime. On average, network lifetime of the networks where \( d = 50\% \) are 68.36\% of that of the ones where \( d = 80\% \), and network lifetime of the networks where \( d = 20\% \) even drops to 36.92\% of that of the ones where \( d = 80\% \). In other words, increasing the sensing range of each sensors can extend network lifetime further.

C. Effects of Energy Differences

In the above simulations, \( \text{gap} \) was always set to 5 unit energy. But in some applications, sensor nodes may either have same energy or the energies of each sensor node shows a great difference. Therefore, in this simulation, we conducted the experiments with different values of \( \text{gap} \), where \( \text{gap} \) was set to 0, 5, and 20 respectively. For all the network instances, 5 targets were deployed and each sensor node covered at least 2 targets. The results are shown in Figure 4.

As shown in Figure 4, the networks with uniform energy at each sensor node have the smallest network lifetime. The networks where \( \text{gap} = 20 \) have the longest network lifetime. On average, network lifetime of the networks where \( \text{gap} \) is set to 5 are 69.82\% of that of the ones where \( \text{gap} = 20 \), and network lifetime of the networks where \( \text{gap} = 0 \) (uniform case) is 57.95\% of that of the ones where \( \text{gap} = 20 \). This indicates that our algorithm can efficiently make use of the energy at each sensor node, since sensors with remaining energy that have already served in some dominating sets may still be selected to be turned on, until this sensor completely runs out of energy.

VI. CONCLUSIONS

In this paper, we have studied the coverage problem with an objective of maximizing network lifetime, which is a critical issue in wireless sensor networks. We first reduced this problem to a related domatic problem, that is to find the maximum lifetime dominating sets in directed graphs. We also presented various distributed algorithms whose approximation ratio is within a factor of \( O(\log n) \) where \( n \) is the number of sensors. These distributed algorithms are totally localized. In particular, we transformed the coverage relationship between sensors and targets to a directed graph whose vertices consist of only sensors (no targets involved). The main property of this directed graph \( G' \) is that each dominating set of \( G' \) is a solution of the coverage problem. In other words, the dominating set completely cover all the targets. Using the probability argument and assigning color scheme, the distributed algorithms return a set of dominating sets that are activated successively to prolong network lifetime. Through the simulations, we also studied which factors may affect network lifetime. The simulations showed that the larger the sensing range, the longer network lifetime. It also indicated that our algorithm can efficiently make use of the energy at each sensor node.

REFERENCES


[17] H. Zhang and J. C. Hou, Maintaining Sensing Coverage and Connectivity in Large Sensor Networks