COLLISIONS IN TWO DIMENSIONS

Introduction: In this lab, we will analyze two separate collisions for “frictionless” pucks on an air table; in one of our two collisions, we will arrange for the two pucks to stick together after the collision. Text Reference: Wolfson 10.3, 11.2-4, 12.3-4.

SPECIAL CONCERNS FOR COLLISIONS IN TWO DIMENSIONS:
1. In this lab, the equipment for data-taking and analysis will be shared between two three-person teams. Teamwork and cooperation are required if you are to finish your data collection and analysis during the allotted lab time.

Procedure: You will be provided with pucks and air tables with which you will create two collisions. Each collision will begin with one puck in motion, the “incident” puck, and one puck at rest near the center of the table, the “target” puck. You will videotape each collision for detailed frame-by-frame analysis.

Before beginning the formal experiment, you will need to check that the air table is level. A puck placed in the center of the table should remain relatively still; it should not experience a constant acceleration in any given direction (thereby indicating that such a direction is downslope). Each table is mounted on three leveling screws by means of which the slope can be adjusted; however, all tables should already be level (or nearly so), so any adjustments that you make should be SMALL. In addition, your analysis will be considerably simplified if the two pucks have the same mass; use the lab balance to check this. If the pucks do not have the same mass, add some small masses to the center of the lighter puck until the masses are equalized. Small washers are provided for this purpose.

Before attempting to make a videotape, practice a few collisions. Your analysis will be simplified if the target puck is truly at rest before the collision; practice minimizing any small drift velocity of that puck. Head-on collisions are not allowed. Your analysis will also be simplified if the incident puck is not rotating before the collision. Rubber bands at the corners of the table can be used to launch the incident puck. The speed of the incident puck should be moderate; when the speed is too slow the pucks can drift due to variations in air pressure, but when the speed is too large the value of frame-by-frame analysis is diminished.

For the formal experiment, you will need to make two videotapes for frame-by-frame analysis. Your group will share an air table and a television monitor with another group; your two groups will need to take turns both in making the videos and in doing the analysis on the monitors. In one of the two collisions you will need to arrange that the two pucks stick together after the collision; this can be done by wrapping the two pucks with velcro
strip, or by wrapping the two pucks with masking tape, sticky side out. If you choose to use the velcro strips, be careful that the edges don’t drag, creating friction.

To analyze each of your collisions, you will make a motion diagram for the moving pucks both before and after the collision during a frame-by-frame playback on the television monitor. Tape a piece of acetate over the monitor on which to record your motion diagram. For the case in which the pucks do not stick together, you will have to track each puck both before and after the collision; for the case in which the pucks stick together, after the collision you will have to track the center-of-mass of the spinning two-puck system. For each frame, in addition to the positions of the pucks (or two-puck system), you will also need to record the orientations of the pucks (or two-puck system); the pucks are marked with radial lines and an off-center dot for this purpose. The standard video frame rate is 30 frames per second; you may choose to record positions at every frame, or at every other frame, depending on the speeds and rotation rates of your pucks. Finally, you will also need to indicate the radius of the pucks somewhere on the acetate; we will do our calculations in “acetate dimensions” which are not the same as real dimensions, but instead different by some scale factor.

Once you have acetates for each of your two collisions, you are ready to proceed with the analysis. These are two-dimensional collisions, so you will need to choose an x-y coordinate system in which to work. Choosing the x-axis in the direction of the incident puck is convenient; after the collision, the velocity directions of the pucks can be measured with respect to this axis. The collision in which the two pucks do not stick together should certainly be the more elastic of the two collisions, although we do not expect that it will be perfectly elastic; nevertheless, we will refer to this as the “elastic” collision. When the two objects stick together after the collision, the collision is classified as perfectly inelastic.

For the elastic collision, make a table with columns of speed, direction, x velocity, y velocity, rotation rate $\omega$ (in rad/s), speed$^2$, and $0.5R^2\omega^2$, where $R$ is the puck radius in acetate dimensions; the table should have four rows, for puck 1 before, puck 2 before, puck 1 after, and puck 2 after, where puck 2 is the target puck. If you write the table by hand, then you will be able to write in your calculated errors for each entry. You may estimate errors in direction. Errors in speed and rotation rate should come from repeated measurements on the acetate; measure over five intervals to get an average value, and measure five individual intervals to get a sample of five values (be sure to show these data on your data sheets). Since the masses of our two pucks are equal they will divide out of the conservation-of-momentum equations (show this in your written analysis). We will compute the fractional loss in kinetic energy, so again the equal masses will divide out (show this in your written analysis). Using the entries in your table, check conservation
of momentum in both the $x$ and $y$ directions. For the $y$ direction, if you have made the suggested choice of axis, the initial $y$ momentum will be zero; the final $y$ momentum (per unit mass) should also be zero to within the uncertainty of your measurements. What fraction of the initial kinetic energy has been “lost” in this collision? Use the entries in your table to answer this question. The puck speed squared is conveniently twice the translational kinetic energy per unit mass, or $2K_{CM}/M$ where $M$ is the puck mass. Don’t forget to include rotational kinetic energies in your calculation. Take the rotational inertia of your puck to be $0.5MR^2$ (model the pucks as uniform cylinders); the entry $0.5R^2\omega^2$ in the data table is thus twice the rotational kinetic energy per unit mass ($2K_{rot}/M$). The total kinetic energy of a puck is its center-of-mass kinetic energy plus its rotational kinetic energy (about the center of mass).

For the perfectly inelastic collision, make a table with columns of speed, direction, $x$ velocity, $y$ velocity, rotation rate $\omega$ (in rad/s), $2K_{CM}/M$, and $2K_{rot}/M$; the table should have three rows, for puck 1 before, puck 2 before, and the combined pucks after, where puck 2 is the target puck. In checking conservation of momentum for both the $x$ and $y$ directions, remember that the combined system of two pucks has twice the mass of a single puck. This doubling of mass also affects your entries in the final two columns of your table. For the individual pucks, $2K_{CM}/M$ will be the speed squared as in the elastic collision; but for the combined pucks, $2K_{CM}/M = 2(speed)^2$. Similarly, for the individual pucks, $2K_{rot}/M$ will be $0.5R^2\omega^2$ as in the elastic collision; however, the rotational inertia of the combined pucks, about their center-of-mass, is $3MR^2$, so $2K_{rot}/M = 3R^2\omega^2$. What fraction of the initial kinetic energy has been “lost” in this collision? How does this collision compare with the “elastic” collision?

Since we have done our calculations in acetate dimensions, our actual numbers for momentum and velocity, or kinetic energy, would be off by a scale factor; for momentum and velocity the scale factor is simply the ratio of puck radius on the acetate to actual puck radius, while for kinetic energy the scale factor is this same ratio squared. If you wish, you may measure the masses and radii of the pucks, compute this scale factor, and then compute the actual momenta and kinetic energies involved in our experiment; however, this is not required.
Prelab Quiz PHY122
Two-Dimensional Collisions

Name
Section Time and Day

1. Given that a puck’s velocity is speed $v$ at an angle $\theta$ (measured in radians) with the $x$ axis, we know that the puck’s $x$ velocity is $v\cos(\theta)$. Given that the error in $v$ is $\sigma_v$ and the error in $\theta$ is $\sigma_\theta$, what is the resulting error in the puck’s $x$ velocity?

2. Given that a puck’s velocity is speed $v$ at an angle $\theta$ (measured in radians) with the $x$ axis, we know that the puck’s $y$ velocity is $v\sin(\theta)$. Given that the error in $v$ is $\sigma_v$ and the error in $\theta$ is $\sigma_\theta$, what is the resulting error in the puck’s $y$ velocity?

3. If we model a puck as a uniform cylinder, the rotational inertia of a puck is $0.5MR^2$, where $M$ is the puck mass and $R$ is the puck radius. Consider two pucks stuck together and spinning about their common center of mass. Use the parallel-axis theorem to find the rotational inertia of the two-puck system about its center-of-mass.