9.1 Linear Momentum

In Chapter 8, we studied situations that are difficult to analyze with Newton's laws. We were able to solve problems involving these situations by identifying a system and applying a conservation principle, conservation of energy. Let us consider another situation and see if we can solve it with the models we have developed so far:

> A 60-kg archer stands at rest on frictionless ice and fires a 0.50-kg arrow horizontally at 50 m/s. With what speed does the archer move across the ice after firing the arrow?

From Newton's third law, we know that the force exerted on the arrow by the bow is matched by a force in the opposite direction on the bow by the arrow. This force causes the bow, with the archer attached, to slide backward on the ice with the speed requested in the problem. We cannot determine this speed using motion models such as the particle under constant acceleration because we don't have any information about the acceleration of the archer. We cannot use force models such as the particle under a net force because we don't know anything about the size of the forces in this situation. Energy models are of no help because we know nothing about the work done in pulling the bowstring back or the elastic potential energy in the system relating to the taut bowstring.

Despite our inability to solve the archer problem using models learned so far, this problem is very simple to solve if we introduce a new quantity that describes motion, *linear momentum*. To generate this new quantity, consider an isolated system of two particles (Fig. 9.1) with masses m_1 and m_2 moving with velocities $\vec{\mathbf{v}}_1$ and $\vec{\mathbf{v}}_2$ at an instant of time. Because the system is isolated, the only force on one particle is that by the other particle. If a force by particle 1 (for example, a gravitational force) acts on particle 2, there must be a second force – equal in magnitude but opposite in direction – exerted on particle 1 by particle 2. That is, the forces on the particles form a Newton's third law pair, and $\vec{\mathbf{F}}_{12} = -\vec{\mathbf{F}}_{21}$. We can express this

$$\vec{\mathbf{F}}_{12} + \vec{\mathbf{F}}_{21} = 0$$

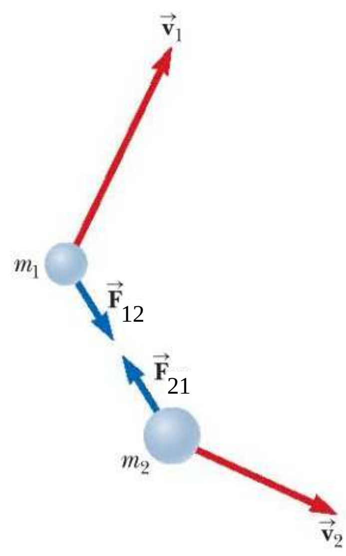

Figure 9.1 Two particles interact with each other. According to Newton's third law, we must have $\vec{\mathbf{F}}_{12} = -\vec{\mathbf{F}}_{21}$.

Return to the book's text now beginning with "Let us further analyze..."