

In this exercise, we will use Excel (or something similar) to compute the motion of a mass (treated as a particle) subject to a net force that is not constant. If we are able to compute this net force (based either on time or on the position of the particle), then the 2nd Law will provide the particle's acceleration. We can then, in turn, calculate the change in the particle's velocity and position during any selected small time interval.

As a first example, assume that the mass is moving in one dimension only, and that the net force on the mass is always directed towards the origin, with a magnitude proportional to distance from the origin, *i.e.* $\Sigma F_x = -Kx$, where K is a proportionality constant in units of N/m. For any time t , the 2nd Law then gives $a_x(t) = -(K/m)x(t)$. We will begin with the mass located somewhere away from the origin and having no speed, and then use Excel to compute and graph the subsequent motion of the mass.

1. Open a blank spreadsheet. The parameters for our calculation will be Δt , K , and m ; enter these parameter names in cells H1-3 respectively (*i.e.* enter deltaT= in cell H1, etc). In cells I1-3, enter our selected values for these parameters. Begin with $\Delta t = 0.1$ s, $K = 4$ N/m, and $m = 1.0$ kg (enter only the numbers, not the units). You will have the freedom to change these parameters in the completed spreadsheet.
2. Make the column for time. Enter the label t in cell A1. In cell A2, enter a zero (our initial value of time). In cell A3, enter a formula which will be applied to all subsequent times, namely that the next time value is equal to the previous time value plus Δt ; so enter $=A2+\$I\1 as the formula for cell A3 (the dollar signs indicate that the cell I1 is the fixed location of the value of Δt), then click in cell A3 and drag the dot at the lower right-hand corner of the cell down to cell A100.
3. Now make the columns for x , v_x , and a_x . Enter the appropriate labels in cells B1, C1, and D1, respectively, then enter the initial conditions in the second row cells for x and v_x , namely that at time zero the mass is at location $x = 1$ m with zero x velocity. For a_x , use a formula, namely $-(K/m)x$ (in the formula bar for cell D2, enter $=(-1)*(\$I\$2/\$I\$3)*B2$, and then apply to cells D2-D100).
4. We next need formulas for x and v_x for each subsequent time. For infinitesimal Δt 's, the correct formula for x would be $x(t + \Delta t) = x(t) + v_x(t) * \Delta t$. For larger values of Δt (such as 0.1 s), our calculation is improved by using an estimate of the average velocity during the time interval t to $t + \Delta t$, namely $v_{x,avg} = v_x(t) + a_x(t) * 0.5 * \Delta t$; enter this formula in cell C3 (*i.e.* $=C2+D2*0.5*\$I\1). In cell C3, we now have an estimated average velocity for the interval from $t=0$ to $t=0.1$ s instead of the computed x -velocity at time $t=0.1$ s. Alternatively, we can think of this entry as the estimated x -velocity at time $t=0.05$ s. Therefore, for the remainder of our velocity column, we will use $v_x(t + 0.5\Delta t) = v_x(t - 0.5\Delta t) + a_x(t) * \Delta t$

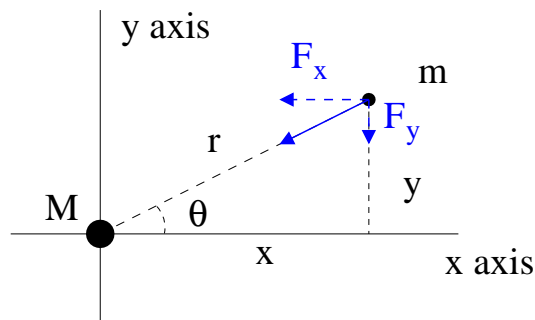
(notice that, in this formula, we are using $a_x(t)$ as an estimate of the average value of a_x in the interval from $t - 0.5\Delta t$ to $t + 0.5\Delta t$). Enter this formula in cell C4 (*i.e.* =C3+D3*\$I\$1) and apply it to cells C4-C100. Finally, we can enter a formula for $x(t)$; in cell B3, enter =B2+C3*\$I\$1, and apply to cells B3-B100.

5. Check your work. If everything is correct, you should have the following results in row 100: $t=9.8$ s, $x=0.708618$ m, $v_x=-1.26239$ m/s, and $a_x=-2.83447$ m/s².
6. Graph your results. Choose INSERT Chart > Scatter with Smooth Lines and Markers, then right click on the chart and Select Data. Remove any data automatically placed on the chart by Excel, then Add data. Click on cell B1 for the Series name, then select cells A2-A100 (time) for Series X values, and cells B2-B100 (position) for Series Y values (for the Y values, you must first delete anything automatically placed there by Excel). Now add a second curve to your graph. Right click to Select Data > Add data then click on cell D1 for the Series name, once again select cells A2-A100 for Series X values, and this time select cells D2-D100 (a_x) for Series Y values. Enable Legend under Chart Elements to label the curves.
7. Your graph should show a simple oscillatory motion for the mass, with slightly more than three cycles in the 9.8-s duration of our calculation. You should observe that a_x is always in the opposite direction of x , as is required by $a_x = -(K/m)x$.
8. You can now try out other possible values for the parameters in our calculation. For example, it is easy to verify that the amplitude of the acceleration depends linearly on the value of K ; but how does the frequency of the oscillatory motion depend on the value of K ? Hint: try at least $K=1$ N/m, 4 N/m, and 16 N/m; and, for each value, use the graph to count the number of cycles which occur in about 6.4 s.

OMIT 9. If you have time, in Column F, add a cosine function of time, with $\sqrt{K/m}$ as the
in conversion factor between time in seconds and angle in radians. Put an appropriate
Fall Class label in cell F1, and put an appropriate formula in cell F2, and then apply that formula to cells F2-F100. If this cosine function is this added to the graph, then you should find that there is little difference between the graph of our computed $x(t)$ and the graph of the cosine function, at least for several cycles of the motion.

Congratulations. You now have a spreadsheet that can be used to determine the motion of particles moving in 1D and subject to varying forces (as, for example, in the HW2 problem entitled “1D Motion with Variable Acceleration”). You can add extra parameters if needed; for example, you could add a parameter for a non-zero initial x -velocity. In our next example, we will expand the spreadsheet to handle 2D motions.

As a second example of solving for the motion of a particle by numerical means, we will solve for the orbital motion of a small mass m around a much larger mass M (taken to be stationary, and at the origin of our coordinate system). This is now a two-dimensional problem, so we will work with both x and y versions of the Second Law. Only gravity is acting, so the x -component of the net force acting on the smaller mass is $\Sigma F_x = -(GMm/r^2)(x/r)$, where (x/r) is the proper expression for the value of $\cos(\theta)$, as defined by the figure to the right. Similarly, the y -component of the net force is $\Sigma F_y = -(GMm/r^2)(y/r)$, and thus the x - and y -components of the Second Law yield $a_x = -(GM/r^3)x$ and $a_y = -(GM/r^3)y$. Begin by converting the value of GM into units more convenient for motions in our solar system, namely years and astronomical units (take one year to be 365.25 days and 1.0 au to be 1.496×10^{11} m).



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1. Show how to find the value of GM in units of years and au. The mass of our Sun (our M) is 1.989×10^{30} kg, and the value of G is 6.674×10^{-11} N·m²/kg².

Fill in
later on

$GM =$

$$= 39.485 \frac{\text{au}^3}{\text{y}^2}$$

2. The parameters for our calculation will be the value of Δt , the value of GM , and the initial x -location and y -velocity of our orbiting mass (we will always use $y_0 = 0$ and $v_{x0} = 0$). Enter the labels for these parameters in cells N1-N4, and then their values in cell O1-O4; begin with $\Delta t = 0.01$ y, $x_0 = 1.0$ au, and $v_{y0} = 2\pi$ au/y; enter the values of GM and v_{y0} to five significant figures.
2. Make the column for time. Follow the instructions from our previous spreadsheet (except here the fixed location of the value of Δt is cell O1). In this case, apply the appropriate formula to cells A3-A101.
3. Make the columns for x , v_x , a_x , y , v_y , a_y , and finally a column for r and one for $1/r^3$. Enter the appropriate labels in cells B1-I1, then enter the initial conditions for v_x and for y in cells C2 and E2, namely that at time zero the mass has zero x velocity and has y -position zero. Our value for x at time zero is the parameter in cell O3; enter this with a formula in cell B2. Our value for v_y at time zero is the parameter in cell O4; enter this with a formula in cell F2. For a_x , use a formula, namely $a_x(t) = -(GM/r^3)x(t)$ (in the formula bar for cell D2, enter $=(-1)*\$O\$2*I2*B2$, and then apply to cells D2-D101). For a_y , use a formula for $a_y(t) = -(GM/r^3)y(t)$, and apply to the entire column. For r use a formula for $r(t) = \sqrt{x^2(t) + y^2(t)}$ and apply to the entire column, and similarly for $1/r^3$.

4. Now we need formulas for x and v_x (and then y and v_y) for each subsequent time. As we did in the 1D spreadsheet, for our velocity values we will use an estimate of the average velocity during each time interval. For cell C3, we use the formula $v_{x,avg} = v_x(t) + a_x(t) * 0.5 * \Delta t$; (*i.e.* =C2+D2*0.5*\$O\$1). For cell C4, use $v_x(t + 0.5\Delta t) = v_x(t - 0.5\Delta t) + a_x(t) * \Delta t$ (*i.e.* =C3+D3*\$O\$1) and apply it to cells C4-C101. Then repeat the process for v_y using the appropriate cells. Finally, we can enter the formulas for x and for y ; for x , in cell B3 we use =B2+C3*\$O\$1 and then apply the formula to all of column B. For y , apply the same idea.
5. Check your work. If you have entered everything correctly, you should have the following results in row 101: $t=0.99$ y, $x=0.997617$ au, $v_x=0.630321$ au/y, $a_x=-39.3907$ au/y², $y=-0.06902$ au, $v_y=6.2546$ au/y, $a_y=2.725416$ au/y², etc.
6. Now make a motion diagram of your results. Choose INSERT Chart > Scatter, then right click on the chart and Select Data. Remove any data automatically placed on the chart by Excel, then Add data. For the Series name, type =“motion diagram”, then select cells B2-B101 (x position) for Series X values, and cells E2-E101 (y position) for Series Y values (for the Y values, you must first delete anything automatically placed there by Excel). Your result should be a motion diagram of a circular orbit; you will probably need to manually adjust the dimensions of the Excel chart to insure that the x and y axes have the same scaling.
7. The actual orbit of Earth is not quite circular; instead the aphelion distance is 1.0167 au, and the speed at aphelion is 6.1796 au/y. Change the parameters for x_0 and v_{y0} so that the calculation produces Earth’s slightly elliptical orbit.
8. The table below contains aphelion distances and speeds for three Earth-crossing asteroids. Use your spreadsheet to display the motion diagrams for the orbits of these asteroids and enter the calculated perihelion distances. Adjust the dimensions of your chart to maintain equal scaling on the x and y axes, and adjust your value of Δt so that approximately one full orbit is shown. You might like to compare your calculated perihelion distances with the accepted distances as posted on Wikipedia.

Asteroid	Aphelion Distance (au)	Aphelion Speed (au/y)	Perihelion Distance (au)
3362 Khufu	1.4531	3.8003	
25143 Itokawa	1.695	4.0948	
3753 Cruithne	1.5114	3.5601	

To make your spreadsheet more flexible, add parameters for the initial values of y position and x velocity. Then, given the location and velocity of launch, and with suitable units for the relevant value of GM , you can calculate the orbit of any $m \ll M$.