Definitions for Current and Resistance

Here is a reasonable definition of current for beginning students:

DEF: The (AVERAGE) CURRENT ($I_{(av)}$ or i) through any area A is

In Symbols: $$I_{(av)} \equiv \frac{\Delta Q}{\Delta t}$$ in Units of C/s $\equiv$ Amperes (A)

where ΔQ is the absolute value of the net charge that flows through area A in time Δt. The "average" is usually understood. For this definition, current is a positive-only scalar; it is most useful for steady currents (i.e. currents that don't change with time). If currents can change with time (as in AC circuits), we need another definition:

DEF: The INSTANTANEOUS CURRENT ($I(t)$) is the limit as Δt goes to zero of the average current.

In Symbols: $$I(t) \equiv \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$ in Units of C/s $\equiv$ Amperes (A)

But this is not quite right, because in this definition Q must be a signed scalar, so ΔQ cannot be an "absolute value" (we are doing calculus). So $I(t)$ is a signed scalar. To understand the source of the sign, you need to see the real definition of current, which starts with CURRENT DENSITY.

DEF: The CURRENT DENSITY ($\vec{J}(\vec{r})$) at location $\vec{r}$ is the time rate of charge flow per unit area through an infinitesimal area (at right angles to the flow) located at $\vec{r}$; the direction of current density is the direction of positive charge flow. $\vec{J}(\vec{r})$ has units of A/m^2. Current density is a VECTOR.

Now we can define current.

DEF: The CURRENT (I_A) THROUGH AREA A is:

In Symbols: $$I_A \equiv \int_A \vec{J}(\vec{r}) \cdot d\vec{A}$$ in Units of C/s $\equiv$ Amperes (A)

i.e., the current is the FLUX of the current density. Thus, current is a signed scalar, but the only significance of the sign is OUR CHOICE of direction for the vector $d\vec{A}$. If the angle between $\vec{J}$ and $d\vec{A}$ is bigger than 90°, then I_A is negative. IF $\vec{J}(\vec{r})$ is constant over some area A, then the magnitude of $\vec{J}$ is simple:

$$|\vec{J}| = \frac{|I|}{A}$$ in Units of A/m^2

RESISTANCE

For an object with a constant resistance, the definition is rather simple:

The RESISTANCE (R) between any two points on a conducting object is

$$R \equiv \frac{V}{I} \qquad \text{in Units of V/A} \equiv \text{Ohms } (\Omega)$$

where V is the absolute value of the applied potential difference (the voltage) between those two points, and

I is the absolute value of the resulting current.

However, if the resistance of the object depends on the amount of current (this is true for all objects to some degree), then the definition becomes:

$$R(I) \equiv \lim_{\Delta I \to 0} \frac{\Delta V}{\Delta I} = \frac{dV}{dI} \qquad \text{in Units of V/A} \equiv \text{Ohms } (\Omega)$$

where ΔV is the small change in voltage which produces the small change in current ΔI, when a current I is already passing between the two points on the object.

CONDUCTIVITY

The CONDUCTIVITY (σ) of a material is the ratio of the magnitude of the current density at some point in the material to the electric field strength that produces that current density.

In Symbols: $$\sigma \equiv \frac{J}{E} \qquad \text{in Units of } (\text{A/m}^2)/(\text{V/m}) = 1/\Omega\cdot\text{m}$$

or in vector form: $$\vec{J} = \sigma \vec{E} \qquad (1)$$

RESISTIVITY

The RESISTIVITY (ρ) of a material is the reciprocal of the conductivity.

In Symbols: $$\rho \equiv \frac{1}{\sigma} \qquad \text{in Units of } \Omega\cdot\text{m}$$

which allows us to write (1) as $\vec{E} = \rho \vec{J}$.