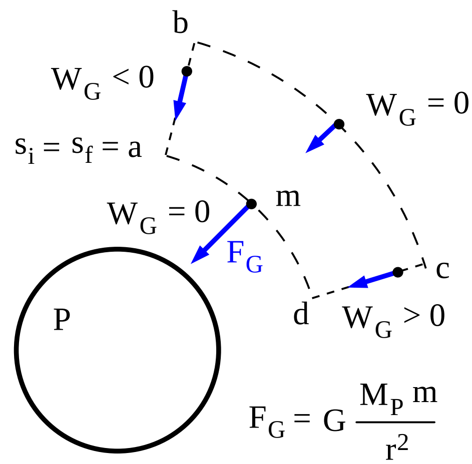
UNIVERSAL GRAVITY IS A CONSERVATIVE FORCE

Our reading on Conservative Forces from page 217, along with our Question 5 from Reading Quiz 11, might leave you with the impression that gravity is a conservative force only because, here near the Earth's surface, gravity is a constant force. This small addition to our reading will show that gravity is always a conservative force, under all conditions.

Newton's Universal Law of Gravity, which we will study in detail in Chapter 13, claims that every particle in the Universe attracts every other particle with a force proportional to the product of the particle masses and inversely proportional to the particle-particle separation distance squared. Consider a planet (P) of mass M_P and a moveable small object of mass m; let the object travel on a path which covers distances comparable in size to the planet radius. The path starts and ends at location a, and the movement of the particle takes it along a radius from a to b, then along an arc of constant radius to c, then inward along a radius to d, and finally along another arc of constant radius and back to a. Along the purely radial legs of the path (from a to b and from c to d) the gravity force on m by P is purely tangential, and changing in size, getting smaller as m moves away from planet P, and getting larger as m moves toward P. Along the purely angular leg of the path from b to c, the force of gravity is constant in size but always perpendicular, or transverse, to the path. The same is true for the angular leg from d back to a; the gravity force is again constant in size and always perpendicular to the path, only for this leg the gravity force is much larger than it was for the b-to-c leg.



The work done by the force of gravity on m by P is easily seen to be zero for the legs b to c and d to a, because the force is always purely transverse for those legs. For each of the two radial legs an integral is required (since the gravity force on those legs, though tangential, is changing in size); however, those two integrals must have an identical absolute value because the size of gravity only depends on the separation distance between m and P. So we will end up with two integrals of equal absolute value, with one negative (the integral from a to b, since the gravity force there is directed opposite to the movement along the path) and the other positive (the integral from c to d). Therefore, when we add up the results for all four legs of the trip along the path, we must find that the work done by gravity for the round trip will always be zero, making gravity a conservative force.

Since we can imagine any possible path being replaced by a set of infinitely small steps which are sometimes purely radial and sometimes purely angular, therefore we can always generalize this argument to demonstrate that the work done by gravity will always be zero for any round trip on any possible path.