CONSERVATION OF ENERGY

By considering a system of objects, with each object being treated as if it were a particle, we arrived at $W_{\text{NC}} = \Delta E_{\text{sys}}$, *i.e.* the work done by all nonconservative forces on all the objects in our system gives us the change in system mechanical energy.

Now suppose we have a universe of true particles (true particles have no possibility of a rotational energy or of a temperature) which interact only by gravity and electromagnetism. Let the electromagnetic interaction between our true particles be well-modeled by a purely spring-like interaction (*i.e.* Hooke's Law). We now wish to consider a system consisting of some subset of N of these true particles; the universe will then consist of our selected N-particle subset plus all the other particles in the universe (*i.e.* those which are not in our subset). What can we learn by applying $W_{\text{NC}} = \Delta E_{\text{sys}}$ to our N-particle system?

Since all interactions between the particles in our selected subset are conservative, what could possibly be the source of any nonconservative work? The answer is that the particles in our subset might still interact with particles outside of our subset. These "external" interactions could still only be gravity and our spring-like electromagnetic interaction; however, since these forces are now external we cannot define a potential energy for them. Recall that our philosophical definition of potential energy is "the energy stored in a system of particles because of the relative positions of the particles in the system"; therefore, only internal forces can possibly be conservative. So, for our system of N true particles, we can write $W_{\text{NC,ext}} = \Delta E_{\text{sys}}$. where the double subscript "NC,ext" is somewhat redundant, since, given our assumptions, only external forces could possibly have been NC forces. This result says, in symbols, that the particle-particle works which are between one particle within our subset and one particle external to our subset result in a change in the mechanical energy of our subset, unless of course these works happen to sum to zero; if these particle-particle works do sum to zero (or if they do not exist), then the mechanical energy of our subset is constant. This argument is as close as we can get to a "proof" of conservation of energy. Ultimately, Conservation of Energy must be taken as an unproveable law of nature which agrees with all repeatable experiments and observations. Let us accept our "proof" as being true.

We have now "proven" Conservation of Energy for a system of true particles; however, this idea is not very useful for us, because we cannot see true particles in everyday life, and even if we could see them, there are uncountable numbers of them. For practical purposes, we still need to consider interactions between objects, objects which are composed of particles. If we can treat those objects as if they were particles (no rotation and no temperature) we already know that $W_{\text{NC}} = \Delta E_{\text{sys}}$, but if energy truly is conserved (as we have "proved"), we should be able to say more about the energies within our system of objects. The interactions between our objects may be either conservative or nonconservative, so now it becomes useful to subdivide all of the nonconservative works into NC works due to internal and external contributions, *i.e.* $W_{\rm NC} = W_{\rm NC,ext} + W_{\rm NC,int}$, where $W_{\rm NC,int}$ refers to all the works done on all the objects in our system by all the NC forces which result from interactions between those objects which are within our system, and $W_{\rm NC,ext}$ refers to all the works done on all the particles in our system by all the forces which result from interactions between a particle in our system and an particle outside of our system. Also, remember that the mechanical energy of our system of objects includes only the kinetic energies of our rigid objects (no rotation and no temperature) and the potential energies which are due to the relative positions of the objects (not the relative positions of the particles within our objects). With these reminders, our result so far is

$$W_{\rm NC,ext} + W_{\rm NC,int} = \Delta E_{\rm sys}, \tag{1}$$

To make sense of this expression, we need to remember that our system of objects is also a system of true particles (*i.e.* all of the particles within all of our objects); and that we have already "proven" a Conservation-of-Energy principle for a system of true particles. Since $W_{\rm NC,ext} = \Delta E_{\rm sys\,of\,true\,particles}$, where we have added the explicit detail in the subscripting so as to keep the observable change in mechanical energy of our system of objects separate from the total change in mechanical energy of the system of true particles that make up all of our objects. By substituting into equation (1) and rearranging we therefore have

$$W_{\rm NC,ext} = \Delta E_{\rm sys\,of\,true\,particles} = \Delta E_{\rm sys,\,observable} - W_{\rm NC,int}. \tag{2}$$

Writing the relationship in this way makes it clear that the term $-W_{\rm NC,int}$ must represent all of the change in mechanical energy of the system of true particles that is not already included in the $\Delta E_{\rm sys,\,observable}$. In other words, $-W_{\rm NC,int}$ must represent the unobservable change in mechanical energy for our system, which we choose to write as $\Delta E_{\rm hidden}$ because this change in mechanical energy is hidden away inside of our objects; therefore

$$W_{\rm NC,ext} = \Delta E_{\rm sys\,of\,true\,particles} = \Delta E_{\rm sys,\,observable} + \Delta E_{\rm hidden}. \tag{3}$$

This "hidden" or invisible change in mechanical energy is usually called the change in the internal energy of the objects in our system, and denoted as $\Delta U_{\rm int}$. These internal energies are just untrackable mechanical energies — we cannot possibly keep track of the kinetic energies of every particle (*i.e.* atom), nor keep track of changes in the electric potential energy of a system of N particles when N is on the order of 10^{24}. However, we can argue that this change in an object's internal energy must necessarily be positive if the object's temperature rises, because a higher temperature must indicate that the object's true particles (atoms) are wiggling around with higher average kinetic energies (*i.e.* the thermal energy inside the object must be larger). So we have finally acknowledged that our objects are composed of particles (atoms) which are always in perpetual motion (changes in the kinetic and potential energies of these true particles are now in $\Delta E_{\rm hidden}$); however, we still have not accounted for the possibility that our objects might rotate, and therefore possess some rotational kinetic energy. We save that topic for our study of rotations.