CONSERVATION OF MECHANICAL ENERGY

Consider a collection of objects, each of which will be treated as if it were a particle. We would like to apply the WK Theorem to each of the objects (treated like a particle), and then try to find a useful principle concerning the energy of the entire system of objects. To make our analysis as simple as possible, consider only a system of two objects, labeled A and B. We begin by applying the WK Theorem to each object separately; :

$$\Sigma W_{\mathrm{on\,A}} \ = \ \Delta K_{\mathrm{of\,A}} \quad \text{and} \quad \Sigma \mathrm{W}_{\mathrm{on\,B}} \ = \ \Delta \mathrm{K}_{\mathrm{of\,B}}.$$

Now we simply add the two equations. The resulting right-hand side is simply the total change in kinetic energy for our system of objects (treating each like a particle). The resulting left-hand side however, is much more complicated. We have the work done on A by every force that is acting on A added to the work done on B by every force that is acting on B. We haven't actually stated with which other objects in the Universe that either A or B is interacting, so let's just number the forces acting on each object. Then we have:

$$W_{1\,\mathrm{on\,A}} \ + \ W_{2\,\mathrm{on\,A}} + \ldots \ + \ W_{1\,\mathrm{on\,B}} \ + \ W_{2\,\mathrm{on\,B}} + \ldots \ = \ \Delta K_{\mathrm{sys}} \tag{1}$$

where $W_{1\,\mathrm{on\,A}}$ indicates the amount of work done on A by the first force acting on A, *etc.* We don't actually know how many forces are acting either A or B, so we just use the three dots to indicate that there will be as many terms for A as there are forces acting on A, and similarly for B.

To make sense of this complicated left-hand side, we choose to classify all forces on any objects composed of particles into two types: conservative and nonconservative forces. (For the conservative forces, we will be able to define a system potential energy, which will indicate the energy stored in our system of objects because of the separation distance between those objects.) Having made this choice, then the left-hand side can be written simply:

$$W_{\mathrm{NC}} \ + \ W_{\mathrm{C}} \ = \ \Delta K_{\mathrm{sys}} \tag{2}$$

where W_{C} now stands for the work done by **all** conservative forces on **all** objects in our system (treating each object as a particle). W_{NC} similarly stands for the work done by **all** nonconservative forces on **all** objects in our system.

TEST YOUR UNDERSTANDING: What is the meaning of the symbol W_{C}? (i) The work done on object A by all conservative forces acting on object A; *i.e.* find the work done on object A (treating A like a particle) by each of those conservative forces and then add all those works together. (ii) The work done on object B by all conservative forces acting on object B; *i.e.* find the work done on object B (treating B like a particle) by each of those conservative forces and then add all those works together. (iii) The combination of items (i) and (ii).

Now we make a scientific definition of the **change** in potential energy of our system, one that will tells us exactly how to calculate the amount of change in system potential energy. Note that only the change in system potential energy is defined. Here's that definition:

$$\Delta U_{\text{sys}} \equiv -W_{\text{C}}$$

Next we subsitute this definition into equation (2) and move the resulting potential energy term over to the right-hand side (which we can now call the energy side of our relation):

$$W_{\text{NC}} = \Delta K_{\text{sys}} + \Delta U_{\text{sys}} \tag{3}$$

Finally, we define the system mechanical energy E by

$$E_{\text{sys}} \equiv K_{\text{sys}} + U_{\text{sys}},$$

and so our final relationship becomes, very simply:

$$W_{\text{NC}} = \Delta E_{\text{sys}} \tag{4}$$

In other words, any changes in the mechanical energy of our system of objects (being treated as if they were particles) can always be attributed to some work that has been done by nonconservative forces on at least some of the objects in our system. Remember that "treated as if they were particles" means that we have not yet considered that our objects might rotate, or that they might have a temperature; so far, our objects can only translate, and so the kinetic energy of each object is simply half the object's mass times the object's speed squared.

If, for any reason, the work done by **all** nonconservative forces on **all** objects in our system (each object being treated as a particle) turns out to total to zero, then the mechanical energy of our system cannot change, and therefore must be constant, or "conserved". Therefore, equation (4), with $W_{\text{NC}} = 0$, expresses the Conservation of Mechanical Energy.

ANSWERS TO TEST YOUR UNDERSTANDING QUESTIONS: **(iii)** The symbol W_{C} was explicitly defined as the work done by **all** conservative forces on **all** objects in the system.