Definitions for Current and Resistance

AVERAGE CURRENT

The AVERAGE CURRENT \((I_{av})\) through any area \(A\) is the total charge \(\Delta Q\) that passes through that area in time \(\Delta t\), divided by \(\Delta t\).

In Symbols: \[ I_{av} = \frac{\Delta Q}{\Delta t} \] in Units of C/s = Amperes (A)

INSTANTANEOUS CURRENT

The INSTANTANEOUS CURRENT \((I(t))\) is the limit as \(\Delta t\) goes to zero of the average current.

In Symbols: \[ I(t) = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} \] in Units of C/s = Amperes (A)

CURRENT DENSITY

The CURRENT DENSITY \((\vec{J}(\vec{r}))\) at location \(\vec{r}\) is the time rate of charge flow per unit area through an infinitesimal area located at \(\vec{r}\); the direction of current density is the direction of positive charge flow.

In Symbols: \[ \text{IF } \vec{J}(\vec{r}) \text{ is constant over some area } A \] \[ |\vec{J}| = \frac{|I|}{A} \] in Units of A/m²

in general for area \(A\) \[ I_A \equiv \int_A \vec{J}(\vec{r}) \cdot d\vec{A}. \]

This last equation is often used as the definition of current. Note that current density is a VECTOR while current is a signed scalar.
RESISTANCE
The RESISTANCE \( R \) between any two points on a conducting object is

\[
R \equiv \frac{V}{I} \quad \text{in Units of } \text{V/A} = \text{Ohms (}\Omega\text{)}
\]

where \( V \) is the absolute value of the applied potential difference between those two points, and

\( I \) is the absolute value of the resulting current.

CONDUCTIVITY
The CONDUCTIVITY \( \sigma \) of a material is the ratio of the magnitude of the current density at some point in the material to the electric field strength that produces that current density.

In Symbols: \( \sigma \equiv \frac{J}{E} \quad \text{in Units of } (\text{A/m}^2)/(\text{V/m}) = 1/\Omega \cdot \text{m} \)

or in vector form: \( \vec{J} = \sigma \vec{E} \) \hspace{1cm} (1)

RESISTIVITY
The RESISTIVITY \( \rho \) of a material is the reciprocal of the conductivity.

In Symbols: \( \rho \equiv \frac{1}{\sigma} \quad \text{in Units of } \Omega \cdot \text{m} \)

which allows us to write (1) as \( \vec{E} = \rho \vec{J} \).