PHY 132 – Summer 2000
LAB 3: Electric Potential and Electric Fields[1]

Introduction

A vector field is a map of vectors at every point in space. This is one of the more difficult concepts in introductory physics courses. In this lab we will visualize and measure a 2-dimensional electric field for conductors immersed in water. We'll see that field lines are:

1. Perpendicular to a conductor
2. Zero inside a closed conductor
3. Exhibit a symmetry that matches the shape of the conductors.

We will also measure equipotential lines for the same field, and combine the concepts of field and potential maps. In this lab, we will make a map of the electric potential surrounding electrodes in a water tank. From these measurements you can determine the associated electric field. You will also confirm that the potential inside a coaxial capacitor varies as ln(r). You should be familiar already with the conceptual background for these experiments from the PHY131 course.

1. Electric Fields

Equipment:

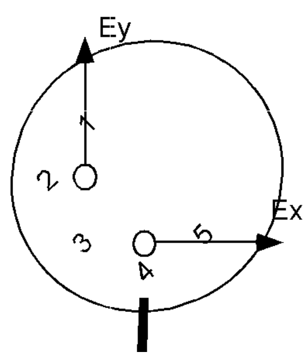

Fig. 1: 3-point field probe.

We use a 3-point probe to measure two orthogonal components of electric field (E_x and E_y) at the same time. This probe is shown in Fig. 1. We use pins 1,3,5, which form an orthogonal pattern. The voltage $V_{3,5}$ is proportional to E_x and $V_{3,1}$ is proportional to E_y. These voltage readings correspond physically to the energy lost by charges in moving

---

[1] Adapted by R. J. Jacob from P. Bennett, PHY-132 Lab Manual© (ASU)

between the two pins, which is proportional to the average electric field between the pins. This is closely equal to its value at the midpoint between pins. Note that E_x and E_y are at slightly different locations, and both are offset slightly from the center of the plug. Data are taken by *translating* the probe around the tank, taking care to maintain its angular orientation. It is important that the probe reaches to bottom of the tank and not be tilted.

The entire apparatus is shown in Fig. 2. We use an AC source to generate the field in the water tank. This allows use of a scope to continuously monitor the probe output and provide a graphic representation of the E_{xy} vector. The circuit includes an isolation transformer, which is necessary because the scope inputs are grounded, but the conductors in the water tank must be "floating".

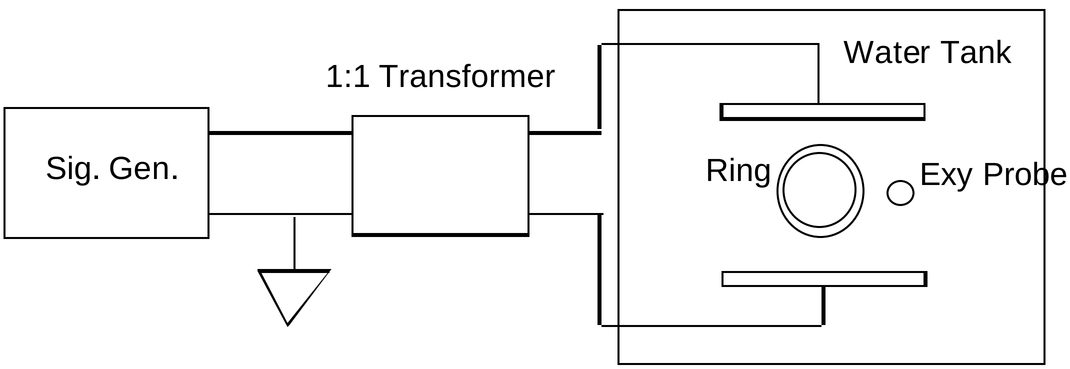

Fig. 2. Apparatus for measuring electric field maps.

Procedure

1. Setup:

Arrange the conductors as shown in Fig. 2, but without the center ring for now. Put the probe in the tank near the center. Set the signal generator at 200 Hz and maximum amplitude. Set the scope inputs on XY (NOT timebase), and matching sensitivities (about 50mV). The trace should fill the screen. Be sure both channels match, so the display scale is square, not rectangular. You should see a tilted line or ellipse. The peak-peak readings of the extremities of the ellipse correspond to E_x and E_y. Because we use AC, there is an ambiguity of ±180 degrees in the field direction. Lets use E_x positive by convention.

2. Calibrate the probe:

Put the probe near the center of the tank with no ring. Take readings of E_x and E_y for about 10 angular orientations from 0 to 180 degrees, without shifting the location. Don't obsess about the angle, you will derive it from the E_{xy} data ($\tan\theta = E_y/E_x$). Hopefully you see a vector of nearly constant magnitude vs angle (see analysis later).

3. Field Map:

Place the ring in the tank, carefully centered between the bars. Take measurements of E_x, E_y at many locations in the tank, sufficient to draw a reasonable map of the field everywhere between the bars. Be careful to hold the probe orientation fixed at zero degrees. Data should be tabulated and simultaneously marked with vectors on the matching paper grid. It is useful to choose symmetric locations (top/bottom/left/right) within the conductor array. Be sure to include a few points as close as possible to the ring, both inside and outside.

Analysis

1. Probe calibration: Plot the *magnitude* of electric field vs. angle. What is the random noise in this measurement? Is there any systematic deviation in the data?
2. Field Map: Based on your vector data, draw a set of field lines that reasonably indicates the field everywhere between the bars. You will be graded on the quality of this field map. It must be derived from your actual data, and it should display correct physical behavior. Is there any pronounced asymmetry in the field?
3.

4. Mapping Equipotentials

Theory

Electric potential V(r) is an example of a scalar field. It takes a scalar value at all points in space. This can be charted as a family of equipotential lines. Since this is a conservative field, there is an associated vector field, E(r) which we studied last time. The E-field takes a vector value at all points in space. The two are related by

$$\underline{E}(\underline{r}) = -\text{grad}[V(\underline{r})] \qquad \text{Eq. 1}$$

or, in X-Y components

$$E_x = -[dV/dx]\ \underline{i}$$
$$E_y = -[dV/dy]\ j \qquad \text{Eq. 2ab}$$

where these are partial derivatives. The gravitational field provides a useful mechanical analog. Here, $V_g(r)$ is work done against gravity, and can be charted as a contour map of elevation. Then –grad[V] corresponds to the local slope (downhill), which has both magnitude and direction. Indeed, this represents the force per unit mass pulling downhill, as any snow-boarder could tell you!

For any given (conservative) scalar field the associated vector field can be derived from Eq. 1. It may be useful to construct a map of field lines derived from the potential. This is done using the principle that field lines are everywhere perpendicular to the equipotential lines. They start and end on +/- charges, respectively, and do not cross. Their density (number per unit area) is proportional to field strength, hence inverse to the spacing between equipotential lines. In practice, one can evaluate E at any location on a potential map, working in polar coordinates. That is, the direction of E is normal to the EP lines, and the magnitude of E is given by $\Delta V/\Delta S$. Here we use finite difference to evaluate the derivative of Eq. 1. Thus, $\Delta V = V_2 - V_1$ is the difference in potential for a closely spaced pair of lines, and ΔS is the spacing between the lines. The units of E, follow the unit of V and S, of course.

Coaxial cylinders

The field between coaxial cylinders is radial and given as

$$E(r) \sim 1/r. \qquad \text{Eq. 3}$$

Hence the potential is given as

$$V(r) \sim \ln(r) \qquad \text{Eq. 4}$$

Procedure

1. Set up the dipole conductors as used in the previous lab. Measure the potential at many points in space, sufficient to make a decent map. This is best done by following a given potential line. That is, chose a potential, say 3.0 V then map the line of all points with this potential. Repeat for another potential.
2. Choose a location near the center of the tank, then measure the potential with and without the copper ring surrounding the probe. Explain.
3. Place the graphite bar at the center of the tank, and measure the potential next to it with the bar oriented parallel vs. perpendicular to the dipole axis. Explain.
4. Select a location between two nice contour lines, somewhat off the tank center (see class). Determine the field here, based on your data for potential lines. Then measure the field at the same location, using the field probe from last week. Compare the two.
5. (Optional) Set up coaxial cylinders (see class). Using the field probe, qualitatively explore the field between the cylinders, and describe this in your report.
6. Measure V(r) for the coaxial cylinders. This field has radial symmetry: points at a given radius should match V. Take 4 readings (spaced 90 degrees around the circle) at each radius. This serves to establish errors.

Analysis

1. Dipole: How will you characterize errors for the dipole field? We don't really have a model function for this particular field, but we can use the powerful concept of symmetry. We know that the field will have the same symmetry as the conductor shapes. In this case, there are two mirror planes. That is, the potential map should match left-right and top-bottom. You can flip your data over and trace it (inverted) onto itself. Do this for both mirror planes, and turn in this symmetrized data for your report. Based on this plot, estimate the errors (in volts) in your potential map.

2. Coaxial cylinders: Is the model function in Eq. 4 a good fit to the data? Hint: Make a linearized plot. You don't need to set error bars in GA, just plot all the data. That is, errors are apparent from the spread in V values at a given "r".